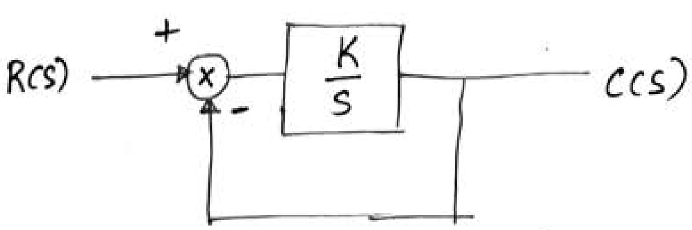
ROOT LOCUS:

Purpose:

1. To find the closed loop system stability.
2. To find the range of 'K' value for system stability.
3. To find the 'K' value to become the system as marginal-lly stable.
4. To find the frequency of oscillations when system is marginal stable.
5. To find the 'K' of a system which is undamped, underdamped, critical damped and overdamped system.
6. To find the relative stability.
7. If the root locus branches moving towards the left then the system is more relatively stable.
8. If the root locus branches moving towards the right then the system is less relatively stable.
9. The word 'roots' means roots of characteristic equation which are called closed loop poles ~~as~~ and the word 'locus' means path.
10. 'Root locus' means closed loop poles path by varying K from 0 to ∞

Problem

(*) Draw the root locus to the following system?



Drawing the root locus means identification of closed loop poles path which is given by characteristic equation.

$$C.E \rightarrow \quad 1 + G(s) = 0$$

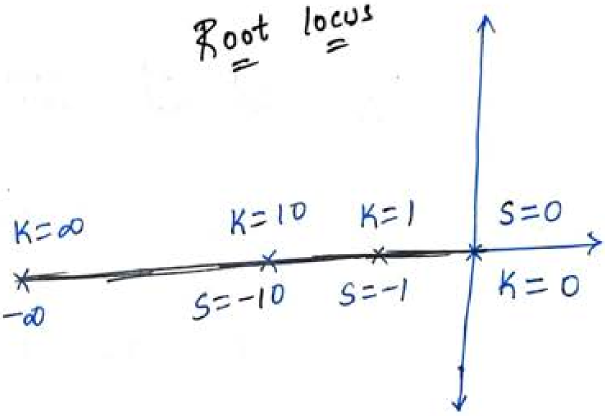
$$1 + \frac{K}{s} = 0$$

$$C.E \rightarrow \quad s + K = 0$$

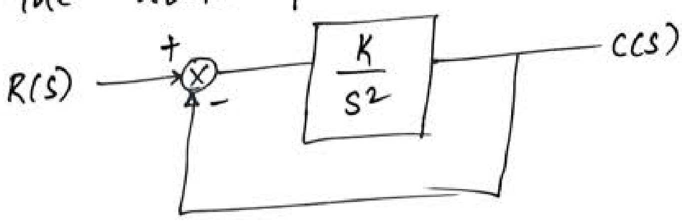
$$s = -K$$

K-value	Pole location ($s = -K$)
0	$\rightarrow$ 0
1	$\rightarrow$ -1
2	$\rightarrow$ -2
5	$\rightarrow$ -5
10	$\rightarrow$ -10
∞	$\rightarrow$ $-\infty$

