$$GH(s)\Big|_{\omega=0.1} = 60$$

$$60 = 20\log K - 40\log\omega^{\,0.1}$$

$$20 = 20\log K \Rightarrow K = 10^{1} = 10$$

$$\boxed{\therefore K = 10}$$

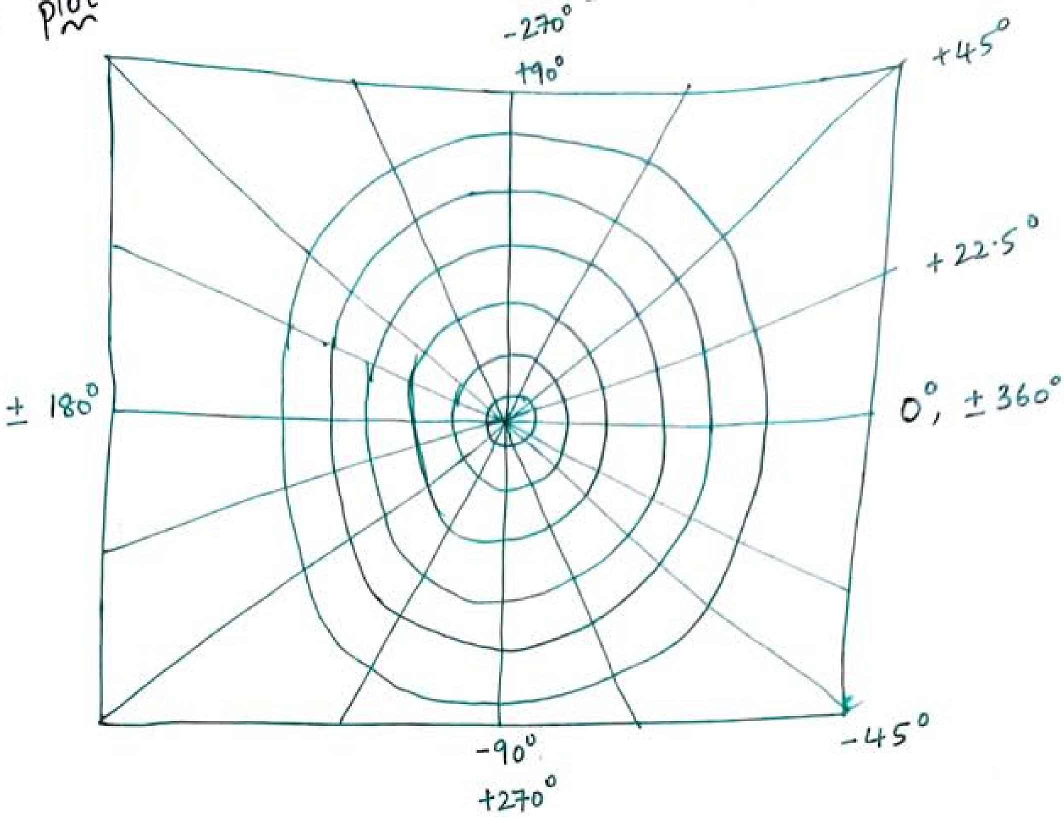
POLAR PLOTS :

Purpose:

1. To draw the frequency response of open loop T/F.
2. To find the closed loop system stability.
3. To find the gain margin, phase margin.
4. To find the relative stability by using gain margin & phase margin.
5. The polar plots are basically developed to use in the nyquist plot
6. The polar plots are not a complete frequency response plots.
7. A complete frequency response plots are nyquist plots
8. The frequency range for the polar plots ~~a~~ is (0 to ∞) whereas for nyquist plots the frequency range is $-\infty$ to $+\infty$.
9. The polar plot is a magnitude Vs phase plot.

