18/08/13

Control Systems: (10 Marks or 12 Marks)

References:- for gate.

1. Control System Engg - NISE
2. Control System Engg - By Nagrath And Gopal
3. Automatic Control system - B.C. KUO
4. Control System Principle & Design - By M. Gopal.
5. Modern control system - Ogata.

Topics

1. Transfer function, Block diagram, Signal flow graph —(1M or 2M).

2. Time domain analysis
   (i) Transient Analysis
   (ii) Steady State Analysis
   } → (2M)

3. Stability (Closed loop)
   (i) Time domain technique - RH / Root locus.
   (ii) freq. domain technique - Bode plots/Nyquist plots
   } → (4M)

4. Compensators / Controllers —(2M)

5. State space Analysis — (2M).

Transfer function :-

It is a mathematical equivalent model of the control system.

Eg:- If transfer function $= \left(\frac{1}{s+1}\right)$.

(i) we can get order = 1

(ii) we can get No. of storage elements (or) no. of time constants by order.

Now Doing analysis with T/F is nothing but dealing with the system.

→ The control systems are basically low pass filters.

Reason:- Main motto of control system is to get desired or accurate output. Noise is very important which effect the system performance to great extent. Noise is having high frequency. Noise is to be eliminated. Low pass filter is used to remove noise. At low frequency all the elements are stable.

General exp. of T/F $= \dfrac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^n(1+s\tau_a)(1+s\tau_b)\cdots}$.

Always there should not be any zeros, and poles must be greater than zeros.

Poles > Zeros (Low pass filter) → strictly proper. T/F

Poles = Zeros (L.P, H.P, B.P, B.S) → proper T/F.

Poles < Zeros (high pass filter) → Improper T/F.

Control system Specifications

1. Speed → Rise time, settling time.
2. Accuracy → Steady state error
3. Stability → Gain margins & phase margin (p.m)
(GM)

If G.M & P.M is more ⇒ more relative stability (adv)
~~more oscillatory~~
⇒ Slow response (disad.).

If G.M & P.M is less → less rel. Stability (disadv)
→ More oscillatory (disadv).

The (best range of optimum rang) Gain margin = 5dB to 10 dB
Phase margin = 30° to 40°.

Generally optimum range is used

4. Senstivity → w.r.t Temperature, disturbance, noise.

Now.
Speed → Time domain analysis
Accuracy → Time domain analysis
Stability – freq. domain analysis
Sensitivity – time domain analysis

The feedback changes the location of poles. As order ↑ finding the location of poles is difficult. Hence we need a stability techquinue for closed loop (order is)

(i) Nyquist plot (ii) Root locus
(iii) ~~R-H Criteria~~ Bode plot (iv) ~~Bode~~ R-H Criteria.

(*) Time domain analysis gives the steady state of transient state analysis

(*) freq domain analysis gives the steady state analysis only.

- Transportation delay / Lag system :-

$$L[g(t-\tau)] = e^{-s\tau} G(s).$$

↳ delay

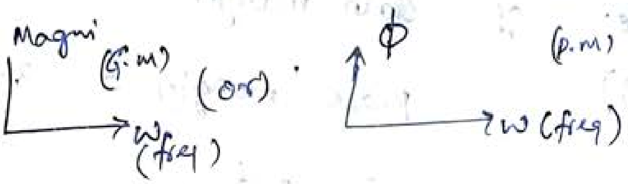
time domain

$$e^{-s\tau} = (1-s\tau) + \frac{s\tau^2}{2!} - \dots \infty$$

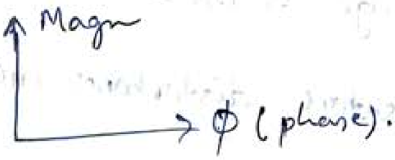
Higher order is neglected.

freq domain :-

Bode plot →



Nyquist plot →



1st prioirity is given to freq. domain.

Compensators / Controllers :-

To get the desired specifications.

State Space Analysis :-

Valid for dynamic systems.

i.e, linear

Non linear

time variant

time invariant.

System:

A system is a group of physical components arranged in such a way that it gives the proper output to the given input.

Control System:

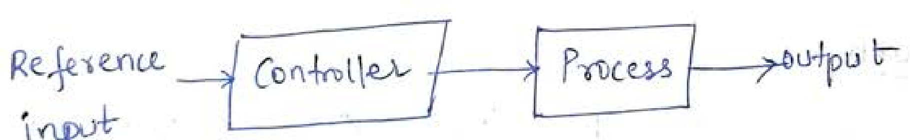
It is a group of physical components arranged in such a way that it gives the desired output by means of control or regulation either direct or indirect method.

For example :-

1. A fan without a blade is not a system → no air flow → No proper output.
2. A fan without a regulator → A system → proper output → air flow.
3. A fan with a regulator → control system → Desired output.

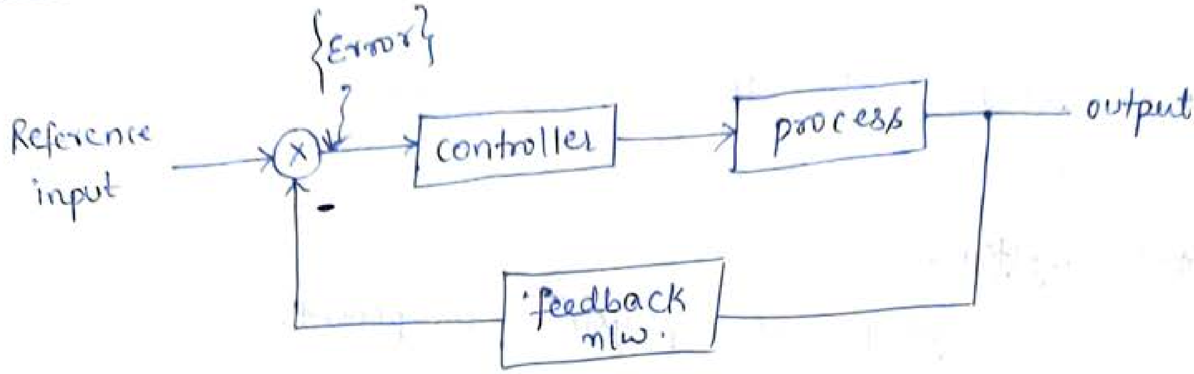
- Control systems are classified into two types based on control action (i) Open loop Control System (OLCS)
(ii) Closed loop Control System (CLCS).

(i) OLCS:



Eg: Manual system.

CLCS :-



Eg:- Automatic system.

Openloop control System:

A system in which the controller action is independent of output then it is called open loop system.

For eg:- Fan, lights, air cooler, traffic lights and so on.

Any system which is not having a sensor & not having provision to select the reference input.

Closed loop control system:

A system in which the controller action depends on the output then it is called closed loop control system.

For eg:- AC, refrigirator, human beings, automatic iron box and so-on. Any system which is having a sensor and provision to select the reference input.

Feedback Network:

It is a property of the closed loop system which brings the output the input and compared with reference input so that the appropriate control action formed to make the error equal to zero.

Error is zero means the system is stable and it gives the desired output.

→ The basic components in a f/b n/w are passive components.

→ The max value of f/b n/w ratio is 1

→ The best feedback is unity negative feedback because the negative feedback improves the relative stability (loop gain >0)

→ The steady state errors are calculated to only unity feedback systems. If non-unity f/b system is given it must be converted into unity feedback.

→ The feedback n/w may consists the transducer which converts the energy from one form to another form.

Transfer Function :-

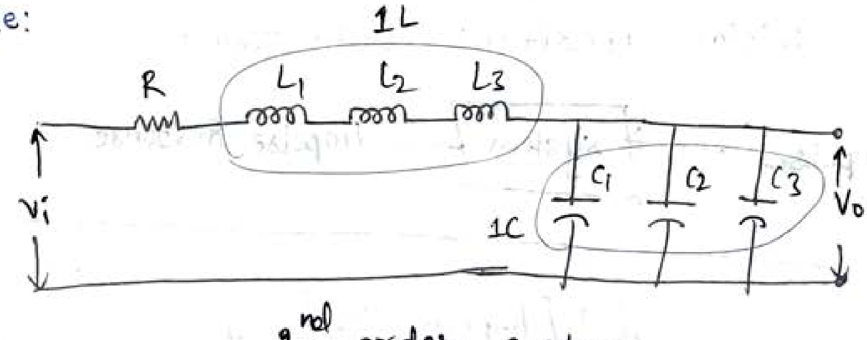
→ A transfer function is basically mathematical equivalent model for the system.

→ The order of the transfer function represents the no. of storage elements or no. of time constants.

Note :

Whenever same kind of elements connected either series or parallel then it should be treated as a single component.

For example:



2^{nd} order system.

1st definition of transfer function :

The transfer function of a linear time invariant (LTI) system is defined as the ratio of laplace transform output to laplace transform input with all initial conditions are zero.

$$\boxed{\text{Transfer function } (T/F) = \frac{\mathcal{L}\cdot T\,[\text{Output}]}{\mathcal{L}\cdot T\,[\text{Input}]}\Big/_{I_i=0}}$$

LTI system :

The LTI system is nothing but RLC circuit because the RLC circuit gives the linear transfer charact- esitics and the RLC component values ~~are~~ will not change with respect to time.

→ In the transfer function analysis the initial condition should be zero because the o/p should not depends on the past history of the system.

→ The T/F analysis will be valid only for LTI system.

2nd definition of transfer function:

The transfer functn of a linear time invariant system is defined as the laplace transform impulse response with all initial conditions to be zero.

Impulse → System → Impulse Response

$$\boxed{\text{Transfer function } (T/F) = \frac{\mathcal{L}\cdot T\,[\text{Impulse Res}]}{\mathcal{L}\cdot T\,[\text{Imp}]} = \mathcal{L}\,[\text{Impulse Response}]_{I_i=0}}$$

$[\because \mathcal{L}T\,[\text{Imp}] = 1]$

Impulse response means system response or natural response

Impulse Response :

→ The system behaviour or characteristics are given by impulse response.

→ The impulse response consists only system parameters (K, τ).

→ The impulse response response is also known as system response or natural response or free forced response (impulse response does not contain any input term).

→ If input signals are step, parabolic, ramp, then the response is called forced response.

→ Any device which is having on & off characteristics is called non linear devices.

Basics :

→ Basically any system described in the form of open loop transfer function.

→ The standard form of the open loop system is represented as G(s) & it is known as open loop transfer function of a system.

$$\boxed{\frac{C(s)}{R(s)} = G(s) = \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^n(1+s\tau_a)(1+s\tau_b)\cdots}}$$ → (time const form).

where K & τ are called system parameters

K = system gain

τ = System time constant.

n = type of the system

→ type gives the no. of poles at origin

→ Order gives the total no. of poles in the s-plane

1. Find the system gain, type & order of the given system. $\frac{C(s)}{R(s)} = \frac{10(s+5)^2}{s^3(s+2)^2(s+10)}$ (pole zero form)

Sol: By comparing with

$$\frac{C(s)}{R(s)} = \frac{K(1+s\tau_1)(1+s\tau_2)\cdots}{s^n(1+s\tau_a)(1+s\tau_b)\cdots}$$

$$\frac{C(s)}{R(s)} = \frac{10\times 5^2(1+0.2s)^2}{2^2\times 10\times s^3(1+0.5s)^2(1+0.1s)}$$

$$\frac{C(s)}{R(s)} = \frac{6.25(1+0.2s)^2}{s^3(1+0.5s)^2(1+0.1s)}$$

Note:

$K = \frac{\text{Nr. const}}{\text{Dr. const}}$

Gain $= K = 6.25$, type $= 3$, order $= 6$.

type & order are same for both pole zero & time const form

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2. Find the type and order of the given CLT/F of a unity f/b system. $\frac{C(s)}{R(s)} = \frac{2s+5}{s^4+5s^3+7s^2+2s+5}$.

Sol. Note: The type & order is not defined to the closed loop T/F. To get the type & order of a closed loop system required open loop T/F of a unity f/b system i.e, G(s) · H(s) should be equal to 1

i.e, $H(s)=1$.

