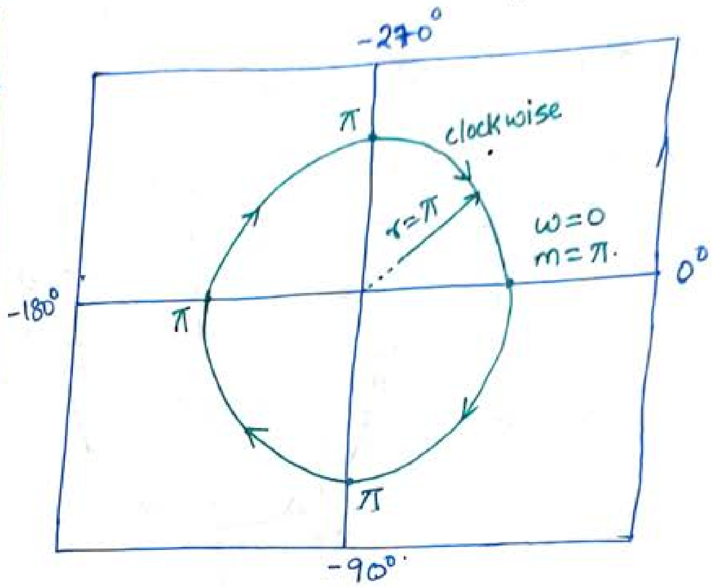
H.W: Q) $GH(s) = \frac{e^{-s}}{s+1}$.

---

* Draw the polar plot of $GH(s) = \pi e^{-2s}$?

Sol. $GH(s) = \pi e^{-2s}$; $M = \pi$; $\phi = -2\omega \times \frac{180}{\pi} = -114.59^\circ \omega$

ω	M	$\angle\phi$
0 →	π →	0°
$\frac{\pi}{4}$ →	π →	-90° ~~-114.59°~~
$\frac{\pi}{2}$ →	π →	-180°
$\frac{3\pi}{4}$ →	π →	-270°
π →	π →	-360°
⋮		⋮
∞	π	



NYQUIST PLOTS:

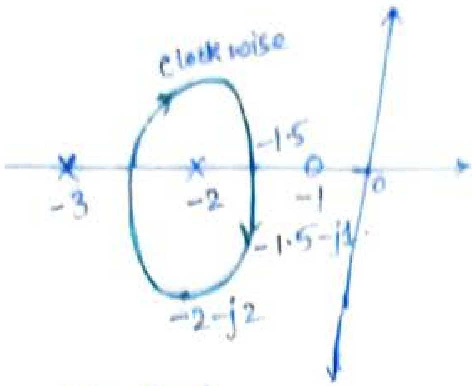
Purpose:

1. To draw the complete frequency response of openloop T/F.
2. To find the number of closed loop poles in the right hand side.
3. To find the range of 'K' value for system stability
4. To find the gain margin and phase margin, gain cross over frequency and phase crossover frequency.
5. To find the relative stability
6. The Nyquist plots are developed by using the mathematical concept known as principle of arguments.

Principle of arguments:

I. Statement: It states that if there are 'P' poles & 'Z' zeros are enclosed by the random selected closed path in the s-plane then the corresponding $G(s)H(s)$ plane encircles the origin with $(P-Z)$ times i.e, $\boxed{N = P - Z}$

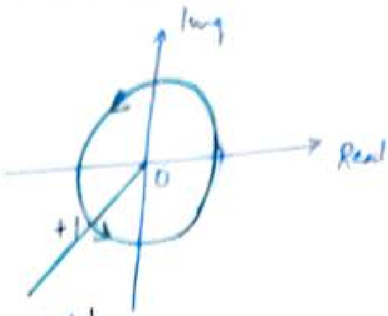
S- plane :



