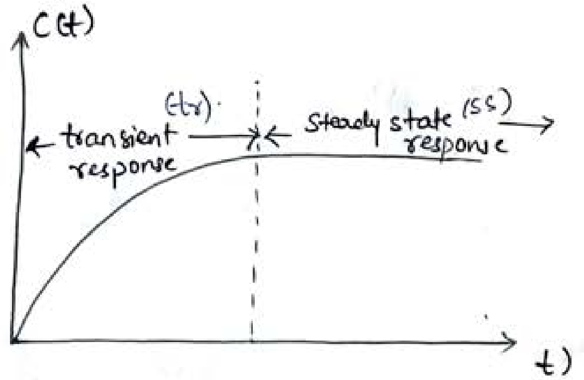
Time Domain Analysis :

Purpose :- To evaluate the performance of the system w.r.t time

Time Response :- If the response of the system varies w.r.t time then it is called time response.

- The time response is nothing but sum of transient response & steady state response.



$$\boxed{\therefore \text{ time response } = c(t) = c_{tr}(t) + c_{ss}(t)}$$

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1. Identify the transient and steady state terms in the given time response. $c(t) = \underbrace{10 + 2\sin 2t + 3\cos 3t}_{\text{Steady state (s.s)}} + \underbrace{4te^{-4t} + 5te^{-5t}\sin 5t + 6e^{-6t}\cos 6t}_{\text{transient state (tr)}}$?

Sol

Condition of transient state is $\boxed{\lim_{t\to\infty} c_{tr}(t) = 0}$.

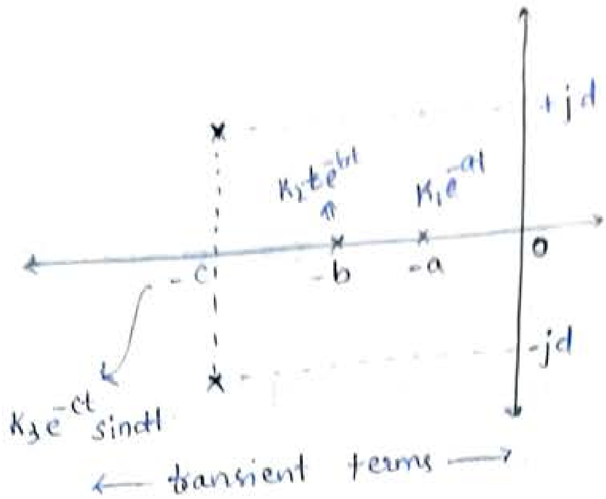
The term which consists a exponential decay is called transients.

The poles which lies in the left will always give transients

Transient Term :

It is a part of time response that becomes the zero as time becomes very large that means $\lim_{t\to\infty} c_{tr}(t) = 0$.

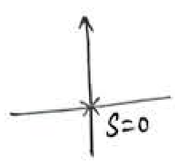
- The transient term consists the exponential decay term.
- The poles which lies in the left hand side gives the transient term.



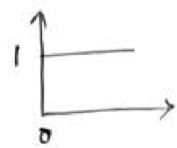
Steady State Term:

It is a part of the time response that remains after the transients become the zero.

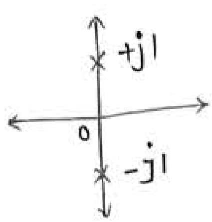
· The poles which lies on the imaginary axes gives the steady state term.

→ 

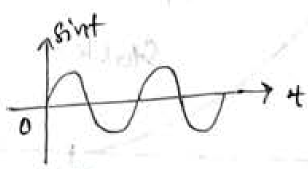
$\frac{1}{s} \Rightarrow 1.$



(one pole at origin)

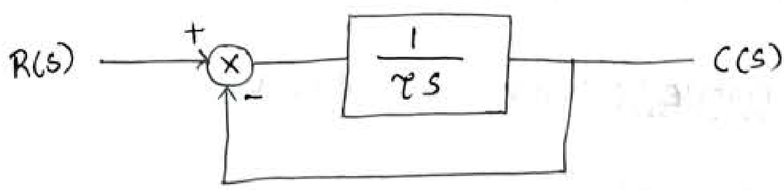
→ 

$\frac{1}{s^2+1} \Rightarrow \sin t$



(two poles are imaginary).