~~Rb~~ **Polar plot graph Sheet:**

Polar plot



Problem:

(*) Draw the polar plot for $G(s)H(s) = \frac{1}{s}$

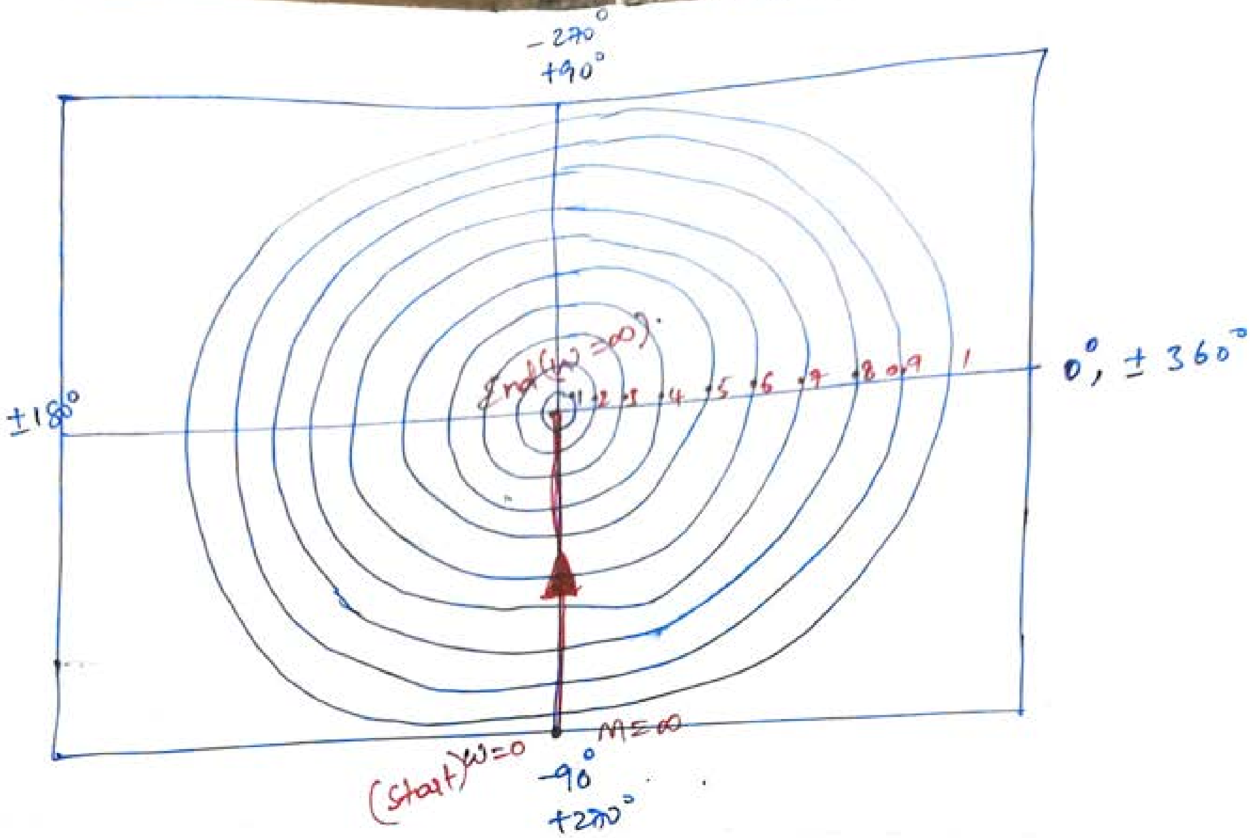
Sol. $G(s)H(s) = \frac{1}{s}$

$s \to j\omega$

$GH(j\omega) = \frac{1}{j\omega}$

Magnitude $= \frac{1}{\omega} = M$; Phase Angle $\angle\phi = \frac{\angle 1}{\angle j\omega} = -90°$.