Give CLTF $= \frac{2s+5}{s^4+5s^3+7s^2+2s+5} = \frac{G}{1+G}$.

$$\frac{1+G}{G} = \frac{s^4+5s^3+7s^2+2s+5}{2s+5}.$$

$$\frac{1}{G}+1 = \frac{s^4+5s^3+7s^2+2s+5}{2s+5}$$

$$\frac{1}{G} = \frac{s^4+5s^3+7s^2+2s+5}{2s+5} - 1$$

$$G = \frac{2s+5}{s^4+5s^3+7s^2} = \frac{2s+5}{s^2(s^2+5s+7)}$$

$\boxed{\text{Type} = 2 \quad , \quad \text{Order} = 4.}$

Note :- To get the open loop transfer function from closed loop substract the numerator term in the denominator when the feedback is unity.

i.e, $C.L.T.F = \dfrac{G}{1+G}$

$$O.L.T.F = \frac{G}{1+G-G} = G.$$

→ To get the closed loop transfer function from the open loop add the numerator term in the denominator when the feedback is unity.

$$O.L.T.F = \frac{G}{1}$$

$$C.L.T.F = \frac{G}{1+G}.$$

Characteristic Equation :

The denominator of the transfer function make, equal to zero then it is called characteristic eq.n, because the denominator of the function gives the system behaviour or system characteristics.

→ For a closed loop system characteristic eqn is

$$1 + G(s)H(s) = 0 \cdot$$

→ The roots of characteristic equation are poles

Pole :

The pole is nothing but negative of inverse of system time constant at which magnitude of the transfer function becomes infinity.

Zero :

Zero is nothing but negative of inverse of system time constant at which the magnitude of the transfer function is zero.

→ The poles effect the system stability & system response but not zeros.

Time Constant :

The time constant gives the system behaviour. If the time constant is very very large then it is called slow response system because it takes the large time to reach the steady state. Practically any system takes five time constants i.e.(5τ) to reach the steady state.

$$\boxed{\text{Time constant } \tau = \frac{-1}{\text{Real part of the dominant pole}}}$$

Dominant pole : The pole which is very closed to imaginary axis

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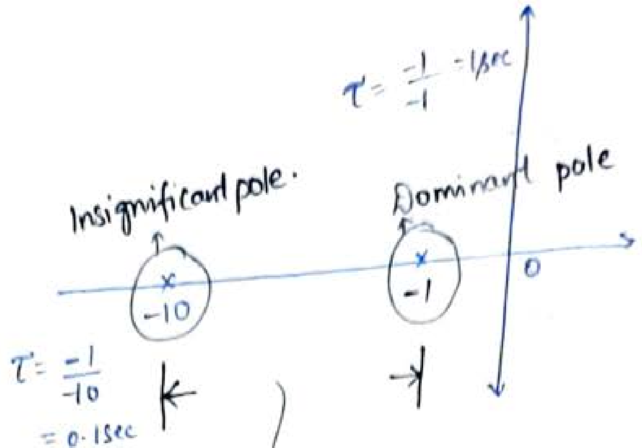
(*)(a) Find the equivalent transfer function to the following

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)},$$

b. Find the time constant

Sol



$\boxed{\text{Insignificant pole}(\tau) \leq 5 \text{ times of dominant pole}(\tau)}$

The above condition is used to identify insignificant pole when dominant pole is already identified.

$\therefore$ Insignificant pole$(\tau) \leq \frac{\text{dominant pole}(\tau)}{5}$

Insignificant pole:-

The pole which lies on the left most side. The insignificant pole time constant must be less than or equal to 5 times of the dominant pole time const.

→ The best pole is the insignificant pole because it gives very quick response and more relatively stable. Because of the dominant pole the system response become slow & less relative stable.

→ The insignificant poles are neglected because even if the insignificant poles are neglected there is no much change in the response.

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}$$

$$\frac{C(s)}{1} = \frac{1}{(s+1)(s+10)} \qquad (\because R(s) = 1)$$

$$C(s) = \frac{1}{9(s+1)} - \frac{1}{9(s+10)}$$

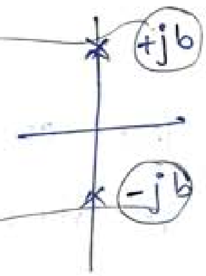
Apply inverse laplace transform.

$$c(t) = \underbrace{\frac{1}{9}\left[e^{-1t}\right]}_{\text{Dominant pole response}} - \underbrace{\frac{1}{9}\left[e^{-10t}\right]}_{\text{Insignificant pole response.}}$$

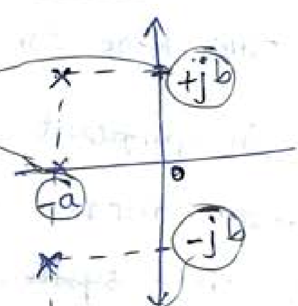
→ In the response the exponential powers are the real parts of the pole, sine or cosine functions are imaginary parts of the pole. If 't' represents the repeated nature of poles.

$$L\left[e^{-at}\right] = \frac{1}{s+a}$$

$$L\left[\sin \text{ or } \cos bt\right] = \frac{b \text{ or } s}{s^2+b^2}$$



$$L\left[e^{-at}\sin bt\right]$$



$$L\left[t^n e^{-at}\right] = \frac{n!}{(s+a)^{n+1}}$$

(n+1) poles repeated

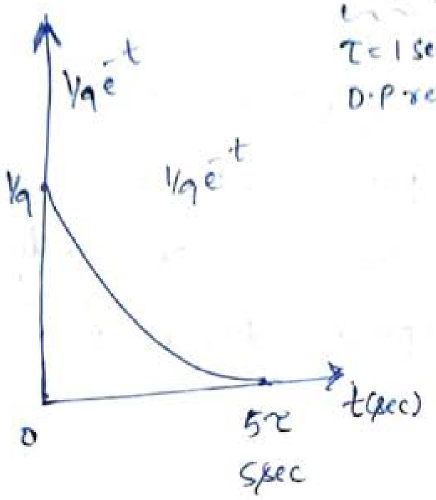
→ To get the system time constant from the response compare the response with $e^{-t/\tau}$.

→ The system time constant is nothing but dominant pole & time constant (fixed on the largest value.)

→ Response of $c(t) = \frac{1}{9}e^{-1t} - \frac{1}{9}e^{-10t} \approx \frac{1}{9}e^{-t}$

$\tau = 1\text{ sec}$ D.P respon

$\tau = 0.1\text{sec}$ ISP response



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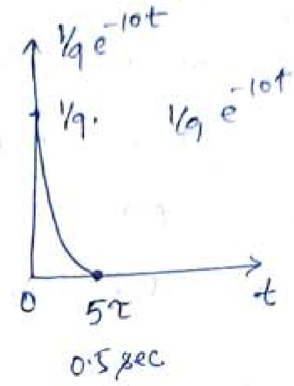

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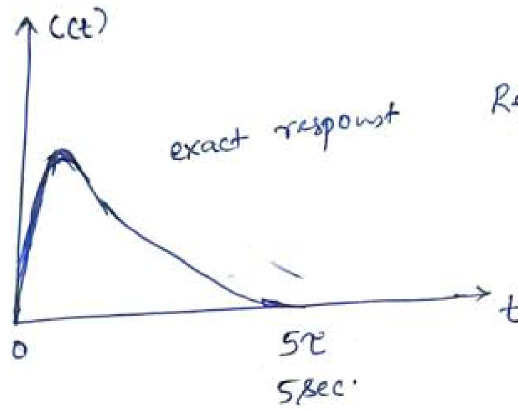


Response of c(t) is approx equal to that of dominant pole response.

Now $\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)}$

Note:- The insignificant poles neglected only in the time constant form because the system should not be effected.

$$\frac{C(s)}{1} = \frac{1}{10(s+1)(1+0.1s)} \qquad (\because R(s)=1)$$

$$\boxed{\frac{C(s)}{R(s)} = \left(\frac{0.1}{s+1}\right), \quad \tau = 1\text{ sec}}$$

Apply I.L.T

$$\boxed{c(t) = 0.1e^{-t}} \qquad (\because R(s) = 1).$$

(*) Find the equivalent T/F for $\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+10)(s+100)}$.

(a) $\frac{1}{s+1}$ (b) $\frac{1}{(s+1)(s+10)}$ (c) $\frac{0.01}{(s+1)(s+10)}$ (d) $\frac{0.001}{(s+1)}$

(A) While finding system response, system stability, system time constant, consider only poles but not the zeros because the system response consists of only poles response terms. No zero response terms present (exist) in the system response. For example.

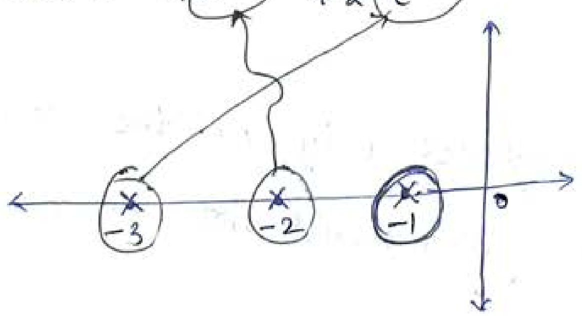
Eg:
$$\frac{C(s)}{R(s)} = \frac{(s+1)}{(s+2)(s+3)}$$

Syst. Res: $R(s) = 1$

$$C(s) = \frac{-1}{s+2} + \frac{2}{s+3}$$

Apply inverse laplace transforms

$$C(t) = -1\,e^{-2t} + 2\,e^{-3t}$$



System response consits of only poles response term. No zero response term exist in the system response.

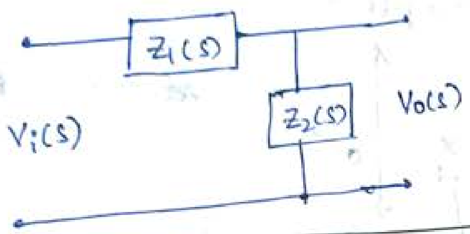
→ To find the system stability: Substitute $t \to \infty$ in the response if it gives the finite value then the system is stable. If it gives the infinite value the system is unstable

→ A time constant is mainly required to draw the response.

→ The open loop zero never effect the open loop system stability

→ A open loop zero effect the closed loop system stability, because closed loop poles are nothing but sum of open loop poles & open loop zero with the function of system gain 'K'.

Transfer function to the Electrical Networks :

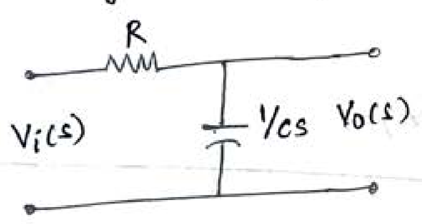


$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{\text{Impedance across output}}{\text{Total circuit impedance}} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}}$$

Problem

1\. (a) Find the transfer function to the given electrical n/w and locate the poles in the 's' plane.

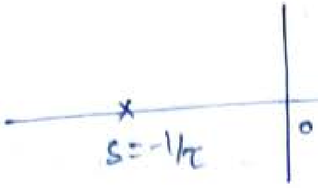
(b) Find the system response.

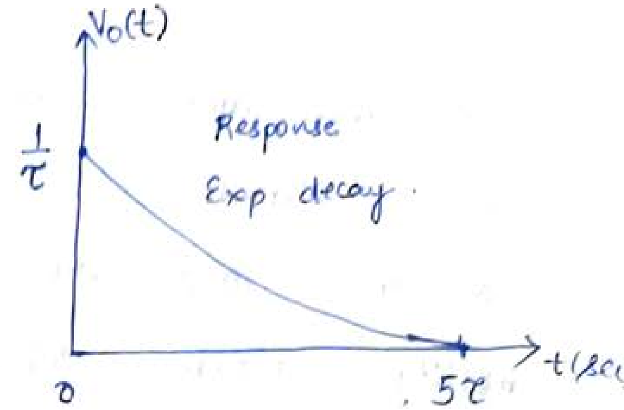


Sol

$$\frac{V_o(s)}{V_i(s)} = \frac{1/sc}{R+\frac{1}{sc}} = \frac{1}{scR+1} = \frac{1}{s\tau+1} \quad (\because \tau = RC).$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s\tau+1}.$$



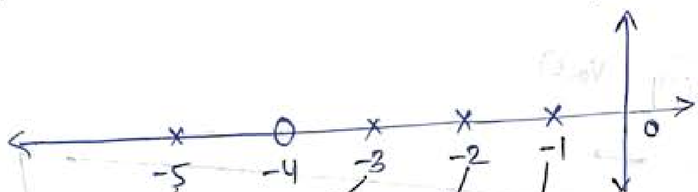


System response $V_i(s) = 1$.

Apply inverse laplace transform.