$N = P - Z$

$N = 1 - 0 = +1$

GH(s) - plane



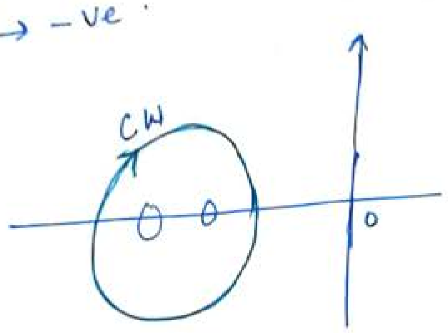
N=+1

P, change in direction

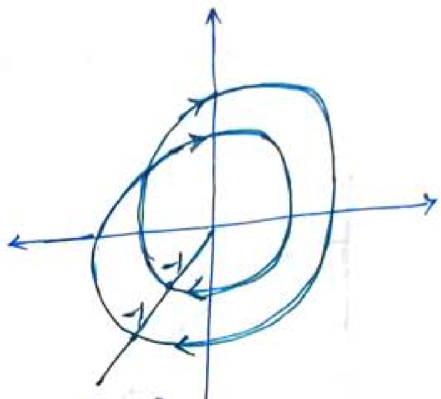
Z, No change in direction

ACW → +ve

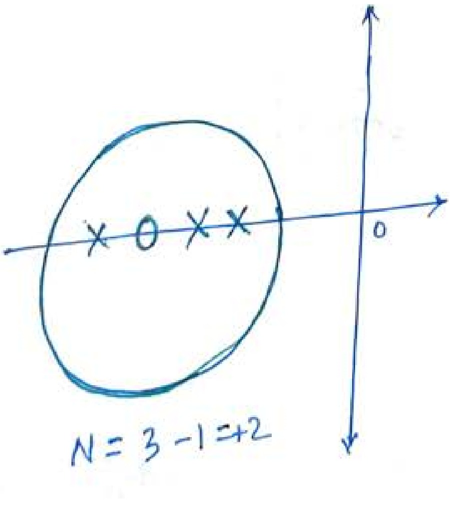
CW → -ve



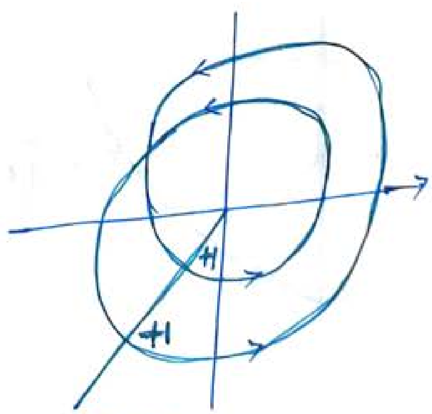
$N = 0 - 2 = 2$



$N = -2$

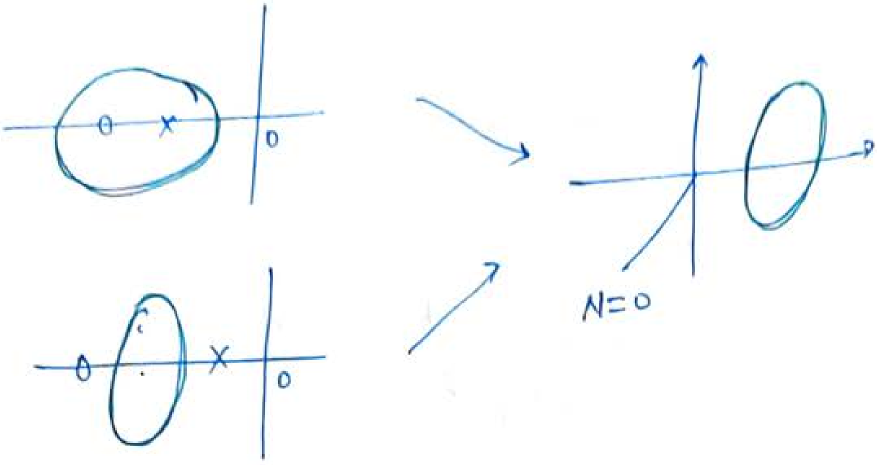


$N = 3 - 1 = +2$



$N = +2$

S-plane

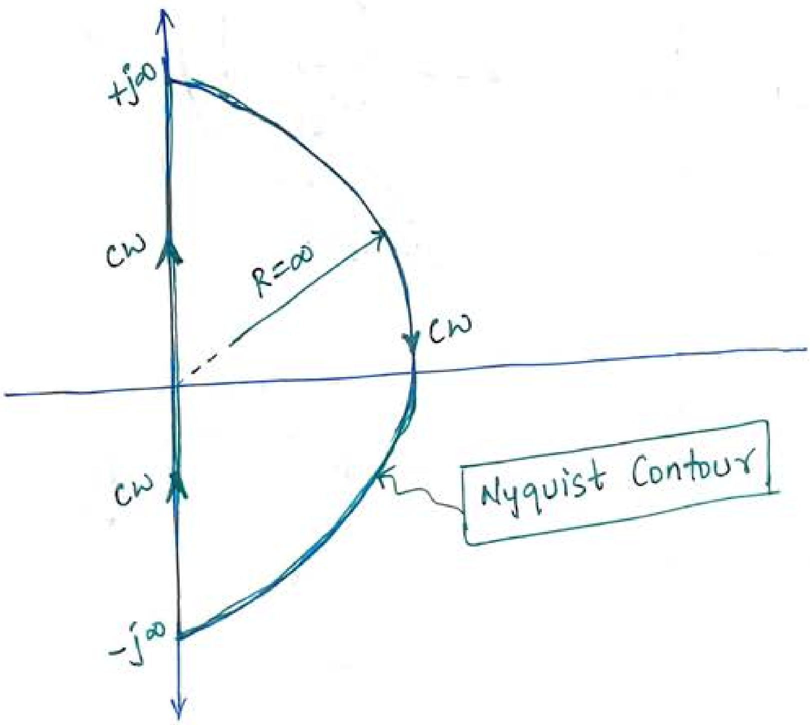


→ A random selected closed path should not pass through any pole or zero.

→ The principle of argument concept is applied to total right half of the s-plane with the radius of infinity.

→ The selected total right half of the s-plane with a radius of infinity is called Nyquist Contour.

→ The nyquist stability analysis is (right-half ~~of~~ 's' plane (or) (right of) analysis



→ To get the no. of encirclements about the origin use

$N = P - Z$; where

N = no. of encirclements about origin.

P = no. of open loop poles

Z = no. of open loop poles.

for OLTF $\boxed{N = P - Z}$.