Time Response to the 1st order systems:



1st order system is type-1 & order-1.

$$\frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}$$

Impulse response:

$r(t) = \delta(t)$.

$R(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{1}{(s\tau+1)}$$

$$C(s) = \frac{1}{(s\tau+1)}$$

$$C(s) = \frac{1}{\tau(s+\frac{1}{\tau})}$$

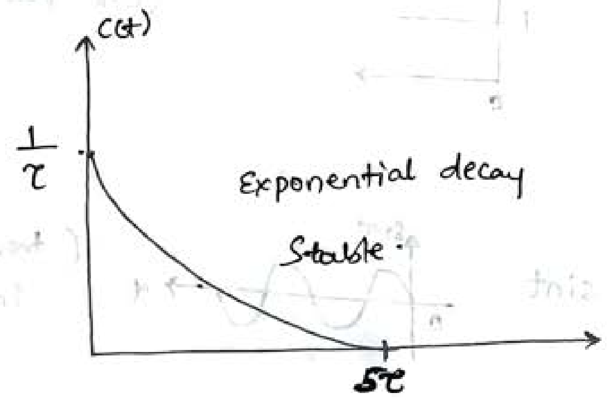
→ The impulse response consists only transient team

→ The transient team consists the system parameters.

→ Hence the impulse response is called system response, natural response or free forced response.

Apply inverse laplace on both sides.

$$\boxed{c(t) = \frac{1}{\tau} e^{-t/\tau} = \text{transient team.}}$$ → Impulse response.



for 1st order system.

Error: Error is nothing but deviation of the o/p from the i/p. i.e, $e(t) = r(t) - c(t)$.

Steady state error (e_{ss}): Error at $t \to \infty$.

$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$= \lim_{t\to\infty} [r(t) - c(t)]$$

- There is no steady state in the imp. response. So we cannot perform the steady state analysis

· For impulse input the steady state errors are not defined because there is no input exist at $t \to \infty$, hence we cannot compare output with input at $t \to \infty$ (or)

· As the impulse response not consists the steady state term hence we cannot perform steady analysis.

Unit step Response :-

$r(t) = 1 \cdot u(t)$.

$R(s) = \frac{1}{s}$

$$\frac{C(s)}{R(s)} = \frac{1}{(s\tau+1)}$$

$$C(s) = \frac{1}{s(s\tau+1)}$$

$$C(s) = \frac{1}{s} - \frac{\tau}{s\tau+1}$$

Apply inverse laplace.

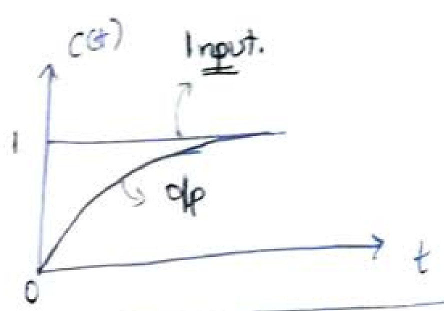
$$\boxed{c(t) = (\underbrace{1}_{\text{steady state term}} - \underbrace{e^{-t/\tau}}_{\text{transient term}})}$$

· In the response the steady state term is because of the i/p where as the transient term is because of the system.

$\rightarrow$ e_{ss} = Steady state error $= \lim_{t\to\infty} [1 - (1 - e^{-t/\tau})] = \lim_{t\to\infty} e^{-t/\tau}$

$\left[\because e_{ss} = \lim_{t\to\infty} (r(t) - \underbrace{c(t)}_{\text{output}})\right]$.

$\therefore e_{ss} = 0$.