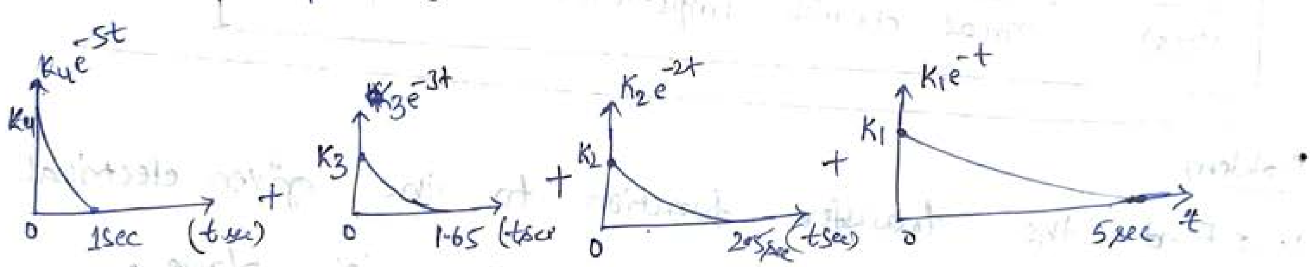
$$V_o(t) = \frac{1}{\tau} e^{-t/\tau}.$$

Note: If one or many poles lies in the left of 's' plane at different locations then the system response is exponential decay irrespective of position of zeros and the system is stable. For example.



~~at~~ -4 is ~~p~~ zero

$$K_4 e^{-5t} + K_3 e^{-3t} + K_2 e^{-2t} + K_1 e^{-t}$$



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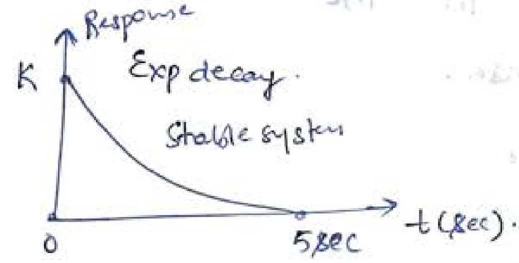


Stability:-

The movement of pole in the 's' plane is nothing but varying the system component ~~&~~ values.

- <u>Absolutely stable system:-</u>

A system which is stable for all the values of system paramaters like system gain K from 0 to ∞.

→ Conditional stable system :-

Here the system is stable for certain range of system components like K from 0 → 100

→ Addition of pole & zero to the transfer function is nothing but adding RLC components to the system.

→ The RLC components are added to the system in two ways (i) Series Connection

(ii) Parallel Connection.

→ Series Connection :-

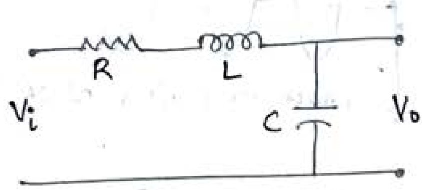
Here the components are added only in the forward path.

Parallel Connection :-

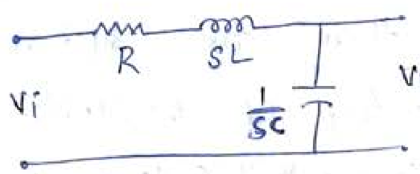
Here the components are added in the feedback path.

(a) Find the transfer function to the given RLC circuit and locate the poles in the s-plane, by considering $R = 0\Omega$, $L = 1H$, $C = 1F$.

(b). Find the system response

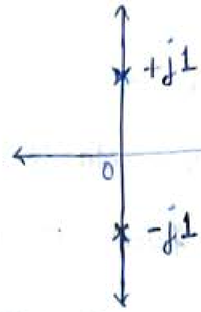


Sol



$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SC}}{R+SL+\frac{1}{SC}} = \frac{1}{s^2LC+1} \quad (\because R=0)$$

$$\therefore \frac{V_o(S)}{V_i(S)} = \frac{1}{s^2+1} \quad (\because L = 1H, C = 1F).$$



$0 + j(1)$

↓ Real part ↘ Img part

Time const $= \left|\frac{-1}{0}\right| = \infty$.

freq of oscillation = 1 rad/s.

$\frac{-1}{\text{Real part}}$ = time const

Img part = freq of oscillation.

Non-repeated poles on $j\omega$ axis which is marginal stable.

Note :-

In complex conjugate poles the real part gives the system time constant & imaginary part gives the freq of oscillation.

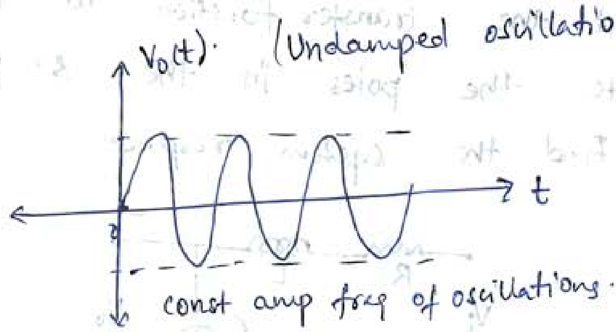
Now system respons.

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2+1}$$

$V_i(s) = 1$

$$V_o(s) = \frac{1}{s^2+1}.$$

$$V_o(t) = \sin t$$



Note :- whenever the poles lies on imaginary axis which are not repeated then the system response is constant amplitude & freq of oscillations ~~which~~ are called undamped oscillations. Such system is marginal stable

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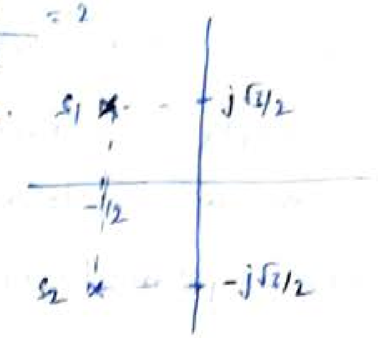
(*) Repeat the above problem by considering $R = 1\,\Omega$, $L = 1H$, $C = 1F$.

Sol. $\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC + sCR + 1}$ $\quad R = 1\Omega, \; L = 1H, \; C = 1F$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2+s+1}$$

time const $= \frac{-1}{-1/2} = 2$

freq $= \frac{\sqrt{3}}{2}$ rad/sec.



$s_1 = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$ $\quad s_2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$.

System response: $V_i(s) = 0$.

$$V_o(s) = \frac{1}{s^2+s+1}.$$

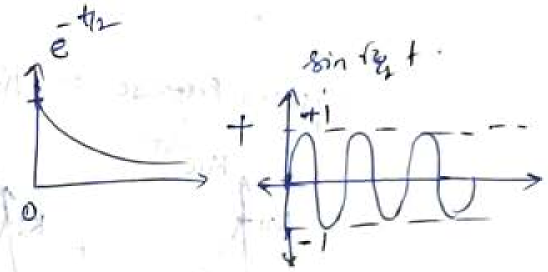
Apply I.L.T

$$v_o(t) = L^{-1}\left[\frac{1}{s^2+s+\frac{1}{4}+\frac{3}{4}}\right] = L^{-1}\left[\frac{1}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right]$$

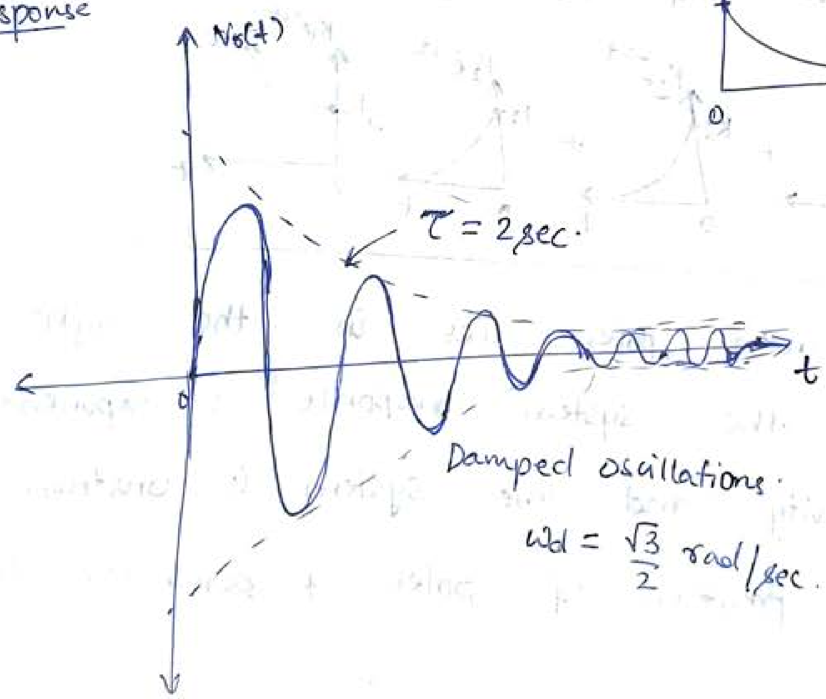
$$v_o(t) = \frac{2}{\sqrt{3}} L^{-1}\left[\frac{(\frac{\sqrt{3}}{2})}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right]$$

$$v_o(t) = \frac{2}{\sqrt{3}} e^{-t/2} \cdot \sin\frac{\sqrt{3}}{2} \cdot t$$

$$\boxed{\therefore \; V_o(t) = \frac{2}{\sqrt{3}} e^{-t/2} \sin\frac{\sqrt{3}}{2}t}$$ → damped oscillations.



Response



General response of system is $Ke^{(\text{Real part})t}\sin(\text{Img part})t$

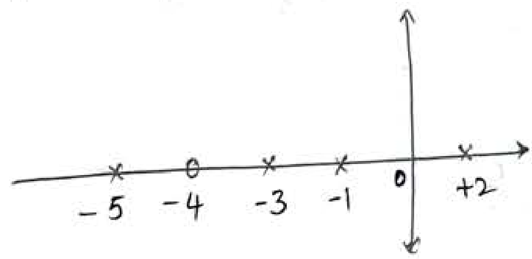
Note :-

Whenever a pole lies in the left of s plane which are complex conjugate then the system response is exponential decay freq of oscillations which are called damped oscillations and the system becomes stable.

→ Any system which produce damped oscillations is called underdamped system

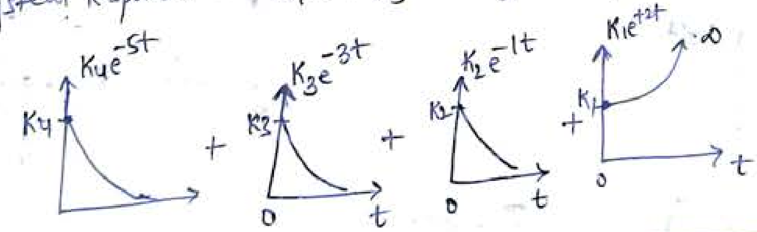
---

(*) Find the system time constant and a system response to the given poles locations in the s-plane.



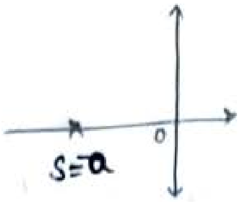
Sol. A time const is defined for only stable system. The above system is unstable. Hence time const is not defined.

$$\text{System Response} = K_4e^{-5t} + K_3e^{-3t} + K_2e^{-1t} + K_1e^{+2t}$$



(*) → If one or more poles lies in the right of s-plane then the system response is exponential rise to infinity and the system is unstable irrespective of position of poles & zeros in the left of s-plane.

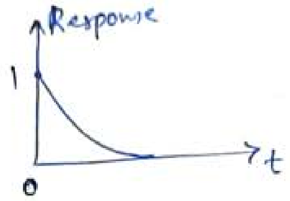
(4) Given position of the poles find T/F, Sys response. Draw the response.



Sol Transfer functions $= \frac{1}{S+a}$.

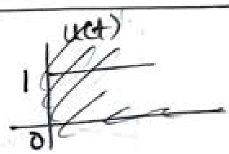
System Response $= e^{-at}$.

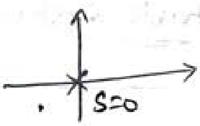
Response =



when the system response follows the input then that system is said to be stable. In the above case i/p is impulse function. Response as $t \to \infty$ the curve approaches to 0.

(10) Repeat the above problem for

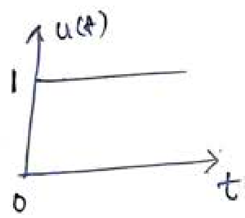




Sol. Transfer function $= \frac{1}{s}$

System Response $= 1\,u(t)$.