Pole zero configuration :

OLTF → $G(s)H(s) = K\dfrac{N(s)}{D(s)} \longrightarrow ①$

CLTF → $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+K\dfrac{N(s)}{D(s)}}.$$

for CLTF $\boxed{\dfrac{C(s)}{R(s)} = \dfrac{G(s)D(s)}{D(s)+K\,N(s)}} \longrightarrow ②$

→ The closed loop poles are given by characteristic equation

CE → $q(s) = 1 + G(s)H(s)$.

$$q(s) = 1 + \frac{K\,N(s)}{D(s)}.$$

$$q(s) = \left[\frac{D(s)+K\,N(s)}{D(s)}\right] \longrightarrow ③.$$

Compare eq.n ① & ③

i.e, $\boxed{\text{Poles of CE = OLTF poles}}$

compare eq.n ② & ③

vvimp $\boxed{\text{Zero's of CE = CLTF poles}}$

$N = P - Z$ → zeros of CE in the RH-s-plane → CLTF poles in the RH-s-plane

N → No. of encirclements about critical point $(-1+j0)$

P → poles of CE in the RH-s-plane ⇓ OLTF poles in the RH-s-plane

→ For closed loop system stability there should not be, any closed loop pole in the right of 's' plane.

→ The closed loop pole is nothing but zeros of characteristic equation which must be zero in the right side that means $Z = 0 \Rightarrow N = P$.