Unit Ramp Response:

$$r(t) = t\,u(t)$$

$$R(s) = \frac{1}{s^2}$$

$$\frac{C(s)}{R(s)} = \frac{1}{(\tau s + 1)}$$

$$C(s) = \frac{1}{s^2(\tau s+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s\tau + 1}$$

$$C(s) = \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau^2}{s\tau + 1}$$

$$C(s) = \frac{1}{s^2} - \tau \cdot \frac{1}{s} + \frac{\tau}{\left(s + \frac{1}{\tau}\right)}$$

Apply inverse laplace transform.

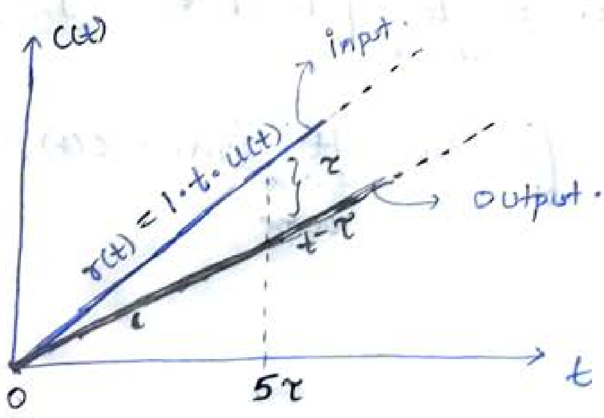
$$c(t) = t - \tau + \tau e^{-t/\tau}$$

$$\boxed{c(t) = \Big(\underbrace{t - \tau}_{\text{Steady State term}} + \underbrace{\tau e^{-t/\tau}}_{\text{transient state term}} \Big)}$$

Steady state error (e_{ss}) $= \lim\limits_{t \to \infty} \left(t - (t - \tau + \tau e^{-t/\tau}) \right)$

$$= \lim_{t \to \infty} \left(\tau - \tau e^{-t/\tau} \right) = \tau$$

$\therefore\ e_{ss} = \tau$



Unit Parabolic Response :

$$r(t) = 1 \cdot \frac{t^2}{2} \cdot u(t)$$

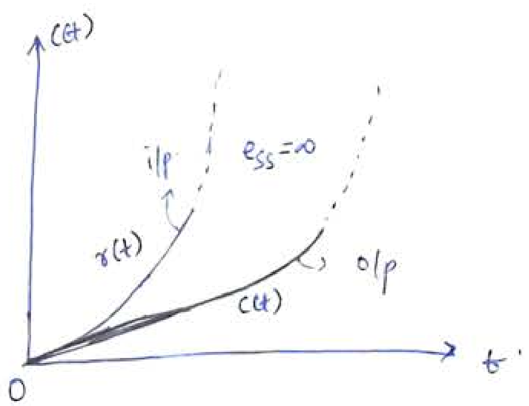
$$R(s) = \frac{1}{s^3}.$$

$$\frac{C(s)}{R(s)} = \frac{1}{\tau s+1}$$

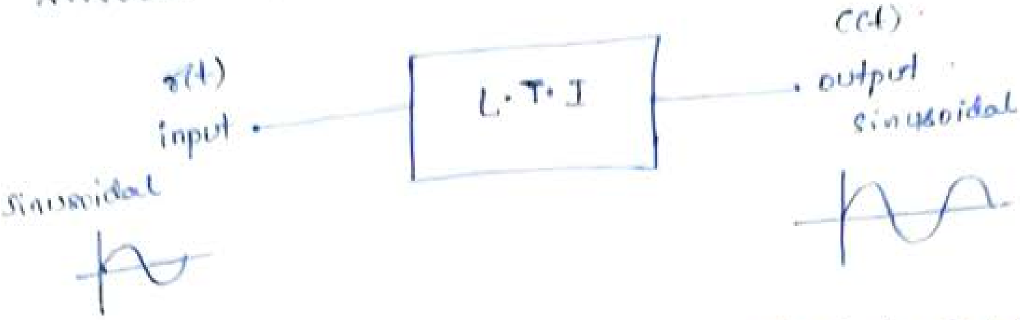
$$C(s) = \frac{1}{s^3(\tau s+1)}$$

$$C(s) = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{D}{s\tau+1}$$

Steady state error $e_{ss} = \infty$.