Response. =

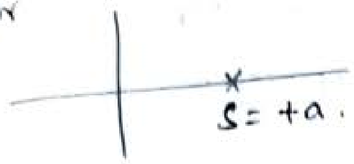


Input $= \delta(t)$
Resp o/p $= u(t)$

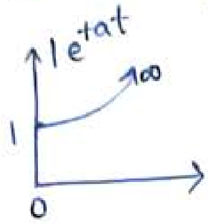
Response does not follow the input. So the system is unstable. i.e as $t \to \infty$ resp = const = 1.

Doubt.

for

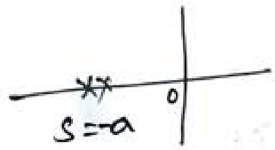
→ 

Transfer function $= \frac{1}{s-a}$

Response = 

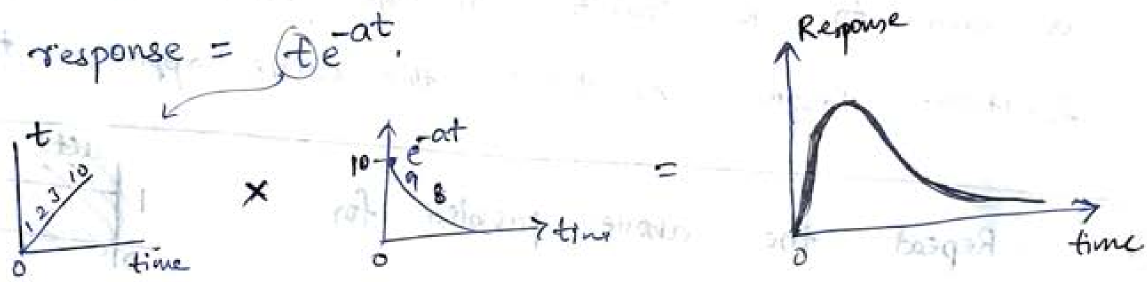
system response is away from i/p so unstable.

→ for pole $s=-a$ repeated two times.



Sol. Transfer function $= \frac{1}{(s+a)^2}$.

System response $= t e^{-at}$.

Sys. Resp = 

stable system

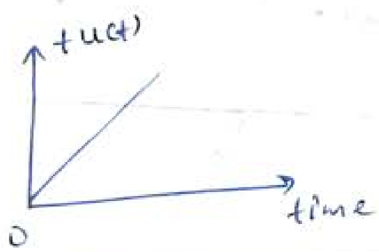
→ for pole $s=0$ repeated 2 times.

Repeated poles on $\pm j\omega$ axis.

sol. 

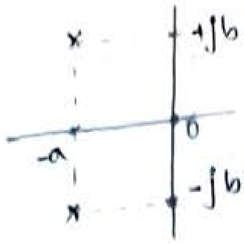
Transfer function $= \frac{1}{s^2}$

System response $= t\,u(t)$



(unstable)

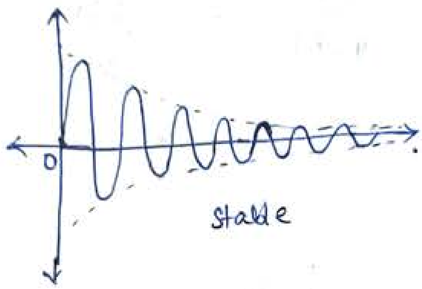
→ for



Sol: T.F = $\frac{1}{(s+a)^2+b^2}$

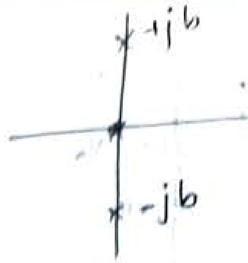
System Res = $\frac{1}{b}e^{-at}\sin bt$

Response



Stable

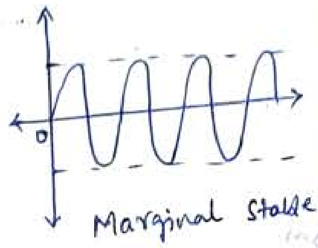
for



T.F = $\frac{1}{s^2+b^2}$

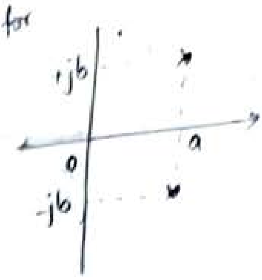
Sys. Res = $\frac{1}{b}\sin bt$

Response



Marginal Stable

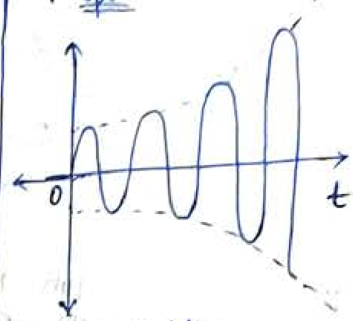
for



T.F = $\frac{1}{(s-a)^2+b^2}$

Sys. Res = $\frac{1}{b}e^{at}\sin bt$

Response.

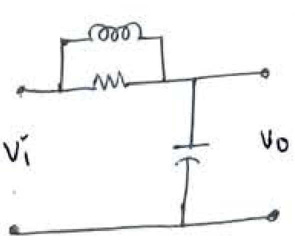


unstable.

Summary :-

S-plane:	System response:
Real part	exponential term.
Imag. part	sin or cosine term.
(Real + Img part)	product of (exp term) & (sin or cosine term)
Repeated Real	product of 't' & exp. term.

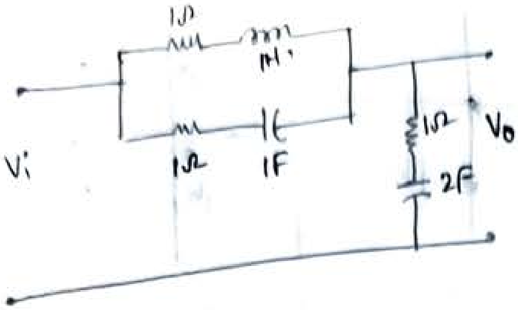
(*) Find the transfer function to the given electrical networks.



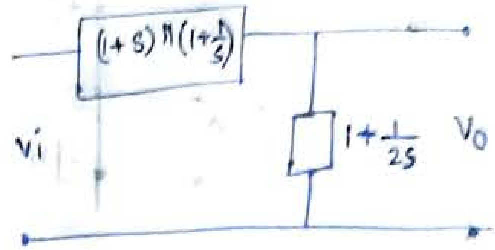
$$\frac{V_o}{V_i} = \frac{\frac{1}{sC}}{\frac{1}{sC}+\frac{sRL}{R+sL}}$$

$$\frac{V_o}{V_i} = \frac{R+sL}{R+sL+s^2RCL} = \frac{R+sL}{s^2RCL+sL+R}$$

(ii)



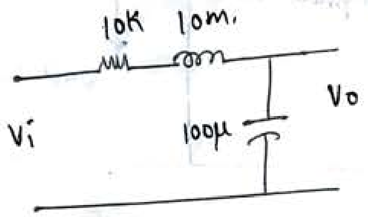
⇒



$$\frac{V_o}{V_i} = \frac{1+\frac{1}{2s}}{\left(1+\frac{1}{2s}\right)+\frac{(1+s)(1+\frac{1}{s})}{(1+s+1+\frac{1}{s})}} = \frac{\left(\frac{2s+1}{2s}\right)}{\cancel{1+}\left(\frac{2s+1}{2s}\right)+\frac{(1+\frac{1}{s}+s+1)}{(1+1+s+\frac{1}{s})}}$$

$$\frac{V_o}{V_i} = \frac{\left(\frac{2s+1}{2s}\right)}{\frac{2s+1}{2s}+1} = \frac{2s+1}{2s}\times\frac{2s}{4s+1} = \frac{2s+1}{4s+1}$$

(iii)



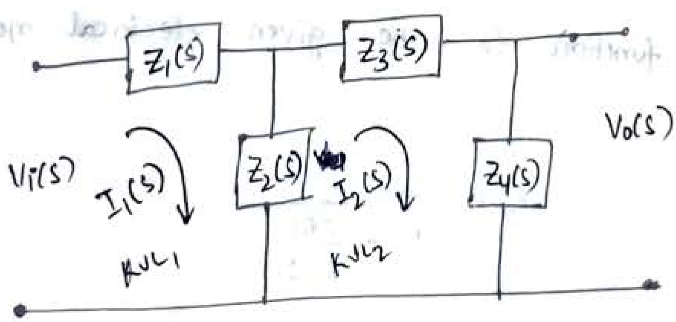
$$\frac{V_o}{V_i} = \frac{\frac{1}{s\times10^{-4}}}{\frac{1}{s\times10^{-4}}+10^{4}+s\times10^{-2}}$$

$$\frac{V_o}{V_i} = \frac{1}{(s\times10^{-4})}\times\frac{(s\times10^{-4})}{s^2\times10^{-6}+s+1}$$

$$\frac{V_o}{V_i} = \frac{1}{(s^2\times10^{-6})+s+1} = \frac{10^{6}}{s^2+10^{6}s+10^{6}}$$

$$\therefore \frac{V_o}{V_i} = \frac{10^{6}}{s^2+10^{6}s+10^{6}}.$$

(*) find the transfer function of



$$V_i(s) = \frac{V_o}{V_i} = \frac{V_o}{V_{o1}} \times \frac{V_{o1}}{V_i}$$

Apply KVL-1

$$V_i(s) = I_1(s)\left[Z_1(s) + Z_2(s)\right] - I_2 Z_1(s) \quad \text{--- (1)}$$

Apply KVL-2

$$0 = -I_1 Z_2 + I_2(Z_2 + Z_3 + Z_4) \quad \text{--- (2)}$$

$$V_o = I_2 Z_4 \quad \text{--- (3)}$$

$$I_1 = \frac{(Z_2 + Z_3 + Z_4)\, I_2}{Z_2}$$

$$V_i(s) = \left[\frac{(Z_2 + Z_3 + Z_4)(Z_1 + Z_2)}{Z_2} - Z_2\right] I_2$$

$$V_i(s) = \left[\frac{Z_1Z_2 + Z_1Z_3 + Z_1Z_4 + \cancel{Z_2^2} + Z_2Z_3 + Z_2Z_4 - \cancel{Z_2^2}}{Z_2}\right] I_2$$

$$V_i(s) = \left[\frac{Z_1Z_2 + Z_1Z_3 + Z_1Z_4 + Z_2Z_3 + Z_2Z_4}{Z_2}\right] I_2 \quad \text{--- (4)}$$

from (3) & (4)

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_2 Z_4}{Z_1Z_2 + Z_1Z_3 + Z_1Z_4 + Z_2Z_3 + Z_2Z_4}}$$

Aliter:-

$$\begin{bmatrix} V_i \\ 0 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} I_1 & \qquad I_2 \end{array} \\ \begin{bmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 + Z_4 \end{bmatrix} \end{array} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Using cramer's rule

$$I_2 = \frac{\begin{vmatrix} Z_1 + Z_2 & V_i \\ -Z_2 & 0 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 + Z_4 \end{vmatrix}}$$

$$I_2 = \frac{Z_2 V_i}{(Z_1+Z_2)(Z_2+Z_3+Z_4) - Z_2^2}$$

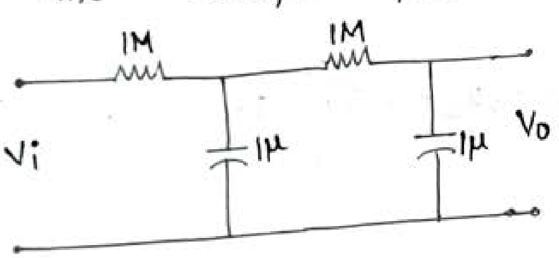
$$I_2 = \frac{Z_2 V_i}{Z_1Z_2 + Z_1Z_3 + Z_1Z_4 + Z_2Z_3 + Z_2Z_4} \quad \text{——} \quad (4)$$

$$\boxed{\frac{V_o}{V_i} = \frac{Z_2 Z_4}{Z_1Z_2 + Z_1Z_3 + Z_1Z_4 + Z_2Z_3 + Z_2Z_4}}$$

Short cut to Remember :-

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_2 Z_4}{Z_1(Z_2+Z_3+Z_4) + Z_2(Z_3+Z_4)}}$$

(*) Find the transfer function.