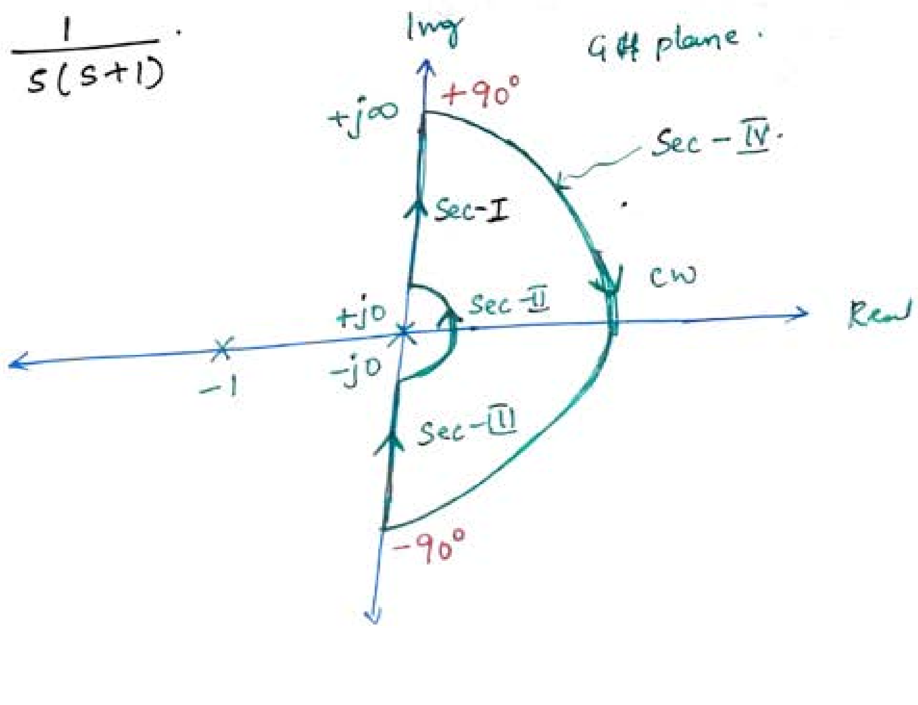
Nyquist Stability Criteria:

statement: It states that the number of encirclements (N) about the critical point $(-1+j0)$ must be equal to poles of characteristic equation which are called open loop T/F poles in the right of 's' plane i.e, $\boxed{N = P}$

(*) Draw the Nyquist plot & find the system stability for

$$G(s)H(s) = \frac{1}{s(s+1)}.$$

Sol.

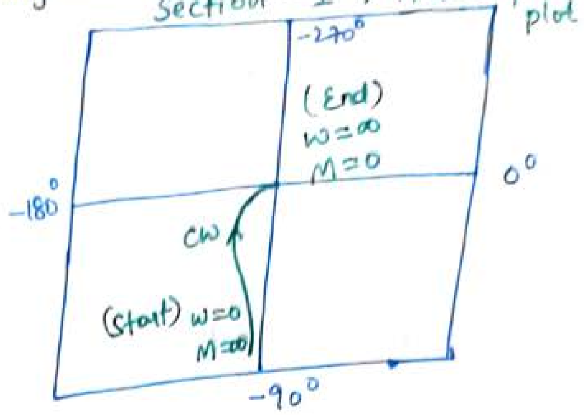


Section- I.

$$M = \frac{1}{\omega\sqrt{\omega^2+1}} \quad ; \quad \phi = -90^\circ - \tan^{-1}\omega$$

only -ve tan ⇒ clockwise.

Section - I ; Actual polar plot.



$\omega = 0^+ \Rightarrow M = \infty \ , \ \phi = -90^\circ$

$\omega = \infty \Rightarrow M = 0 \ , \ \phi = -180^\circ$

Section- II:

$s \Rightarrow Re^{j\theta}$.

$$GH(s) = \frac{1}{Re^{j\theta}(Re^{j\theta}+1)}.$$

$R \to 0$

θ varies from -90° to 90°.

$$GH(s) = \frac{1}{Re^{j\theta}(\underbrace{Re^{j\theta}}_{0}+1)} = \frac{1}{R}e^{j(-\theta)} = \frac{1}{R}\angle -\theta.$$

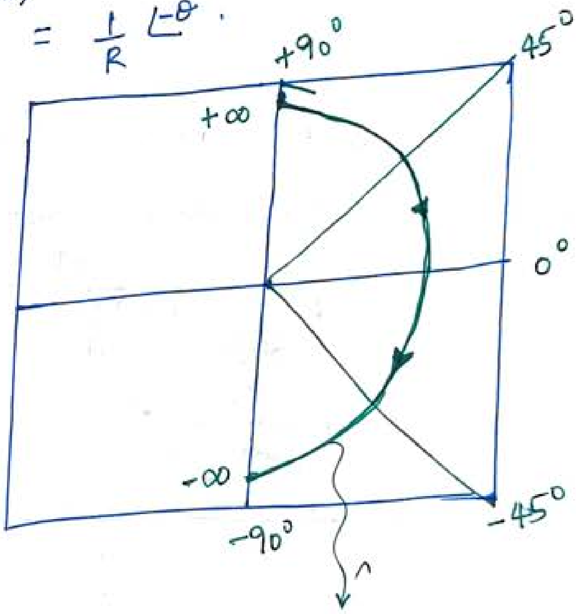
$R=0, \ \theta = -90^\circ \Rightarrow \infty \angle +90^\circ$