ω	Magnitude (M).	Phase (ϕ)
Start → 0 →	∞ →	$-90°$
1 →	1 →	$-90°$
2 →	0.5 →	$-90°$
5 →	0.2 →	$-90°$
10 →	0.1 →	$-90°$
⋮	⋮	⋮
End → ∞	0	$-90°$



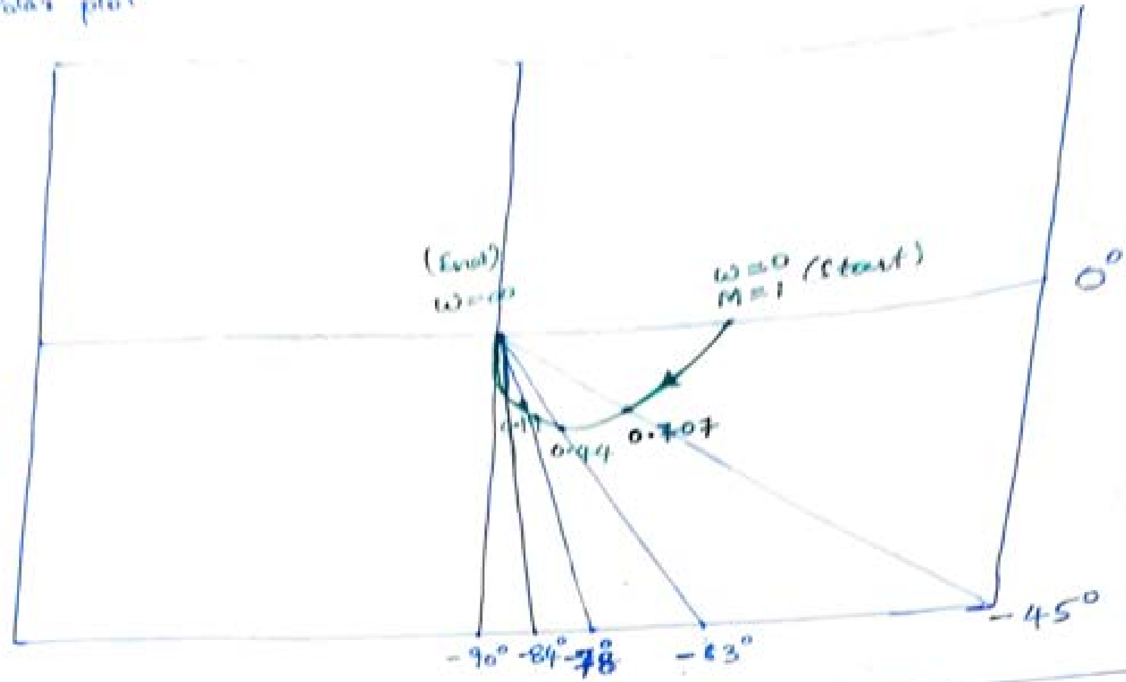
(*) Draw the polar plot for $GH(s) = \frac{1}{s\tau+1}$

<u>Sol</u>. $GH(s) = \frac{1}{s\tau+1}$; Magnitude $M = \frac{1}{\sqrt{1+\omega^2\tau^2}}$

$s \to j\omega$ $GH(j\omega) = \frac{1}{j\omega\tau+1}$; Phase angle $\angle\phi = -\tan^{-1}(\omega\tau)$.

ω		Magnitude (M)		Phase Angle ϕ
0	$\longrightarrow$	1	$\longrightarrow$	0°
$\frac{1}{\tau}$	$\longrightarrow$	$\frac{1}{\sqrt{2}}$	$\longrightarrow$	-45°
$\frac{2}{\tau}$	$\longrightarrow$	0.44	$\longrightarrow$	-63°
$\frac{5}{\tau}$	$\longrightarrow$	0.19	$\longrightarrow$	-78°
$\frac{10}{\tau}$	$\longrightarrow$	0.1	$\longrightarrow$	-84°
$\vdots$				
∞	$\longrightarrow$	0	$\longrightarrow$	-90°