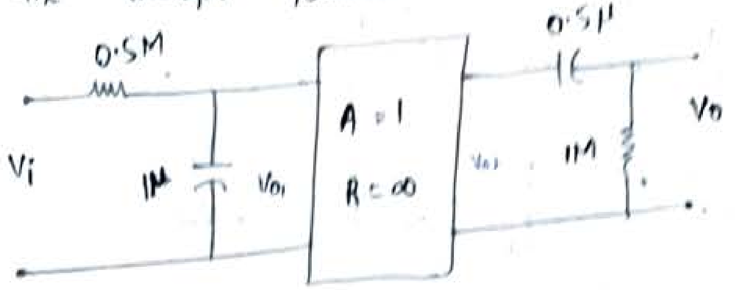
Sol.

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{s^2} \times 10^{+12}}{1M\left(\frac{1}{s\times10^{-6}} + 1M + \frac{1}{s\times10^{-6}}\right) + \frac{1}{s1\mu}\left(1M + \frac{1}{s\times10^{-6}}\right)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{10^{12}}{s^2}}{\frac{10^{12}}{s} + 10^{12} + \frac{10^{12}}{s} + \frac{10^{12}}{s} + \frac{10^{12}}{s^2}}$$

$$= \frac{\frac{1}{s^2}}{\frac{1}{s} + \frac{1}{s} + \frac{1}{s} + \frac{1}{s^2} + 1} = \frac{\frac{1}{s^2}}{\frac{3}{s} + \frac{1}{s^2} + 1} = \frac{1}{s^2+3s+1}$$

(*) Find the transfer function.



Sol

$$\frac{V_o}{V_i} = \frac{V_o}{V_{o2}} \times \frac{V_{o1}}{V_i} \times \frac{V_{o2}}{V_{o1}} = \frac{V_o}{V_{o2}} \times \frac{V_{o1}}{V_i} \quad (\because A = 1)$$

$$\frac{V_{o1}}{V_i} = \frac{\frac{1}{s \cdot 1\mu}}{\frac{1}{s \cdot 1\mu} + 0.5M} \qquad \frac{V_o}{V_{o2}} = \frac{1M}{1M + \frac{1}{s \times 0.5\mu}}$$

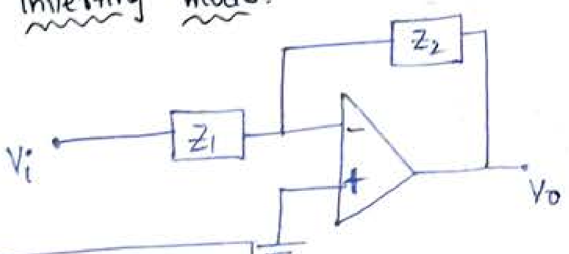
$$\frac{V_{o1}}{V_i} = \frac{1}{1 + 0.5s} \qquad \frac{V_o}{V_{o2}} = \frac{0.5s \times \cancel{10^{12}}}{s \times 0.5 + 1}$$

$$\frac{V_{o1}}{V_i} = \frac{2}{s+2} \qquad \frac{V_o}{V_{o2}} = \frac{s \times \cancel{10^{12}}}{s+2}$$

$$\boxed{\frac{V_o}{V_i} = \frac{2s}{(s+2)^2}}$$

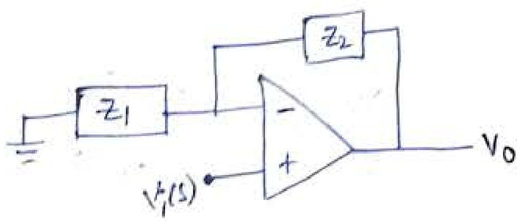
Transfer function to the op-amps:-

Inverting mode:



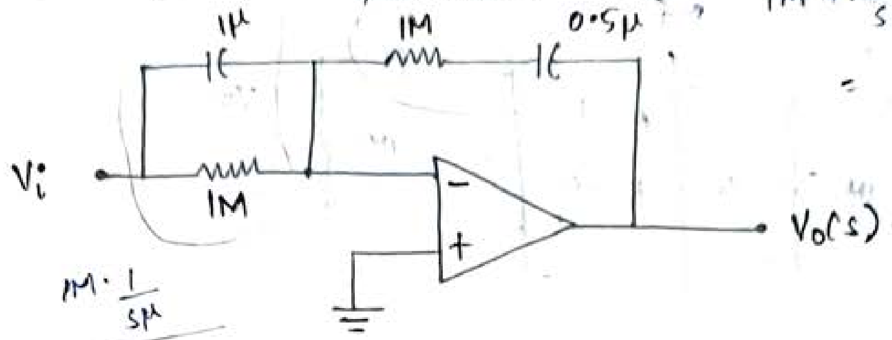
$$\boxed{\frac{V_o(s)}{V_i(s)} = -\frac{Z_2}{Z_1}}$$

Non inverting mode



$$\boxed{\frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2(s)}{Z_1(s)}}$$

(*) Find the transfer function.



$1M + \frac{1}{s \cdot 0.5\mu}$

$= \left(\frac{s0.5+1}{s0.5\mu}\right)$.

Sol.

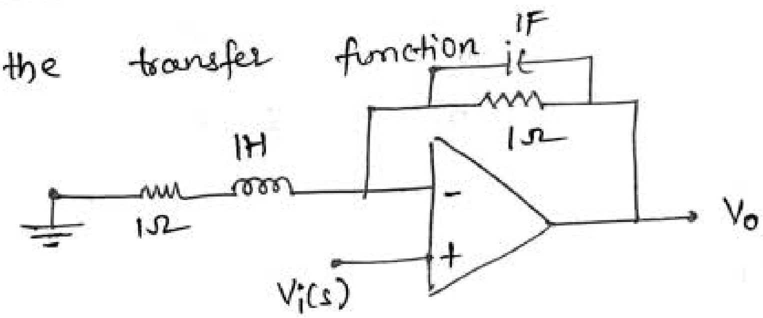
$$\frac{1M \cdot \frac{1}{s\mu}}{1M + \left(\frac{1}{s\mu}\right)}$$

$$= \left(\frac{1M}{s+1}\right)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{-\left[\frac{(0.5s)+1}{s \times (0.5\mu)}\right]}{\left[\frac{1M}{s+1}\right]}.$$

$$\frac{V_o(s)}{V_i(s)} = \frac{-(s+1)(0.5s+1)}{0.5s} = \frac{-(s+1)(s+2)}{s}.$$

(*) Find the transfer function



Sol

$$\frac{V_o}{V_i} = 1 + \frac{Z_2}{Z_1} = 1 + \left[\frac{\left(\frac{1\Omega \times \frac{1}{s}}{1+\frac{1}{s}}\right)}{1+s}\right] = 1 + \left(\frac{1}{s+1}\right) \times \frac{1}{(s+1)}.$$

$$\frac{V_o}{V_i} = \frac{(s+1)^2+1}{(s+1)^2} = \frac{s^2+2s+2}{s^2+2s+1}.$$

Transfer function to the differential equations:

(*) Write the transfer function to the following system where 'x' is input 'y' is output.

$$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y = 2\frac{dx}{dt} + x(t-\tau).$$

Sol Apply laplace.

$$s^3 Y(s) + 5s^2 Y(s) + 7s\,Y(s) + 10Y(s) = 2sX(s) + X(s)e^{-s\tau}.$$

$$Y(s)\left[s^3 + 5s^2 + 7s + 10\right] = X(s)\left[2s + e^{-s\tau}\right]$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{2s + e^{-s\tau}}{s^3 + 5s^2 + 7s + 10}}$$

(*) $$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10 = 2\frac{dx}{dt} + x.$$

Sol $$s^3 Y(s) + 2s^2 Y(s) + 7s\,Y(s) + \frac{10}{s} = 2s\,X(s) + X(s).$$

The given eq-n is a non-linear eq-n. The transfer function is not defined.

(*) Write the differential eq-n to the given transfer function

$$\frac{Y(s)}{X(s)} = \frac{2s+3}{s^2+5s+6}.$$

Sol $$s^2 Y(s) + 5s\,Y(s) + 6Y(s) = 2s\,X(s) + 3X(s)$$

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy}{dt} + 6y(t) = 2\frac{dx(t)}{dt} + 3x(t)$$

Transfer function to the signal response :

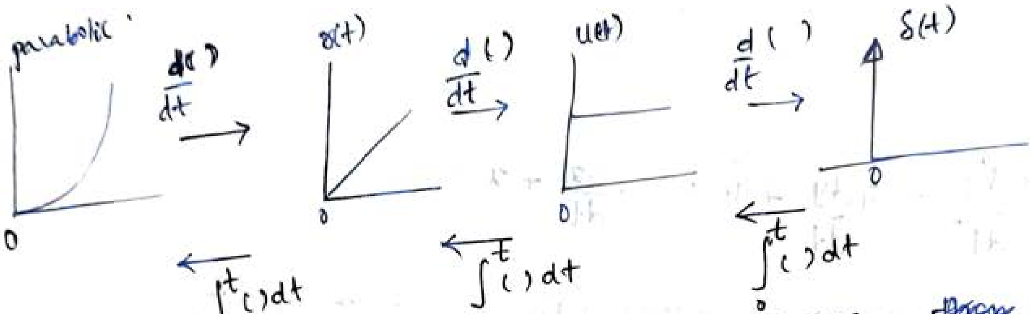
$$T.F = \frac{L[o/p]}{L[i/p]} \quad \Big| \quad I_i \text{ (Initial cond)} = 0$$

$$T.F = \frac{\mathcal{L}[\text{Unit Ramp Response}]}{\mathcal{L}[\text{Unit Ramp}]}.$$

Always to get a transfer function from signal response use c

$$T.F = \frac{L[o/p]}{L[i/p]}$$ with initial conditions are zero.

Conversion of Responses:



To get a T/F from the signal response ~~~~ use the formula.

$$T.F = \frac{L[o/p]}{L[i/p]}$$

---

(7) The unit step response of the system is $y(t) = \left(\frac{5}{2} - \frac{5}{2}e^{-2t} + 5t\right)$. The transfer function is

Sol:

$$\text{Transfer function} = \frac{L\left[\frac{5}{2} - \frac{5}{2}e^{-2t} + 5t\right]}{L[u(t)]}.$$

$$= \frac{\frac{5}{2s} - \frac{5}{2(s+2)} + \frac{5}{s^2}}{\frac{1}{s}}$$

$$= \frac{\frac{5s(s+2) - 5s^2 + 5\cancel{2}(s+2)}{2s^{\cancel{2}}(s+2)}}{\frac{1}{\cancel{s}}}$$

$$= \frac{5s^2 + 10s - 5s^2 + 10s + 20}{2s^2 + 4s}$$

$$T/F = \frac{15s + 10 + 5s + 10}{2s^2 + 4s} = \frac{20s + 20}{2s^2 + 4s} = \frac{10s + 10}{s^2 + 2s}$$

$$\boxed{\therefore \text{Transfer functn} = \frac{10s+10}{s^2+2s}}$$

* The impulse response of the system $c(t) = (-4e^{-t} + 6e^{-2t})$.
The eq. step response of the system is

Sol. step response $= \int (\text{impulse response})$

$$= \int_0^t \left(-4e^{-t} + 6e^{-2t}\right) dt = \left(+4e^{-t} - \frac{6e^{-2t}}{2}\right)_0^t$$

$$= \left[\left(4e^{-t} - 3e^{-2t}\right) - (4-3)\right]$$

$$\text{Step response} = \left[4e^{-t} - 3e^{-2t} - 1\right]$$

Sensitivity with respect to Noise / disturbance / Environmental conditions:

The system sensitivity is used to describe the relative variations in the output due to the variations in system parameters like $G(s)$, $H(s)$

→ Sensitivity of the transfer function w.r.t to variations in $G(s)$ is denoted by S_G^T

$$S_G^T = \frac{\text{\% of change in transfer function}}{\text{\% of change in } G(s)} = \frac{\partial T/T}{\partial G/G}$$

$$S_G^T = \left(\frac{G}{T}\right)\left(\frac{\partial T}{\partial G}\right).$$

$$\text{III}^{ly} \quad S_H^T = \left(\frac{H}{T}\right)\left(\frac{\partial T}{\partial H}\right)$$

④ find the sensitivity of open loop and closed loop system w.r.t variations (i) G(s) (ii) H(s).