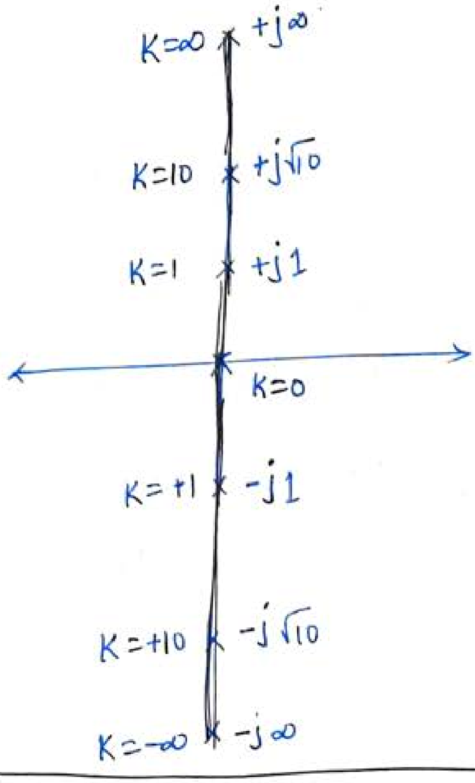
Root locus



Repeat the above problem for



Root locus



Sol

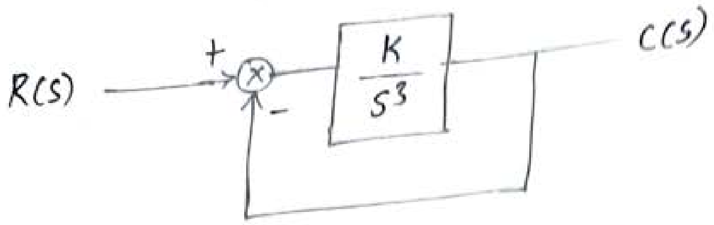
$$C.E \rightarrow \quad 1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s^2} = 0$$

$$C.E \rightarrow \quad s^2 + K = 0$$

$$s = +j\sqrt{K}$$

(*) Repeat the above procedure for



Sol. As order increases finding the roots for CE is very difficult. Hence we cannot draw a root locus diagram by using C.E. To draw a root locus diagram we use open loop transfer function of a unity or non unity feedback system, but for the analysis for of closed loop system.

Reln. b/w open loop and closed loop tranfer function for poles & zeros :-

Open loop transfer function (OLTF) $\Rightarrow G(s)H(s) = K\frac{N(s)}{D(s)}$.

Open loop poles $\Rightarrow D(s) = 0 \rightarrow$ OLP

Open loop zeros $\Rightarrow N(s) = 0 \rightarrow$ OLZ.

Closed loop poles are given by characteristic eq'n (C.E).

i.e, $1 + G(s)H(s) = 0$

$$1 + K\frac{N(s)}{D(s)} = 0$$

CE $\rightarrow$ $\boxed{\underbrace{D(s)}_{OLP} + \underbrace{K}_{0 \text{ to } \infty}\underbrace{N(s)}_{OLZ} = 0}$

The closed loop poles are nothing but sum of open loop poles, open loop zeros for the function of system gain K.

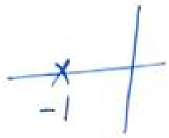
Case-(i): $K = 0$:

$$K = 0 \Rightarrow K = \left|-\frac{D(s)}{N(s)}\right|$$

$D(s) = 0$ if $K = 0$ i.e, closed loop poles.

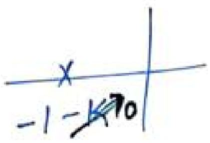
$$\boxed{\text{when } K = 0 \text{ ; closed loop poles} = \text{open loop poles}}$$

Eg: $G(s) = \dfrac{K}{s+1}$, $H(s) = 1$



open loop pole $= -1$.

Closed loop T/F $\longrightarrow$ $= \cancel{\dfrac{K}{s}} \rightarrow \dfrac{K}{s+1+K}$.



closed loop pole $= -1$.

Case-(ii): $K = \infty$.

Closed loop pole $\rightarrow$ $N(s) = 0$ if $K = \pm\infty$. i.e, $K = \left(- \dfrac{D(s)}{N(s)} \right)$

$$\boxed{\text{when } K = +\infty \text{ ; closed loop pole} = \text{open loop zero}}$$

$$\boxed{\text{when } K = -\infty \text{ ; closed loop pole} = \text{open loop zero}}$$

From above we can conclude that if 'K' is increased from 0 to ∞, the direction of root locus branch is from open loop pole to open loop zeros.

→ If 'K' is increased from $-\infty$ to 0, the direction of the root locus branch is open loop zero to open loop pole.

If $K \uparrow$ (0 to ∞)

$\hookrightarrow$ $K=0$ $\longrightarrow$ $K=\infty$

$\times \rightarrow \rightarrow \rightarrow \rightarrow \circ$

OL pole $\quad$ OL zero

If $K \uparrow$ ($-\infty$ to 0).

