ROUTH - HURWITZ CRITERIA (R-H Criteria):

Purpose:

- → To find the closed loop system stability.
- → To find the number of closed loop poles in the right, left, on imaginary axes of the s-plane.
- → To find the range of K value for system stability
- → To find the K value to become the system marginal stable
- → To find the natural frequency of oscillations or undamped oscillations when system is marginal stable.
- → To find the relative stability. ~~by~~ :
- → By using the relative stability concept we can find system time constant (τ), settling time (t_s), and time required to reach steady state (t_{ss}).
- → To find the closed loop system stability by using R-H criteria we require characteristic equation whereas in remaining all the stability techniques required OLTF of unity or non-unity f/b system
- → The n^{th} order general form of characteristic equation is

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s^1 + a_n s^0 = 0.$$

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & \cdots \\
s^{n-1} & a_1 & a_3 & a_5 & \cdots \\
s^{n-2} & \left(\dfrac{a_1 a_2 - a_0 a_3}{a_1}\right) & \left(\dfrac{a_1 a_4 - a_0 a_5}{a_1}\right) & & \\
\vdots & \vdots & & & \\
s^0 & a_n & & &
\end{array}$$

→ The conditions for system stabilities are

(i) All the coefficients in the first column should have same sign and no coeff should be zero in the first column.

(ii) If any sign change occurs in first column then the system is unstable.

(iii) The no. of sign changes in the first column is equal to no. of roots or poles are in the right of s-plane (or) right hand poles.

⊛ Check the stability to the following characteristic equations

(i) $s+10=0$

(ii) $s^2+25=0$

(iii) $s^2+10s+10=0$

(iv) $s^3+7s^2+6s+10=0$

(v) $s^3+6s^2+3s+100=0$

(vi) $s^3+8s^2+4s+32=0$.

Sol. (i) $s+10=0$

Stable.

for 1st order CE : $as+b=0$

$$\left.\begin{array}{c|c} s^1 & a \\ s^0 & b \end{array}\right\}$$

If both a&b are +ve & -ve i.e, $a,b>0$ (or) $a,b<0$ then the system is stable

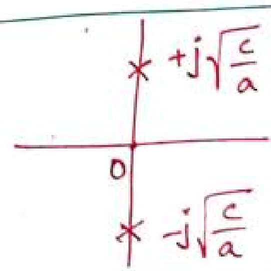
for 2nd order CE : $as^2+bs+c=0$

$$\left.\begin{array}{c|c} s^2 & a \\ s^1 & b \\ s^0 & c \end{array}\right\}$$

- If $a,b,c>0$ (or) $a,b,c<0$ then the system is said to be stable.
- If $b=0$ & $a,c>0$ then it is marginally stable system.

CE → $as^2+c=0$

$s=\pm j\sqrt{\frac{c}{a}}$



Marginally stable

(i) Stable (ii) marginally stable $(b=0)$ (iii) stable.

(iv) $s^3 + 7s^2 + 6s + 10 = 0$

$\overbrace{s^3 + \underbrace{7s^2 + 6s}_{I.P} + 10}^{E.P}$

s^3	1	6
s^2	7	10
s^1	$\frac{7\times 6 - 10}{7}$	
s^0	10	

for 3rd order system

I.P → Internal product $= 7\times 6 = 42$

E.P → External product $= 1\times 10 = 10$

If I.P > E.P ⇒ Stable system.

If I.P = E.P ⇒ Marginally stable system.

If I.P < E.P ⇒ Unstable system.

All the elements in 1st are +ve. So stable

(or) I.P > E.P

$42 > 10$ Stable.

(v) $s^3 + 6s^2 + 3s + 100 = 0$

Sol. $I.P = 6\times 3 = 18$

$E.P = 1\times 100 = 100$

I.P < E.P ⇒ Unstable system.

(vi) $s^3 + 8s^2 + 4s + 32 = 0$

Sol. $I.P = 8\times 4 = 32$

$E.P = 1\times 32 = 32$

I.P = E.P ⇒ Marginally stable.