Sol. Open loop system: (T.F = T = G)

$$S_G^T = \left(\frac{G}{T}\right)\left(\frac{\partial T}{\partial G}\right) = \left(\frac{G}{G}\right)(1) = 1$$

$$\boxed{S_G^T = 1}$$

Closed loop system : $\left(T.F = T = \frac{G}{1+GH} \right)$

$$S_G^T = \left(\frac{G}{T}\right) \times \left(\frac{\partial T}{\partial G}\right)$$

$$T = \frac{G}{1+GH} \Rightarrow \frac{G}{T} = (1+GH)$$

$$\frac{\partial T}{\partial G} = \frac{(1+GH) - G(H)}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

$$S_G^T = (1+GH) \times \frac{1}{(1+GH)^2}$$

$$\boxed{S_G^T = \frac{1}{1+GH}}$$

Note:- Open loop system is more sensitive than closed loop system

$$S_H^T = \left(\frac{H}{T}\right) \times \left(\frac{\partial T}{\partial H}\right)$$

$$T = \frac{G}{(1+GH)}$$

$$\frac{\partial T}{\partial H} = \frac{-G(G)}{(1+GH)^2} = \frac{-G^2}{(1+GH)^2}$$

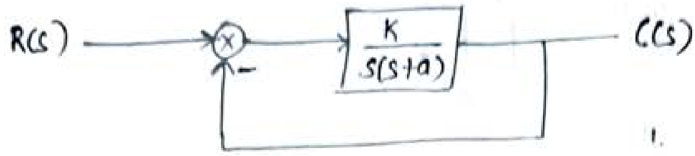
$$= \frac{H}{\left(\frac{G}{1+GH}\right)} \times \frac{-G^2}{(1+GH)^2}$$

$$\boxed{S_H^T = \frac{-GH}{(1+GH)}}$$

$$\boxed{S_H^T > S_G^T}$$

Note: Feedback network is more sensitive than forward path.

(*) Find the sensitivity of the given system with respect to variations (i) a (ii) K.



Sol:
$$\frac{C(s)}{R(s)} = \frac{K}{s(s+a)+K} = \frac{K}{s^2+as+K}$$

$$S_K^T = \frac{\partial T/T}{\partial K/K} = \left(\frac{K}{T}\right)\left(\frac{\partial T}{\partial K}\right)$$

$$S_K^T = (s^2+as+K) \times \frac{(s^2+as+K) - K}{(s^2+as+K)^2}$$

$$\boxed{S_K^T = \frac{s^2+as}{s^2+as+K}}$$

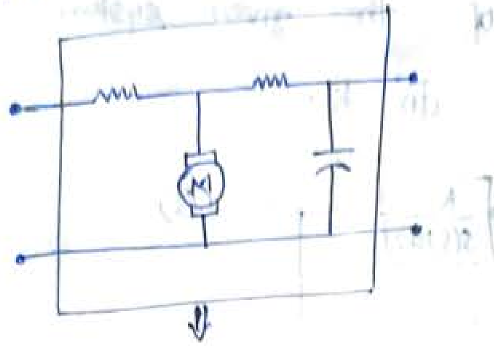
$$S_a^T = \frac{\partial T/T}{\partial a/a} = \left(\frac{a}{T}\right)\left(\frac{\partial T}{\partial a}\right) \qquad T = \frac{K}{s^2+as+K}$$

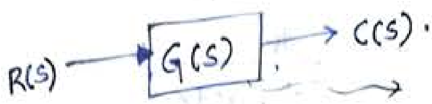
$$S_a^T = \frac{a}{\left(\frac{K}{s^2+as+K}\right)} \times \frac{-Ks}{(s^2+as+K)^2}$$

$$\boxed{S_a^T = \frac{-as}{(s^2+as+K)}}$$

Block diagrams :

Purpose: To find the overall transfer function of the system





→ short hand pictorial representation of the system.

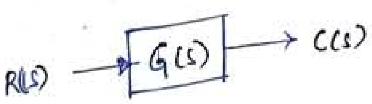
→ Block diagram is nothing but a short hand pictorial representation of the system between input and the ~~opt~~ output.

→ The systems can be represented in two forms.

(i) Open loop form

(ii) Closed loop form.

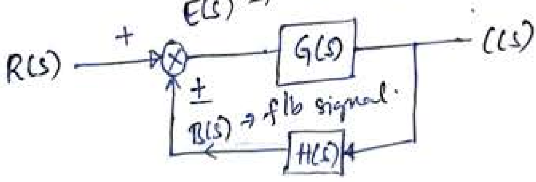
Open loop form :-



$$\boxed{OLTF = \frac{C(S)}{R(S)} = G(S)}$$

Closed loop form :-

E(S) → error signal.



→ $G(s)$ = forward path gain $= \frac{C(s)}{E(s)}$.

→ $H(s)$ = feedback path gain $= \frac{B(s)}{C(s)}$.

→ $G(s)H(s)$ = loop gain (open loop gain).

$G(s)H(s) \rightarrow$ OLTF of a non-unity FB system.

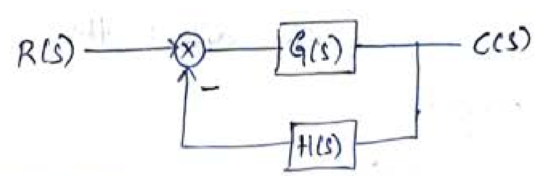
$H(s)=1 \rightarrow G(s)$ OLTF of a unity f/b system.

The factor $G(s)H(s)$ represents the actual closed loop system. It is also known as loop gain.

→ CLTF $\rightarrow$ $\boxed{\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1 \mp G(s)H(s)}}$

→ In a Practically in a +ve f/b system the phase shift b/w input and feedback signal is $(0^\circ$ or $\pm 360^\circ)$, whereas for negative feedback the phase shift b/w input and f/b signal is ($\pm 180^\circ$ or out of phase).

Comparision b/w open loop and closed loop system:

Open loop systems.	Closed loop system.
1.  $\boxed{\text{OLTF} \rightarrow \dfrac{C(s)}{R(s)} = G(s).}$	1.  $\boxed{\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}.}$
2.	2. The main disadvantage of feedback is gain is decreased with $(1+GH)$ factor.
2. The open loop system is more stable because there is no factor to effect the open loop stability	3. The closed loop system stability depends on loop gain. If loop gain is -1 then the closed loop system stability is effected. If the loop gain is equal to zero the the closed

Accuracy :-

→ The open loop system accuracy depends on input and process. The open loop system is less accurate.

Sensitivity.

→ The open loop system is more sensitive w.r.t disturbance, noise and environmental conditions because whatever the changes occurs in $G(s)$. They directly affect the output

Bandwidth :-

→ For any system the gain B.W product must be constant. With feedback the gain is decreased by the factor of $[1+G(s)H(s)]$ that means the bandwidth must be increased by $[1+G(s)H(s)]$. The B.W in a control system represents the speed of the response. Large bandwidth gives the very quick response.

loop system stability is equal to open loop system stability. If loop gain greater than 0 then the closed loop system becomes more stable than open loop system.

→ The closed loop system Accuracy depends on the feedback network ratio. If the feedback network gives the stable value then the closed loop system becomes more accurate than open loop system.

→ With feedback the system becomes less sensitive because the change in output decreased by the factor of $[1+G(s)H(s)]$.

Bandwidth ~~= Speed of response~~.

$$B.W = \frac{0.35}{t_r}$$

→ The closed loop system gives the very quick response compare to open loop system.

Bandwidth is inversely proportional to t_r

$$B.W = \frac{0.35}{t_r}$$

<u>Reliability (Durability)</u>

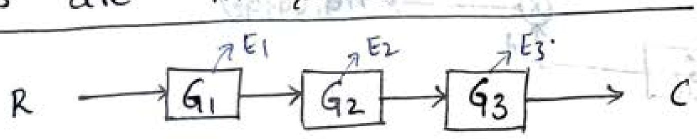
→ The reliability completely depends on no of discrete components. The open loop system is more reliable ~~because it has~~ compared to closed loop system.

→ In open loop system it is not necessary to measure the output errors are not generated. Sensors are not essential

→ Output must be measured. Sensors are essential. Errors are generated. Design is complex.

Block Diagram Reduction Techniques :

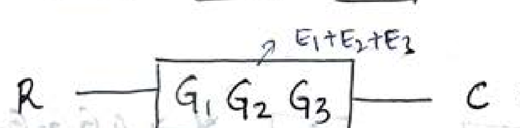
1. <u>Blocks are in series or cascade.</u>



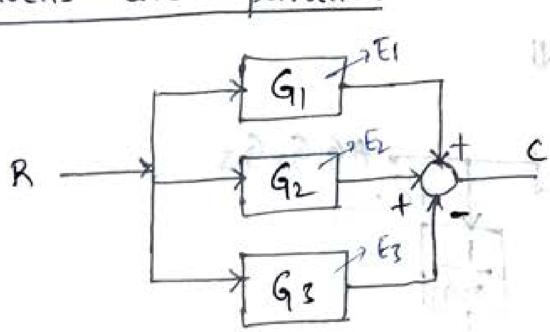
Use logarithmic scale for Error.

~~wether~~ where E_1, E_2, E_3 are errors.

⇒



2. <u>Blocks are parallel.</u>



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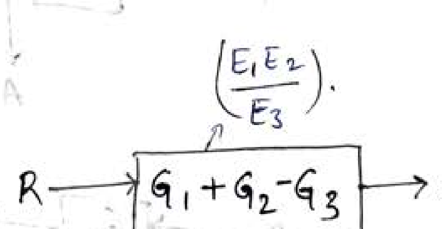
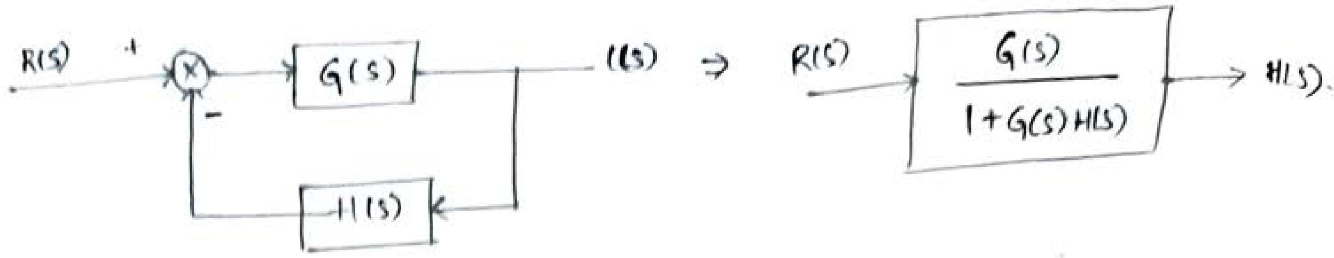
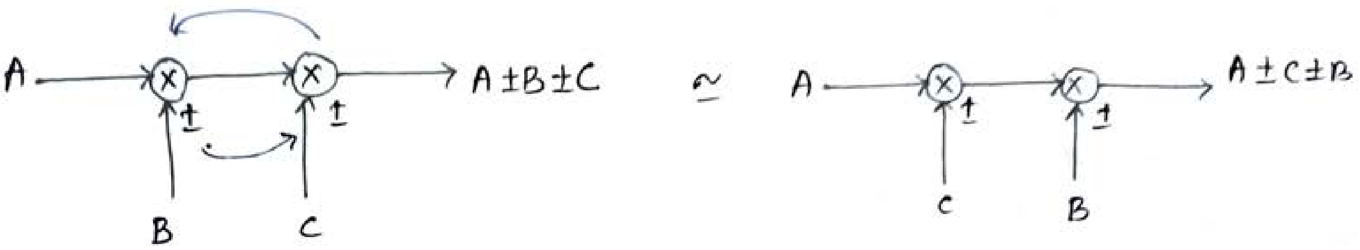