$R=0, \ \theta = -45^\circ \Rightarrow \infty \angle +45^\circ$

$R=0, \ \theta = 0^\circ \Rightarrow \infty \angle 0^\circ$

$R=0, \ \theta = +45^\circ \Rightarrow \infty \angle -45^\circ$

$R=0, \ \theta = +90^\circ \Rightarrow \infty \angle -90^\circ$



for conclusion:

$$GH(s) = \frac{1}{s^3(s+1)}$$

$s \to Re^{j\theta}$

$$GH(s) = \frac{1}{(Re^{j\theta})^3(Re^{j\theta}+1)}$$

$$GH = \frac{1}{R^3} \angle -3\theta$$

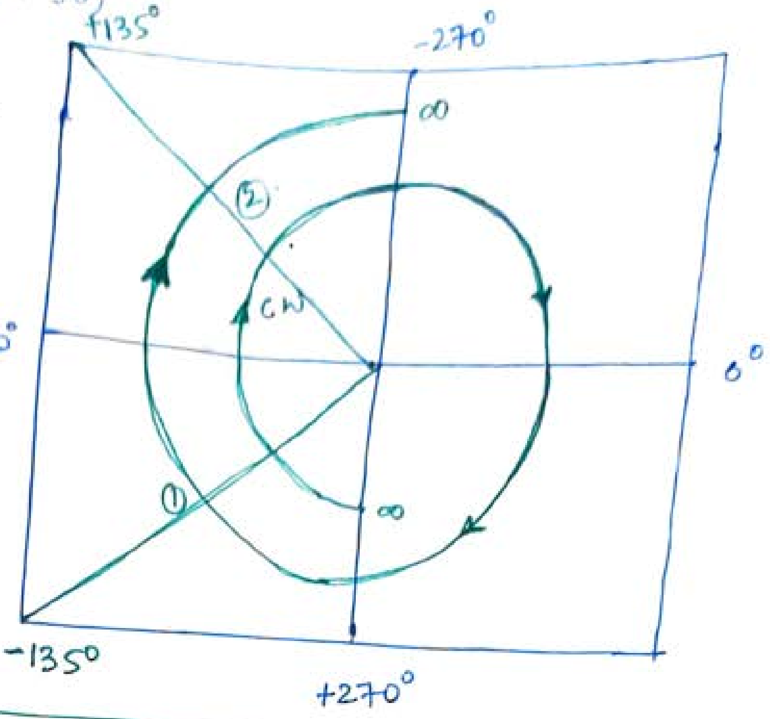
$R = 0,\ \theta = -90^\circ \Rightarrow \infty \angle +270^\circ$

$R = 0,\ \theta = -45^\circ \Rightarrow \infty \angle +135^\circ$

$R = 0,\ \theta = 0^\circ \Rightarrow \infty \angle 0^\circ$.

$R = 0,\ \theta = +45^\circ \Rightarrow \infty \angle -135^\circ$

$R = 0,\ \theta = +90^\circ \Rightarrow \infty \angle -270^\circ$



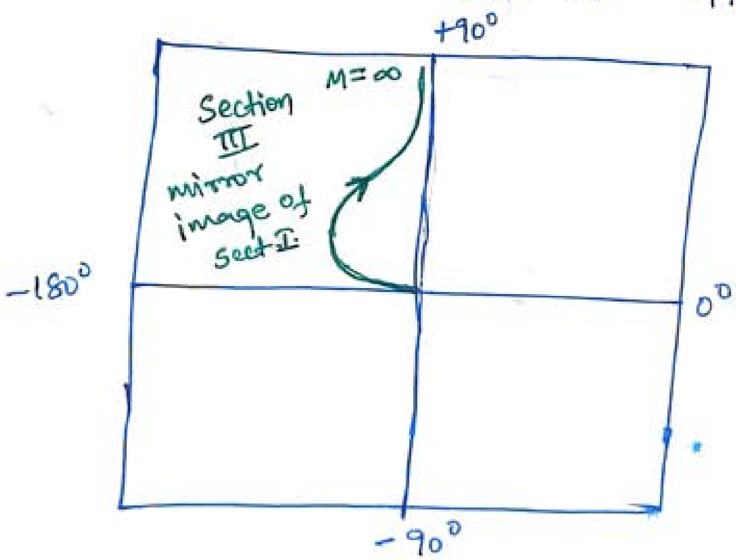
Conclusion:

No. of "∞" radius half (180°) circles = No. of poles at origin.

It is not valid for zeros at origin.

Section - III:

Section III is the mirror image of section I ↗ about the real axis but the direction is continuous but not opposite.



Section - IV:

$s \rightarrow Re^{j\theta}$

$$GH(s) = \frac{1}{Re^{j\theta}(Re^{j\theta}+1)}$$

$R \to \infty$
$\theta = +90^\circ$ to -90° $\Big\}$ $GH(s) = \frac{1}{R} \angle -2\theta^\circ$

The section IV gives the magnitude of zero i.e, it is a point at origin hence neglected the section IV.

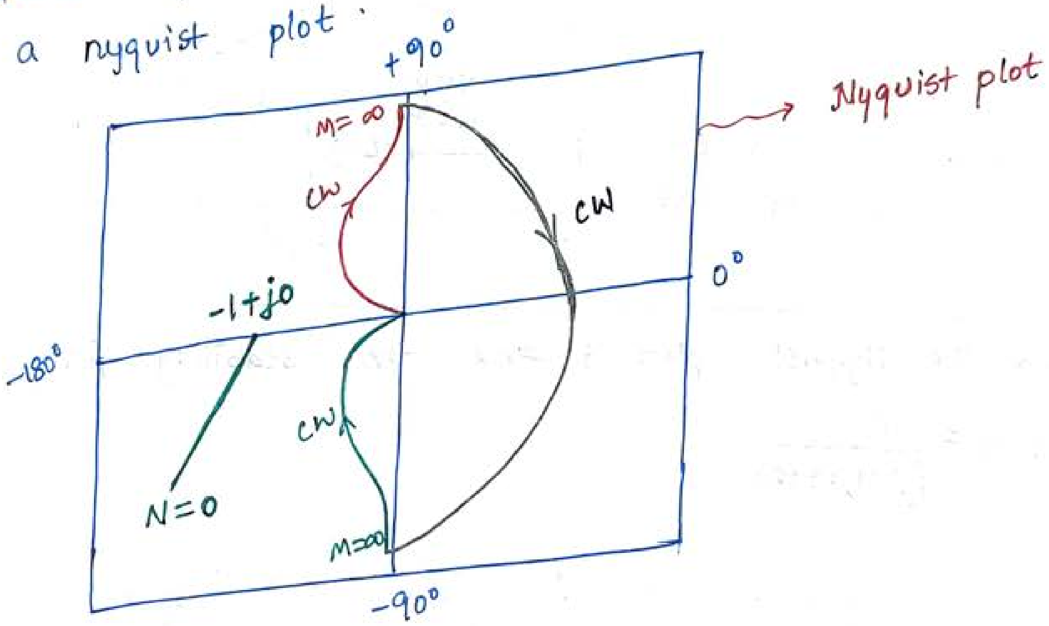
$R = \infty$, $\theta = +90^\circ \Rightarrow 0 \angle -180^\circ$

$R = \infty$, $\theta = +45^\circ \Rightarrow 0 \angle -90^\circ$

$R = \infty$, $\theta = 0^\circ \Rightarrow 0 \angle 0^\circ$

$R = \infty$, $\theta = -45^\circ \Rightarrow 0 \angle +90^\circ$.

Now join all the sections which is a nyquist plot. So combinations of different polar plots in various sections together constitu-tes a nyquist plot.



Stability:

$N = 0$; open loop poles right side $= 0$ i.e, $P = 0$

$\Rightarrow N = P \Rightarrow$ closed loop stability

The infinite radius of circle, starts at where the mirror image end and the infinite radius half circle end at where the actual polar plot is started.