Polar plot



(6) Draw the polar plot for $GH(s) = \left[\frac{s+1}{s+10}\right]$ which is the T/F of (HPF of lead compensator)

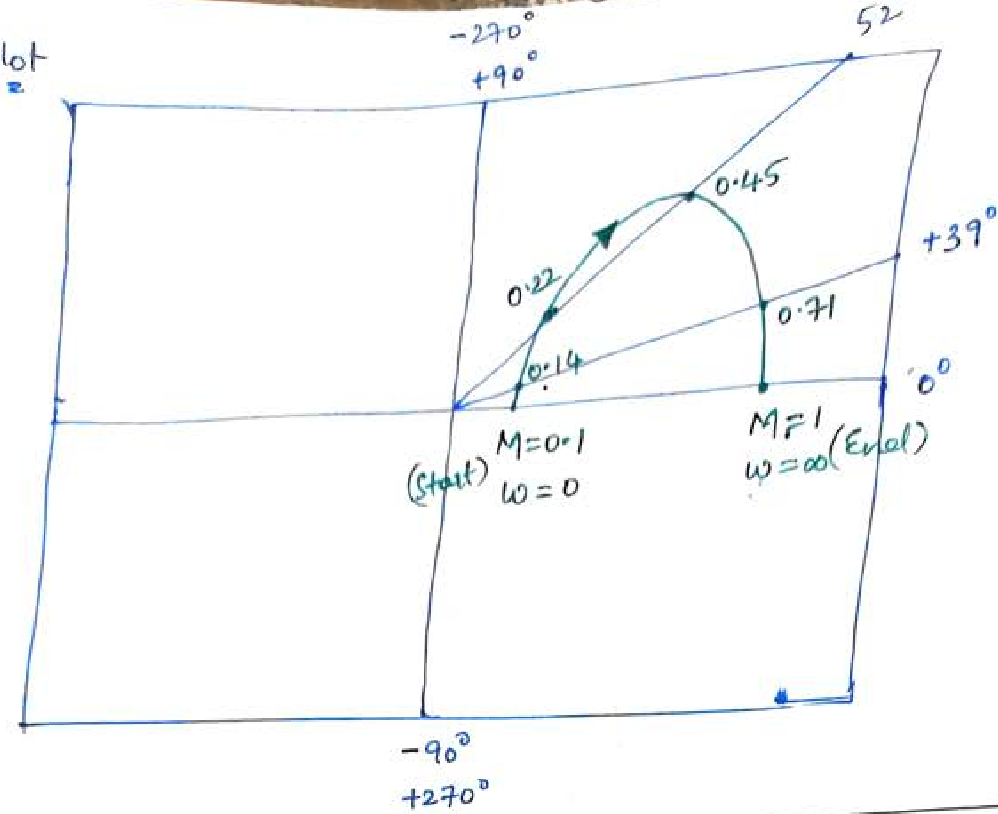
Sol. $GH(s) = \frac{s+1}{s+10}$; Magnitude $M = \sqrt{\frac{\omega^2+1}{\omega^2+100}}$

$s \to j\omega$

$GH(j\omega) = \frac{j\omega+1}{j\omega+10}$; Phase angle $\angle\phi = \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{10}\right)$.

ω	Magnitude (M)	Phase angle φ
0	0.1	0°
1	0.14	39°
2	0.22	52°
5	0.45	52°
10	0.71	39°
⋮	⋮	⋮
∞	1	0°

Polar plot



(*) Draw the polar plots for $GH(s) = \left(\frac{s+10}{s+1}\right)$ which is T/F of the (LPF, lag compensator)?