$\hookrightarrow$ $K=-\infty$ $\quad$ $K=0$

$\circ \rightarrow \rightarrow \rightarrow \rightarrow \times$

O.L zero. $\quad$ O.L pole

⊛ Identify where the root locus branches starts & ends for $G(s)H(s) = \dfrac{K\cdot(s+1)}{s(s+2)(s+5)}$.

Sol. Start $\rightarrow$ at OL pole $\rightarrow$ $(K=0)$ $\rightarrow$ $s=0,-2,-5$.

End $\rightarrow$ at OL zero $\rightarrow$ $(K=\infty)$

$\hookrightarrow$ $s=-1, \underbrace{\infty, \infty}$

(Angle of asymptotes)

$K=0 \longrightarrow \longrightarrow \longrightarrow K=\infty$

OL poles $\quad$ OL zero.

→ To draw a root locus diagram the number of poles must be equal to number of zeros.

→ If the zeros are less we assume zero's at infinity. The direction of ∞ is given by angle of asymptotes.

Angle And Magnitude condition :

The construction rules of root locus are framed by using the angle condition, such that the root locus diagram gives the closed loop poles path and closed loop system stability

The closed loop poles path is given by characteristic equation.

Negative feedback (DRL/180° rules)

CE $\Rightarrow$ $1 + G(s)H(s) = 0$

$G(s)H(s) = (-1 + j0)$.

Angle condition

$\angle G(s)H(s) = \angle(-1+j0)$

= Odd multiples of $\pm 180°$

$= \pm(2q+1)180°, q = 0, 1, 2 \ldots$

Positive Feedback (IRL/CRL/0° rules)

CE $\Rightarrow$ $1 - G(s)H(s) = 0$

$G(s)H(s) = (1 + j0)$.

Angle condition

$\angle G(s)H(s) = \angle(1+j0)$.

= even multiples of ± 180.

$= \pm(2q)180°, q = 0, 1, 2 \ldots$

→ To convert direct root locus to Inverse root locus we should change the following things.

	DRL	IRL
Rule - 3	ODD →	EVEN
Rule - 4	$(2q+1)$ →	$(2q)$
Rule - 6	Left most →	Right most
Rule - 8	$180°$ →	$0°$

Purpose of angle condition:

- To check any point which lie on root locus or not that means all the points on the root locus must satisfy the angle condition.

) Check whether the following points lies on root locus or not for $G(s) = \frac{K}{s(s+5)(s+10)}$; $H(s)=1$ | (1) $s=-3$ (2) $s=-6$.

Sol: (i) Angle condition:

$$\angle G(s) = \frac{\angle K}{\angle s \ \angle s+5 \ \angle s+10}$$

$$\angle G(s)\Big|_{s=-3} = \frac{\angle K}{\angle -3 \ \angle 2 \ \angle 7} = \frac{0^\circ}{\pm 180^\circ + 0^\circ + 0^\circ} = 1(\mp 180).$$

i.e, odd multiples of 180°.

Satisfies angle condition

Therefore the given point $(s=-3)$. is on root locus.

$$\angle G(s)\Big|_{s=-6} = \frac{\angle K}{\angle -6 \ \angle -1 \ \angle 4} = \frac{0^\circ}{\pm 180 \pm 180 + 0^\circ} = 2(\mp 180^\circ).$$

i.e, even multiples of 180°

does not satisfy the angle condition.

$\therefore$ The given point $(s=-6)$ is not on the root locus.

Magnitude Condition:

→ The magnitude condition is the product of magnitude of $G(s)H(s)$ is unity (1) at any point which is on root locus.

$$\boxed{\left|G(s)H(s)\right|_{\substack{\text{at any point} \\ \text{which is on root locus}}} = 1}$$

→ The magnitude condition is valid when the given point is on root locus. The given pt. on the root locus is verified by angle condition.