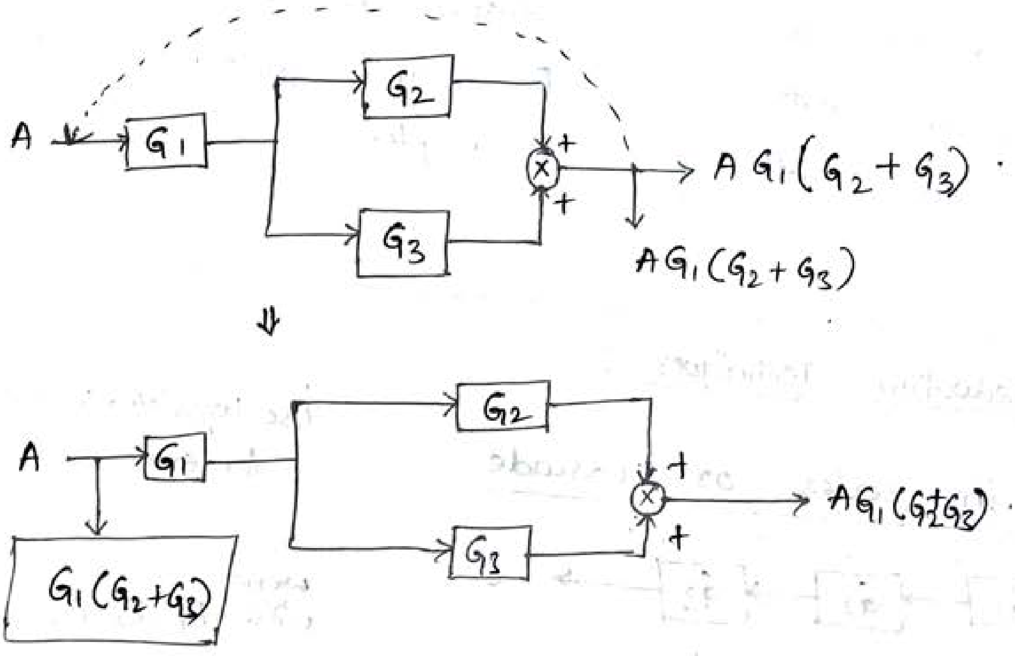
Loop



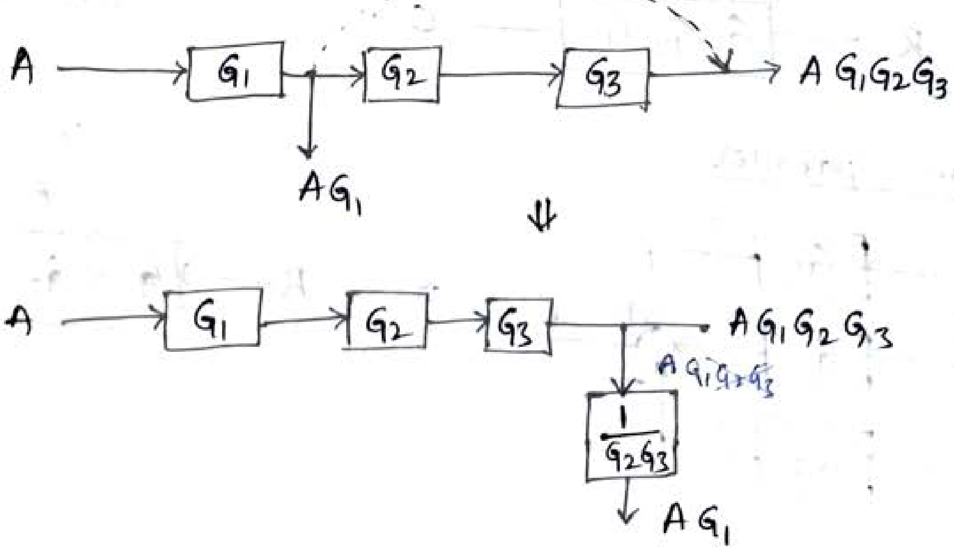
4. Adjusting the summing points :



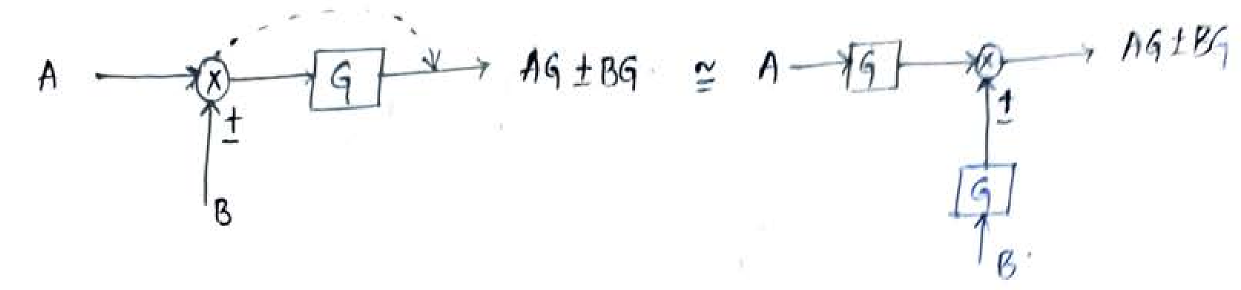
5. Adjusting block gain and take ~~off~~ up point :-

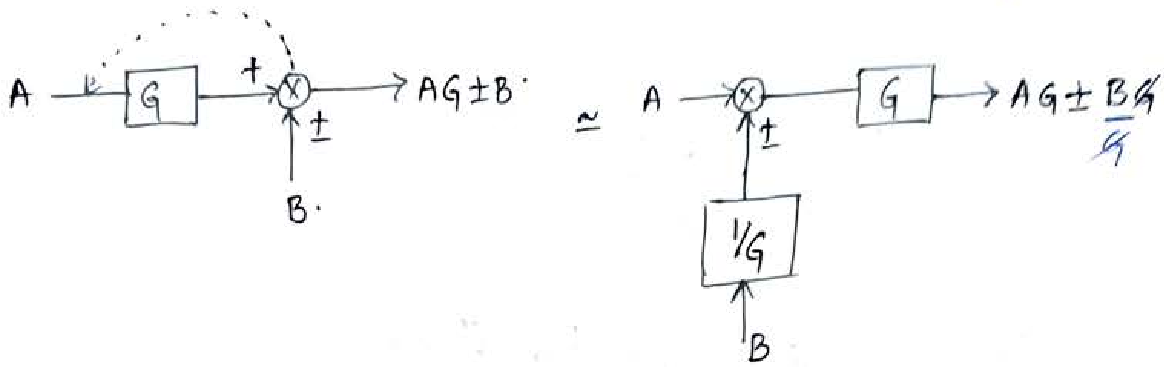


Eg:-

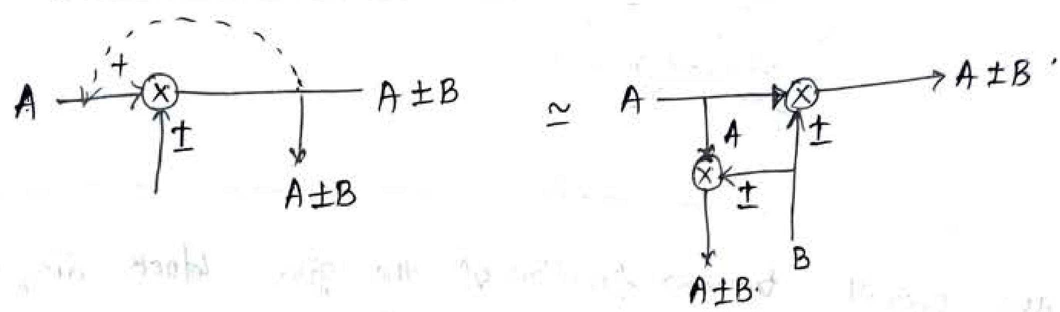


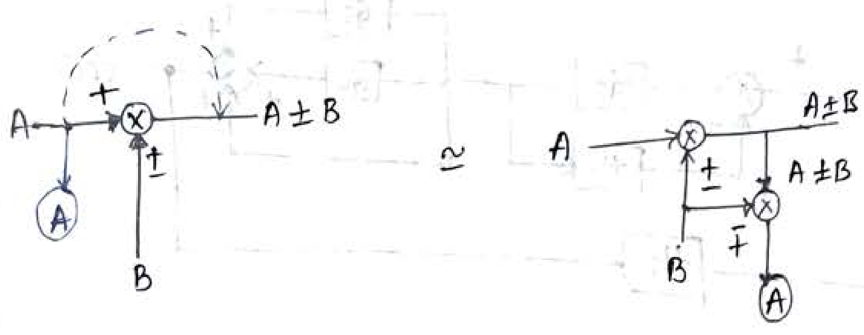
6. Adjusting block gain & summing point :-





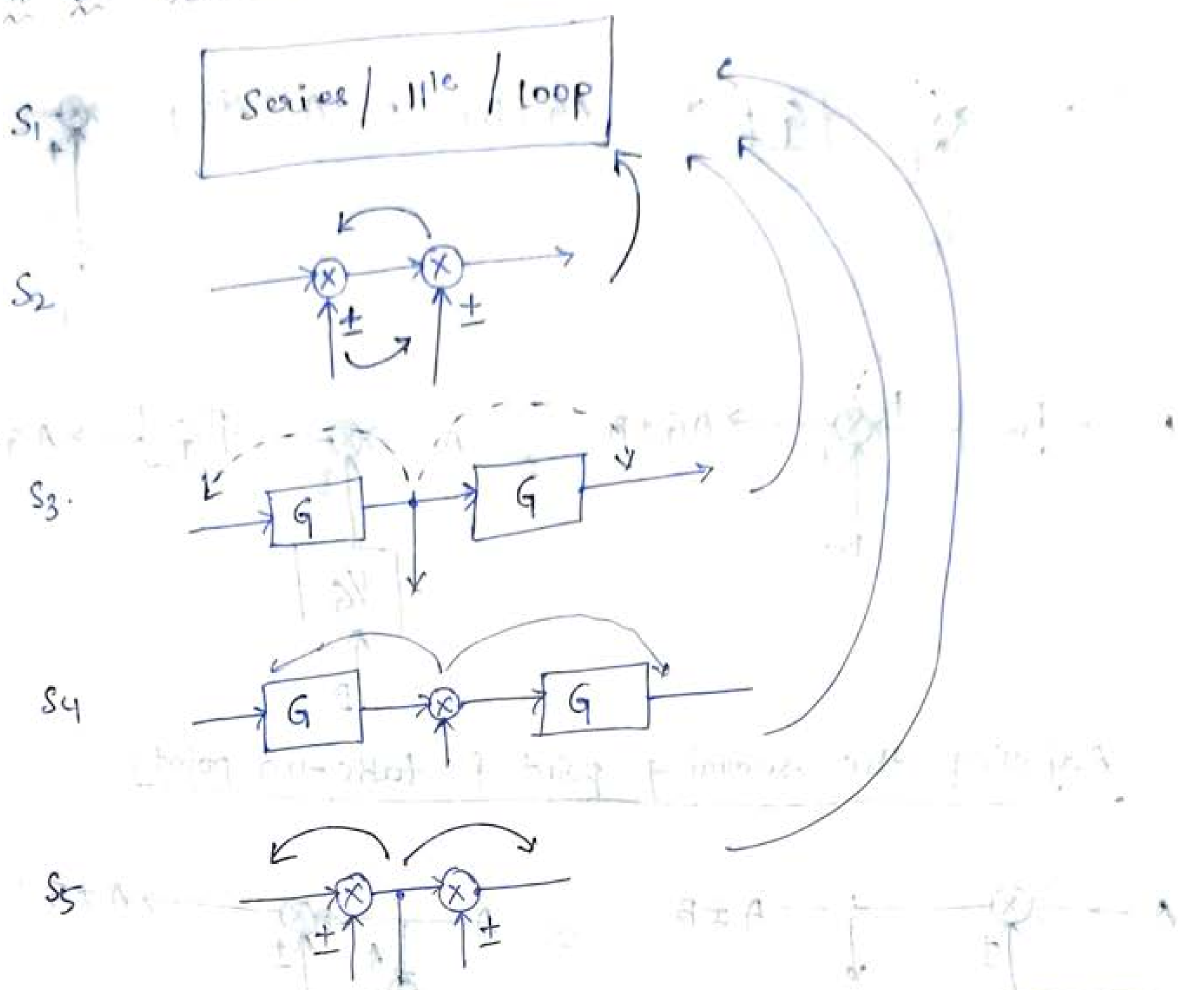
7. Adjusting the summing point & take-up point :-



