STATE SPACE ANALYSIS : (2M to 4M) 100%

State : The state gives the future behaviour of the system based on present input and past history of the system.

→ The past history of the system (initial condition) described by state variables.

→ The resistive circuit is not having any past history because the o/p depends on the i/p but not the past history of the components. (NO past history ⇒ No energy stored ⇒ No state variable).

→ The resistive network is called memoryless system.

→ **Number of State Variables :**

→ If the RLC n/w is given the number # of state variables equal to sum of inductors and capacitors.

→ If the differential equation is given the number of state variables equal to the order of the differential equation.

Standard form of state model :

$\dot{X} = AX + BU$ is called state equation or dynamic equation.

where X = state vector

U = input vector.

$y = CX + DU$ is called output equation.

where A = State matrix

B = Input matrix

C = Output matrix

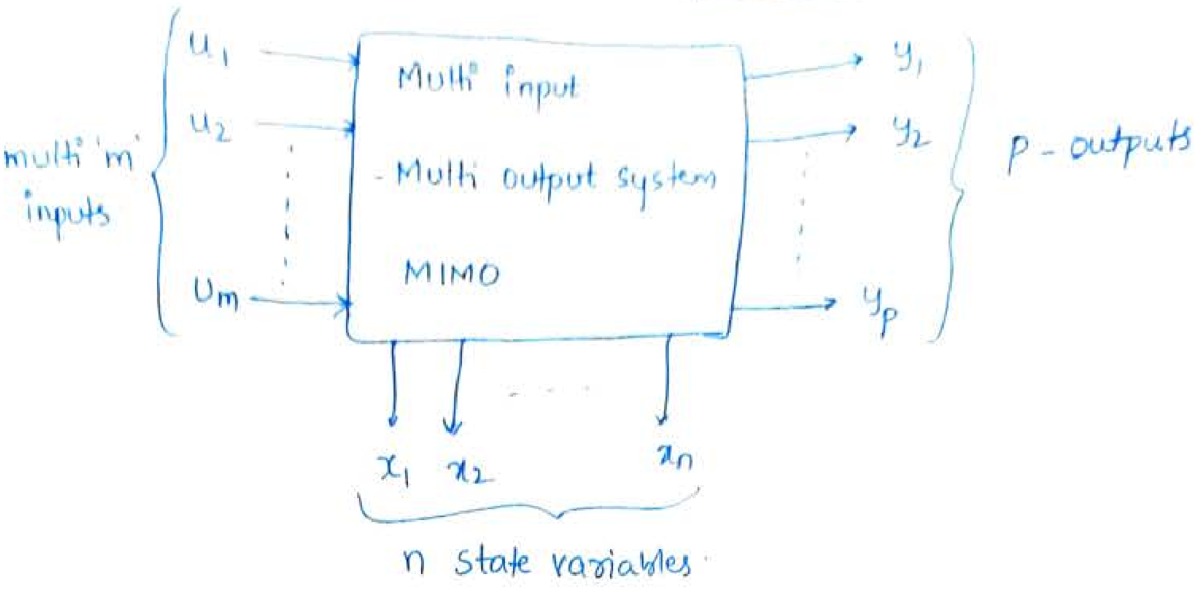
D = Transmission matrix.

y = Output vector

$\dot{x}$ = Differential state vector.

Order of the matrix:

Consider the multi i/p multi o/p system as shown in figure



Input vector $U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}_{m\times 1}$; Output vector $= \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}_{p\times 1}$.

State variable $= X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$; $\underbrace{\dot{X}}_{n\times 1} = \overset{n\times n}{A}\underbrace{X}_{n\times 1} + \underbrace{B}_{n\times m}\underbrace{U}_{m\times 1}$

Note: The order of the differential statevariable must be equal to order of the ~~stable~~ state variable.

Eg: $\underbrace{y}_{p\times 1} = \underbrace{C}_{p\times n}\underbrace{X}_{n\times 1} + \underbrace{D}_{p\times m}\underbrace{U}_{m\times 1}$

(*) Write State model to the differential equations:

(*) Write the state model to the given differential equation

$$\dddot{y} + 3\ddot{y} + 5\dot{y} + 7y = 10U$$

with opp sign of coeff

Sol: No. of state variable 'n' = order of given DE.

i.e, $n = 3$; Let $y = x_1 \longrightarrow (1)$

$\dot{x}_1 = \dot{y} = x_2 \longrightarrow (2)$

$\dot{x}_2 = \ddot{y} = x_3 \longrightarrow (3)$

$\dot{x}_3 = \dddot{y} \longrightarrow (4)$

Sub all the above eq-n in the given system to get $\dot{x}_3$ in terms of state variables

i.e, $\dot{x}_3 + 3x_3 + 5x_2 + 7x_1 = 10U$

$$\boxed{\dot{x}_3 = 10U - 7x_1 - 5x_2 - 3x_3} \rightarrow (5)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} [U] \rightarrow \text{i/p co-eff.}$$

$$(y) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Controllable canonical form (CCF)

State model is not unique, there are 4 forms of state models

1. Controllable canonical form (CCF)
2. Observable canonical form (OCF)
3. Diagonalization (or) Normal form.
4. Jordan Canonical form

To convert CCF into OCF :

$$A_{OCF} = (A_{CCF})^T = \begin{bmatrix} 0 & 0 & -7 \\ 1 & 0 & -5 \\ 0 & 1 & -3 \end{bmatrix}$$

$$B_{CCF} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \begin{matrix} \text{End} \\ \uparrow \\ \text{Start} \end{matrix} \; ; \; B_{OCF} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix} \begin{matrix} \text{Start} \\ \downarrow \\ \text{End} \end{matrix} \; ; \; B_{OCF} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{CCF} = \underset{\text{End} \leftarrow \text{Start}}{\begin{bmatrix} c_0 & c_1 & c_2 & c_3 \end{bmatrix}} \; ; \; C_{OCF} = \underset{\text{Start} \rightarrow \text{End}}{\begin{bmatrix} c_3 & c_2 & c_1 & c_0 \end{bmatrix}} \; ; \; C_{OCF} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

(*) Write the state model to the given

$$\ddddot{y} + 2\dddot{y} + 4\ddot{y} + 6\dot{y} + 8y = 5U$$

Sol: $n = 4$

$\xrightarrow{CCF}$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -8 & -6 & -4 & -2 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} ;$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

State model to the Transfer function (VIMP)

(*) Write the state model to the following system

$$\frac{y(s)}{U(s)} = \frac{2s+3}{s^2+5s+6}.$$

Sol

$$\frac{y(s)}{U(s)} = \frac{2s + 3}{s^2 + 5s' + 6} \longrightarrow x_1$$

$\dot{x}_1 = x_2$

x_1

$\dot{x}_2$

$\dot{x}_1 = x_2 \longrightarrow (1)$

$U = \dot{x}_2 + 5x_2 + 6x_1$

$\dot{x}_2 = U - 6x_1 - 5x_2$ — (2)

$y = 2x_2 + 3x_1. \rightarrow (3)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [U]$$

$$[y] = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Short cut:

with same sign of coeff

C matrix.

$$\frac{y(s)}{K\,U(s)} = \frac{K(2s' + 3)}{s^2 + 5s + 6}$$

A matrix

with opposite sign of coefficient.