Sinusoidal Response :-



$r(t) = A\sin(\omega t \pm \theta) \Rightarrow c(t) = A \times M \sin(\omega t \pm \theta \pm \phi)$ where

M = magnitude

$r(t) = A\cos(\omega t \pm \theta) \Rightarrow c(t) = A \times M \cos(\omega t \pm \theta \pm \phi)$. ϕ = phase.

→ For any L.T.I system if input is sinusoidal the o/p output is also sinusoidal, but difference in magnitude & phase.

→ As standard form of input & output are as follows described in the above figure.

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1. The closed loop transfer function of a LTI system

$\frac{C(S)}{R(S)} = \frac{1}{S+1}$ for the input $r(t) = \sin t$ find the steady state o/p ?

Sol.
$$r(t) = \sin t$$
$$R(S) = \frac{1}{S^2+1}$$
$$\frac{C(S)}{R(S)} = \frac{1}{S+1}.$$
$$C(S) = \frac{1}{(S^2+1)(S+1)}.$$

Now.
$$\frac{C(S)}{R(S)} = \frac{1}{S+1}.$$
$$r(t) = \sin t$$
$$\omega = 1$$
$$s = j\omega = j\cdot 1.$$

$$\frac{C(s)}{R(s)} = \frac{1}{j1+1}$$

$$M = \frac{1}{\sqrt{2}} \;;\; \phi = \frac{\angle 1+j0}{\angle 1+j1} = \frac{0^\circ}{45^\circ} = -\pi/4 .$$

$$\boxed{c(t) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \sin\left(t - \frac{\pi}{4}\right).}$$

2. Repeat the above problem $\frac{C(s)}{R(s)} = \left(\frac{s+1}{s+2}\right)$; $r(t) = 10\cos(2t+45^\circ)$

Sol. $\omega = 2$.

$$s = j\omega = 2j.$$

$$\frac{C(s)}{R(s)} = \left(\frac{1+j2}{2+j2}\right) = \frac{(1+j2)(2-j)}{(4+1)}$$

$$M = \sqrt{\frac{5}{8}} \;;\; \phi = \tan^{-1}2 - \tan^{-1}1 = 63.43^\circ - 45^\circ = 18.43^\circ .$$

$$c(t) = 10 \times \sqrt{\frac{5}{8}} \cos(2t + 45^\circ + 18.43^\circ).$$

$$\boxed{c(t) = 8 \cos(2t + 63.43^\circ).}$$

3. A system $\frac{Y(s)}{X(s)} = \frac{s}{s+P}$. As an output $y(t) = 1\cdot\cos\left(2t - \frac{\pi}{3}\right)$. to an input $x(t) = p\cos\left(2t - \frac{\pi}{2}\right)$. Then the system parameter p is ?

Sol $\omega = 2$

$$s = j\omega = j2$$

$$\frac{C(s)}{R(s)} = \frac{j2}{j2+P} = \frac{Y(s)}{X(s)}$$

$$M = \frac{2}{\sqrt{2^2+p^2}} \;;\; \phi = 90^\circ - \tan^{-1}\left(\frac{2}{p}\right).$$

$$y(t) = P \cdot \frac{2}{\sqrt{4+p^2}} \cdot \cos\left(2t + \frac{\pi}{2} + \frac{\pi}{2} - \tan^{-1}(2/p)\right).$$

By comparing $y(t) = 1\cdot\cos\left(2t - \frac{\pi}{3}\right)$.