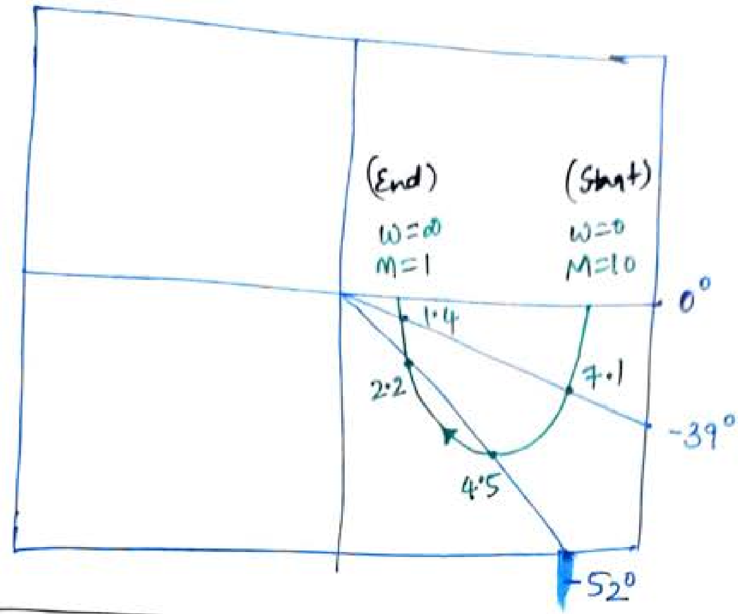
Sol. $GH(s) = \left(\frac{s+10}{s+1}\right)$; Magnitude $M = \sqrt{\frac{\omega^2+100}{\omega^2+1}}$

$s \to j\omega$

$GH(j\omega) = \frac{j\omega+10}{j\omega+1}$; Phase angle $\angle\phi = -\tan^{-1}\omega + \tan^{-1}\left(\frac{\omega}{10}\right)$.

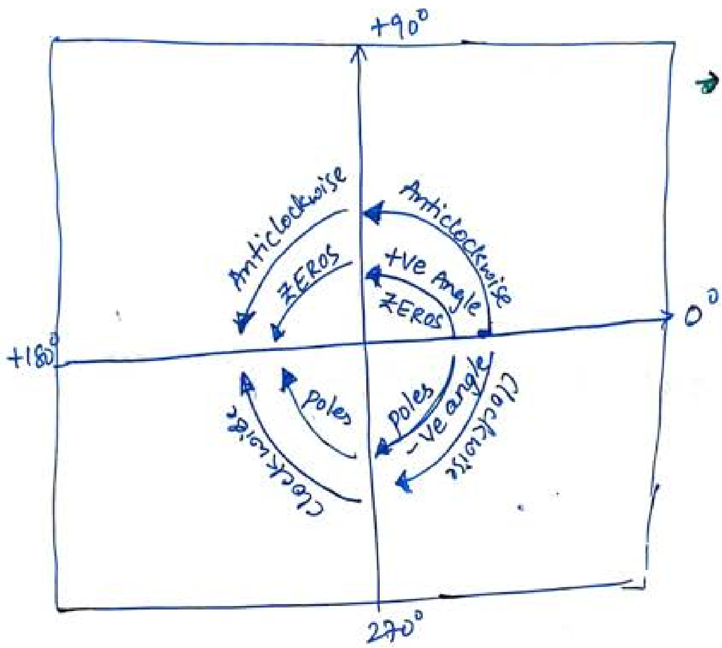
ω	Magnitude (M)	Phase angle ∠(φ)
0	10	0°
1	7.1	-39°
2	4.5	-52°
5	2.2	-52°
10	1.4	-39°
⋮	⋮	⋮
∞	1	0°

Polar Plot



Procedure to draw the polar plot : (Useful in gate exam):

1.



→ low frequency analysis.

Step-1: low frequency analysis.

Observe the $\angle\phi$ i.e, phase angle . If the positive tan term is large then the direction is anticlockwise

→ If the negative tan term is large then the direction is clockwise.

Step-2: high frequency analysis.

For high freq analysis count the no. of +ve & -ve tan terms.