Purpose: To find the system gain at any point which is on root locus

Find the system gain at a point $s = -5 \pm j5$ to the following system. $G(s) = \frac{K}{s(s+10)}$; $H(s) = 1$

Sol:

$|G(s)H(s)| = 1$

$\left|\frac{K}{s(s+10)}\right| = 1.$

$\left|\frac{K}{(-5+j5)\cdot(5+j5)}\right| = 1$

$\Rightarrow \frac{K}{5\sqrt{2}\cdot 5\sqrt{2}} = 1.$

Angle condition:

$$G(s)\Big|_{s=-5+j5} = \frac{\angle K + j0}{\angle(-5+j5)\,\angle(5+j5)} = \frac{0^\circ}{135+45^\circ} = -180^\circ$$

Satisfies angle condition since odd multiple of 180. The given point is on root locus.

always measure the angle in anti-clock wise direction only

Magnitude condition:

$|G(s)H(s)| = 1.$

$\left|\frac{K}{s(s+10)}\right|_{\text{at } s=-5+j5} = 1$

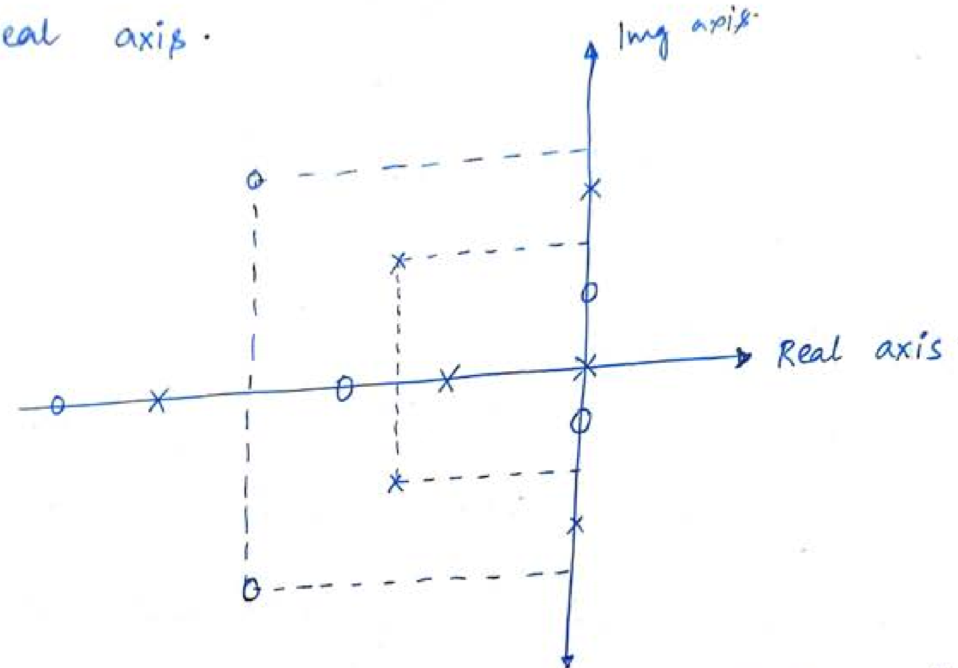
$\left|\frac{K}{(-5+j5)(5+j5)}\right| = 1 \Rightarrow \frac{K}{5\sqrt{2}\cdot 5\sqrt{2}} = 1$

$\boxed{\therefore K = 50} \rightarrow$ system gain

CONSTRUCTION RULES OF ROOT LOCUS :-

Rule -1 : Symmetrical

The root locus diagram is symmetrical about the real axis because the location of the poles and zeros are symmetrical about the real axis.



→ The symmetry not only depends on poles & zero's location. It depends on which graph the plot is constructed.

→ The Nyquist plots are also symmetrical about the real axis but not the bode plots. The bode plot drawn on semi-log sheet which is ~~not~~ non-linear.

Rule - 2 : Number of root locus branches or loci.

The number of root locus branches depends on number of poles and zeros.

Case-(i) : Poles > Zero :

Number of loci = Number of poles.

Case -(ii) Poles < Zeros:

Number of loci = Number of zeros.