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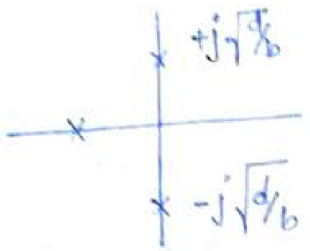
for 3rd order system CE → $as^3 + bs^2 + cs + d = 0$

s^3	a	c
s^2	b	d
s^1	$\left(\frac{bc-ad}{b}\right)$	
s^0	$d > 0$	

If $bc = ad \Rightarrow$ Marginally stable.



for even powers of s-terms = 0

$$bs^2 + d = 0$$

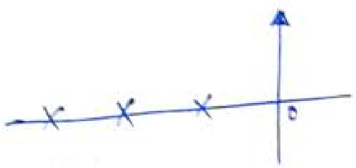
$$s = \pm j\sqrt{\frac{d}{b}}.$$

$$\omega_n = \sqrt{\frac{d}{b}} \text{ rad/sec}$$

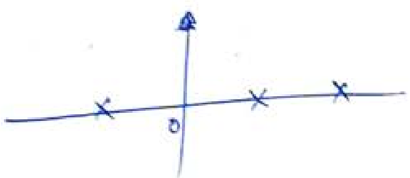
Only marginally stable system undergoes natural frequency (ω_n) oscillations (i.e, even pow. of $s = 0$)

Case-2

If $bc > ad \Rightarrow$ Stable.



Case-3: If $bc < ad \Rightarrow$ Unstable.



(*) Find the number of poles in the right of s-plane

$$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

Sol

s^4	1	3	5
s^3	2	4	
s^2	$\frac{2\times3-4\times1}{2} = 1$	5	
s^1	$\frac{1\times4-5\times2}{1} = -6$		
s^0	5		

Unstable system.

No. of sign changes = 2

Right hand poles = 2

Left hand poles = 4 − 2 = 2.

④ $s^4 + 2s^3 + 3s^2 + 2s + 1 = 0$

Sol

s^4	1	3	1
s^3	2	2	
s^2	$\frac{2\times3-2\times1}{2} = 2$	1	
s^1	$\frac{2\times2-2\times1}{2} = 1$		
s^0	1		

Stable system

Sign changes = 0

Right hand poles = 0

left hand poles = 4.

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④ $s^4 + 2s^3 + 3s^2 + s + 2 = 0$

Sol

s^4	1	3	2
s^3	2	1	
s^2	$\frac{2\times3-1\times1}{2} = \frac{5}{2}$	2	
s	$\frac{\frac{5}{2}\times1-2\times2}{\frac{5}{2}} = \frac{-3}{5}$		
s^0	2		

Unstable system.

Sign changes = 2

Right hand poles = 2

left hand poles = 2

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④ $s^4 + 2s^3 + 2s^2 + 4s + 8 = 0$

Sol.

s^4	1	2	8
s^3	2	4	
s^2	$\frac{2\times2-4\times1}{2} = \cancel{0}\ \varepsilon$	8	
s	$\frac{4\varepsilon-16}{\varepsilon}$		
s^0	8		

$$\lim_{\varepsilon\to0} \frac{4\varepsilon-16}{\varepsilon} = -\infty.$$

Sign changes = 2

Unstable system.

Right hand poles = 2

left hand poles = 2

Difficulty 1:

Whenever any one element is '0' in the first column replace '0' by ϵ and continue the routh stability. Finally find the $\lim_{\epsilon \to 0}$ & check the no. of sign changes.

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3) $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$.

Sol.

s^5	1	2	3
s^4	1	2	15
s^3	$\cancel{0} \to \epsilon$	$\frac{1\times3 - 1\times15}{1} = -12$	
s^2	$\frac{2\epsilon + 12}{\epsilon} = A = +\infty$	15	
s^1	$\frac{-12A - 15\epsilon}{A} = -12$		
s^0	15		

Unstable

Sign changes = 2

Right hand poles = 2

left hand poles = 3

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4) Find the system stability to the given characteristic equation.

$s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0$.

Sol.

s^5	1	3	2
s^4	1	3	2
s^3	$\cancel{0} \to 4$	$\cancel{0} \to 6$	0
s^2	$\frac{3}{2}$	2	
s^1	$\frac{2}{3}$		
s^0	2		

Even power of 's'.

s^4 row $\to$ AE $= 1\cdot s^4 + 3\cdot s^2 + 2 = 0$

s^3 row $\to$ ROZ. (Row of zero).

AE: Auxilary equation

$\frac{dAE}{ds} = 4s^3 + 6s^1 = 0$

Stable.

Sign changes = 0

Right hand poles = 0

left hand poles = 5.