→ If the +ve "tan" terms are more then the direction is anticlockwise

→ If the −ve "tan" terms are more then the direction is clockwise

→ The above procedure is not valid for high pass

→ If the given problem is high pass then follow standard procedure using table & calculator.

→ This method is not valid if the T/F consists of.

(i) Zero's at origin

(ii) Only zeros ~~(No~~ (i.e, no poles)

(iii) $(P = Z)$ ⇒ Check the magnitude ⇒ $\boxed{M|_{\omega=0} < M|_{\omega=\infty}}$

→ In above 3 cases draw the polar plot by using the standard procedure.

(*) Draw the polar plot for $GH(s) = \left[\frac{1}{s+1}\right]$.

Sol. $GH(s) = \frac{1}{s+1}$; Magnitude $M = \frac{1}{\sqrt{\omega^2+1}}$

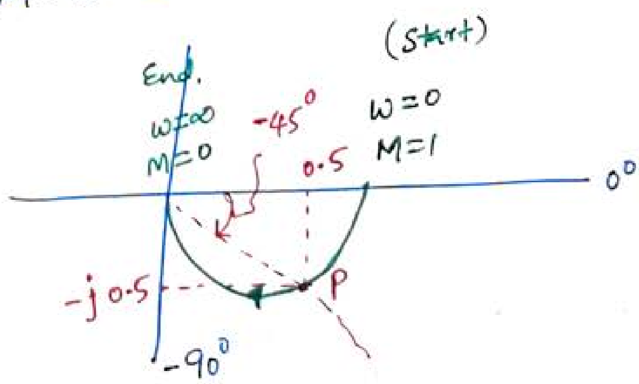
$s \to j\omega$

$GH(j\omega) = \frac{1}{j\omega+1}$; Phase Angle $\angle\phi = -\tan^{-1}(\omega)$.

→ Only −ve tan ⇒ no LF, HF ⇒ clockwise direction

$\omega = 0$ ⇒ $M = 1$ ⇒ $\phi = 0°$

$\omega = \infty$ ⇒ $M = 0$ ⇒ $\phi = -90°$.



Coordinates of pt 'p':

$\angle GH = -45^\circ$.

$-\tan^{-1}\omega = -45^\circ$

$\omega = \tan 45^\circ = 1$ rad/sec.

$M/_{\omega=1} = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$ = Magnitude ; Angle phase $= -45^\circ$.

The cordinates of point $p' = \frac{1}{\sqrt{2}} \angle -45^\circ$ (polar)

Rectangular $\Rightarrow M(\cos\theta, j\sin\theta) = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}, j\left(\frac{-1}{\sqrt{2}}\right)\right] = (0.5, -j0.5)$.

⊛ Draw the polar plot of $GH(s) = \frac{1}{(s+1)(s+2)}$.

Sol $GH(s) = \frac{1}{(s+1)(s+2)}$; Magnitude $M = \frac{1}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})}$.