Magnitude

$$\frac{2p}{\sqrt{4+p^2}} = 1$$

$$2p = \sqrt{4+p^2}$$

$$\boxed{p = \frac{2}{\sqrt{3}}}$$

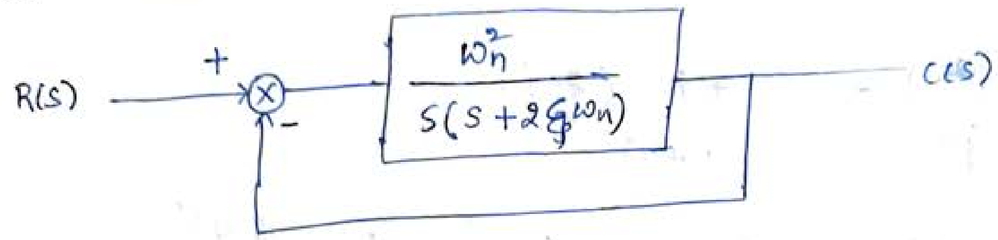
Phase

$$2t - \frac{\pi}{3} = 2t + \tan^{-1}(2/p).$$

$$\frac{2}{p} = \tan 60^\circ = \sqrt{3}$$

$$\boxed{p = 2/\sqrt{3}}$$

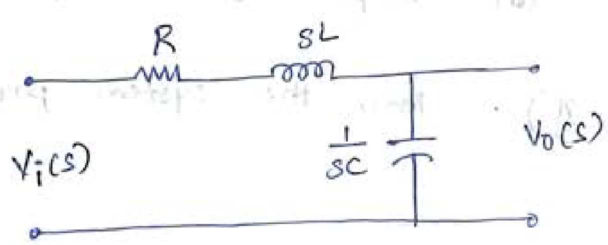
Time Response to the second order systems :-



→ It is type 1, order − 2 system.

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}} \quad \text{— ①}$$

→ The practical circuit to the 2nd order system is RLC circuit.



$$\frac{V_0(s)}{V_i(s)} = \frac{1/sC}{R + sL + 1/sC}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{s^2LC + sCR + 1}.$$

$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{1/LC}{s^2 + s\,R/L + \frac{1}{LC}}} \quad \text{— ②}$$

By comparing ① & ② we get

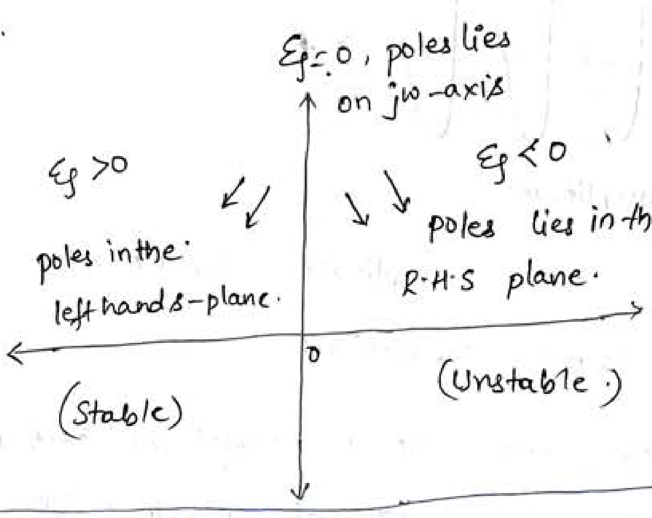
$$\boxed{\omega_n = \frac{1}{\sqrt{LC}} \text{ rad/sec}} = \text{Natural freq of oscillation (or) undamped oscillations (or) sustained oscillations.}$$

$$\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}} = \text{damping ratio}}$$

→ damping ratio gives the ratio of energy lost to energy stored.

$\xi\omega_n$ is called as damping factor (or) actual damping.

→ The 2nd order system nature completely depends on ξ. The 2nd order system is stable for all the values of ξ $(0 < \xi < \infty)$.



Impulse response to 2nd order system :-

$r(t) = \delta(t)$.

$R(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

Case :- 1 :- $\xi = 0$ (Undamped system) :-

$$C(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}$$