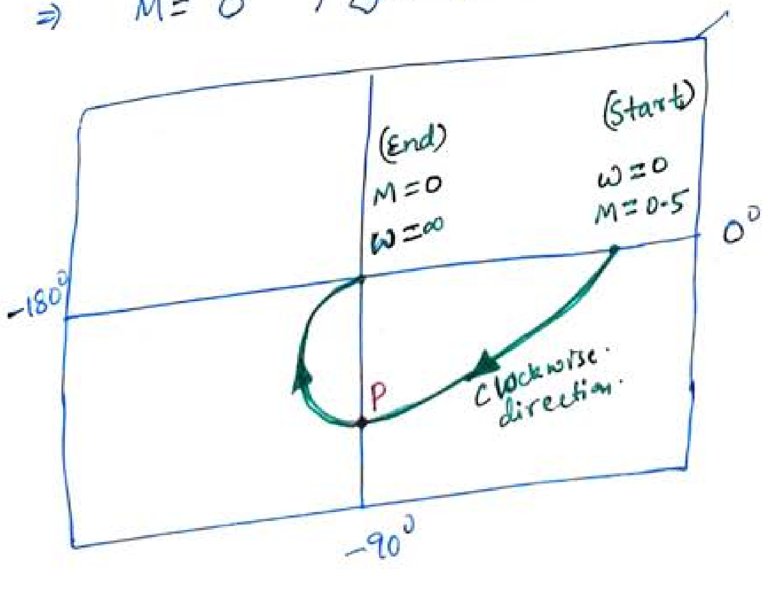
$s \to j\omega$

$GH(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)}$; Phase angle $\angle\phi = \underbrace{-\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}}_{\text{only -ve tan}}$.

Only -ve tan term in phase represents $\rightarrow$ clock wise direction.

$\omega = 0 \Rightarrow M = 0.5$, $\angle\phi = 0^\circ$

$\omega = \infty \Rightarrow M = 0$, $\angle\phi = -180^\circ$



Coordinates of P : $\angle GH = -90^\circ$

$$-90^\circ = -\tan^{-1}\omega - \tan^{-1}\omega/2$$

$$90^\circ = \tan^{-1}\left(\frac{\omega + \omega/2}{1-\omega^2/2}\right) = 0 \Rightarrow \omega = \sqrt{2} \text{ rad/sec.}$$

$(\because = \infty)$

i.e.,
$$\frac{\omega + \frac{\omega}{2}}{1-\omega^2/2} = \infty \Rightarrow 1 - \frac{\omega^2}{2} = 0 \Rightarrow \omega = \sqrt{2} \text{ rad/sec.}$$

$$M/_{\omega=\sqrt{2}} = \frac{1}{\sqrt{3}\times\sqrt{6}} = \frac{1}{\sqrt{18}} \; ; \quad \text{Phase angle} = -90^\circ.$$

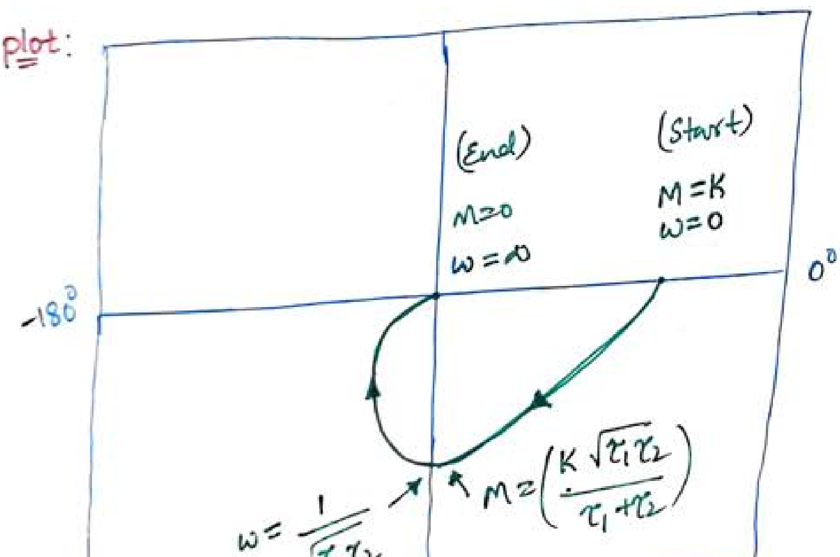
Coordinates of point P = $\frac{1}{\sqrt{18}} \angle -90^\circ$.

Rectangular coordinates = $\left(0, -j\frac{1}{\sqrt{18}}\right)$.

Conclusion:

If the given T/F is $GH(s) = \dfrac{K}{(s\tau_1+1)(s\tau_2+1)}$

Polar plot:



⊛ Draw the polar plot for $GH(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$.

Sol.
$$GH(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

$s \to j\omega$

$$GH(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)(j\omega+3)}$$