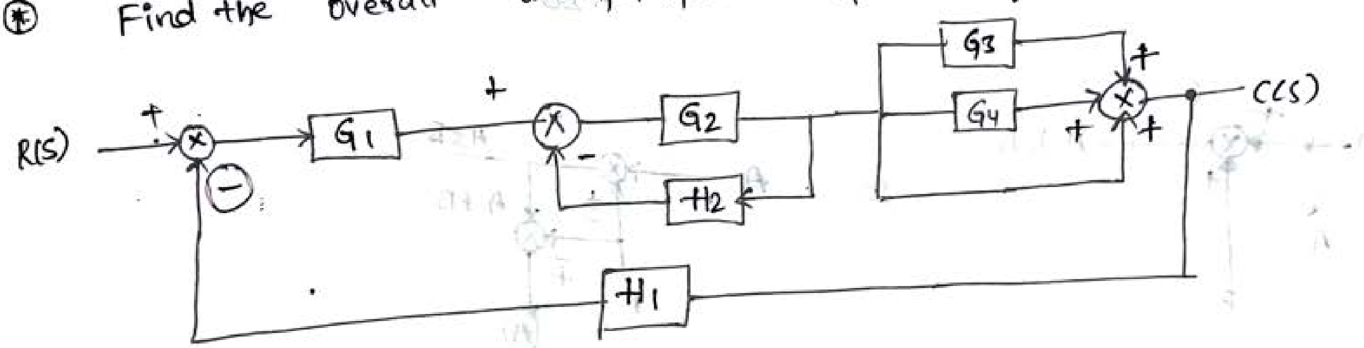


* Find the transfer ~~function of~~

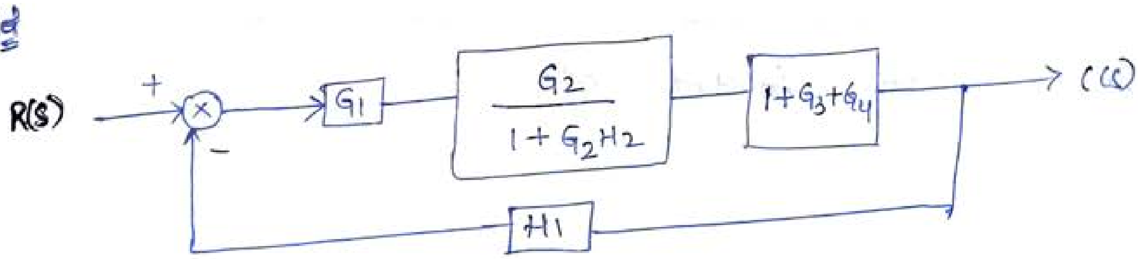
order to be followed :

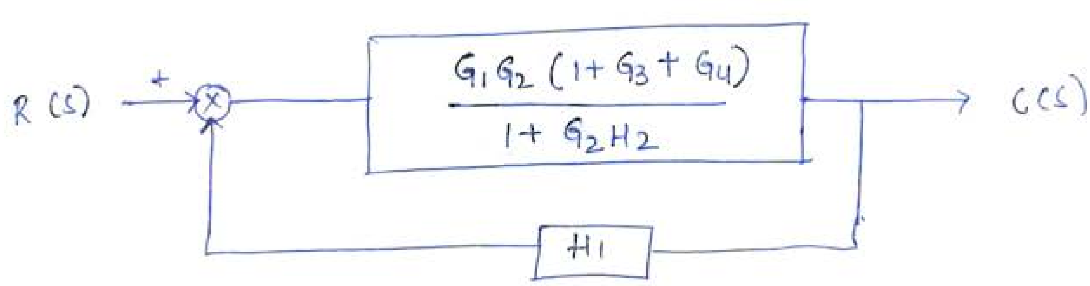


Find the overall transfer function of the give block diagram.



Sol

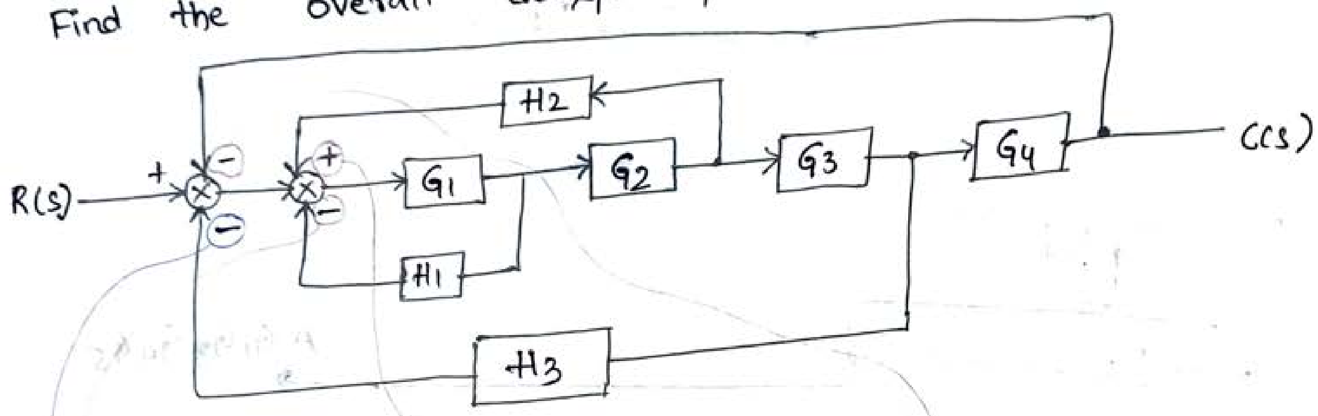




$$\frac{C(s)}{R(s)} = \frac{\left[\dfrac{G_1 G_2 (1+G_3+G_4)}{1+G_2 H_2}\right]}{1+\left[\dfrac{G_1 G_2 (1+G_3+G_4) H_1}{1+G_2 H_2}\right]}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2 (1+G_3+G_4)}{1+G_2 H_2 + G_1 G_2 (1+G_3+G_4) H_1}}$$

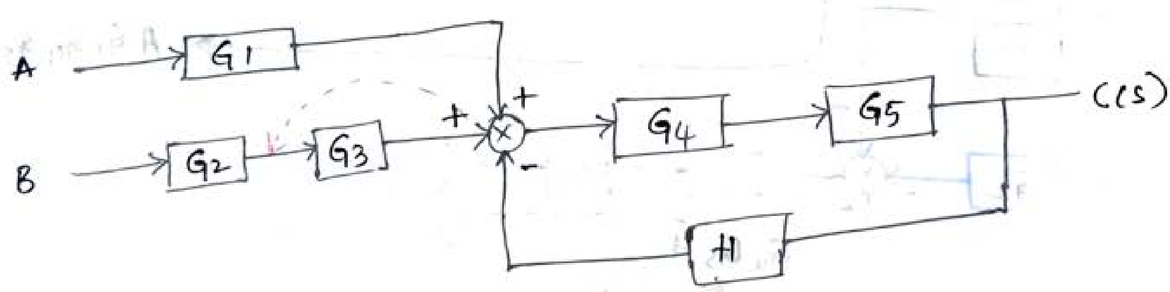
2. Find the overall transfer function of the given block diagra[m]



Sol

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1+G_1 H_1 - G_1 G_2 H_2 + G_1 G_2 G_3 H_3 + G_1 G_2 G_3 G_4 \cdot 1}$$

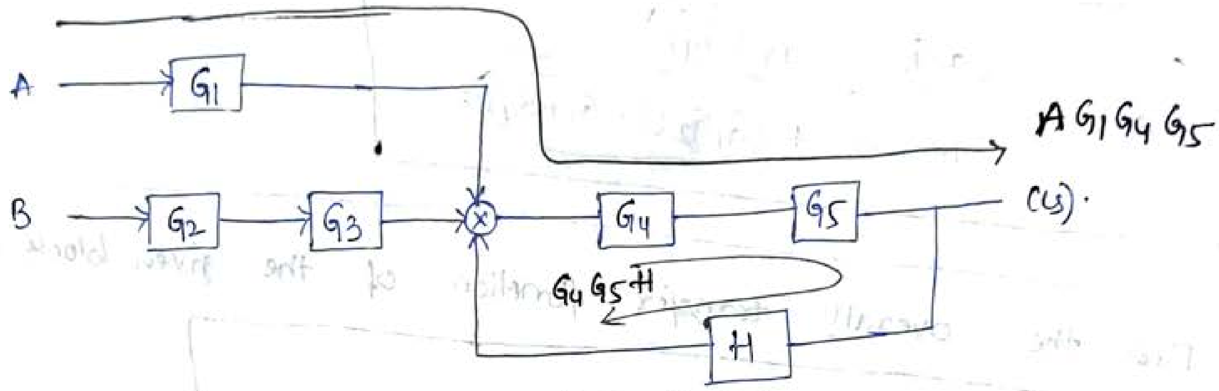
3. Draw the equivalent block diagram to the following.

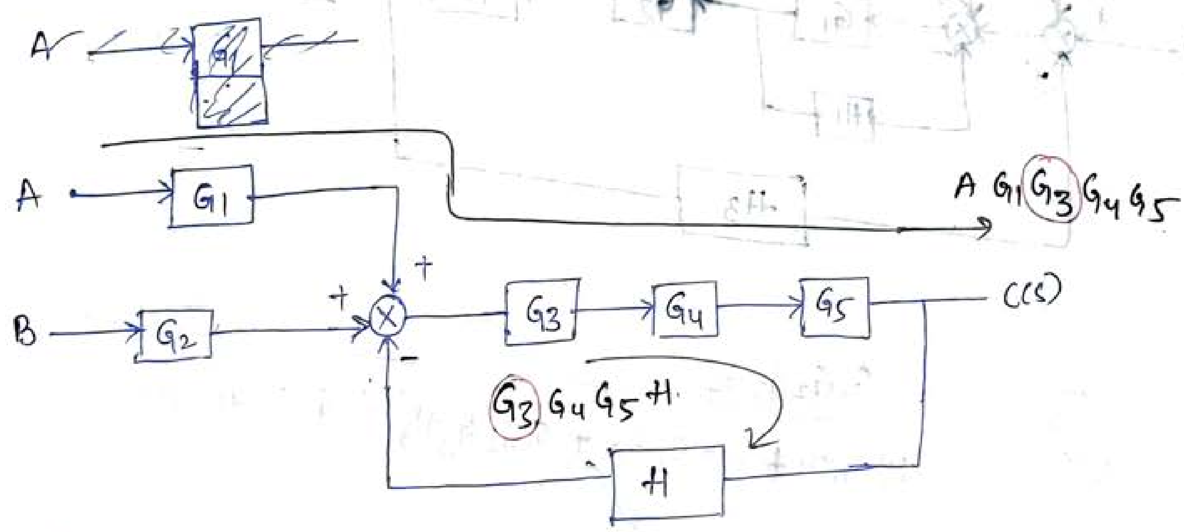


Sol While doing the shifting never change anything in the main forward path.

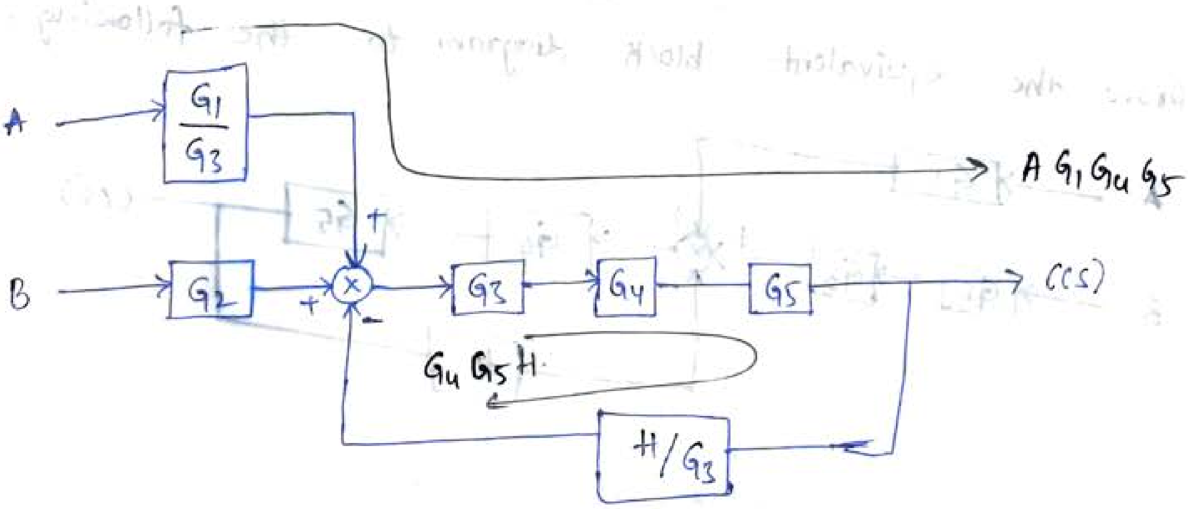
The changes

While doing the shifting operations the changes occurs only in additional forward paths and feedback paths connected to that particular point only.

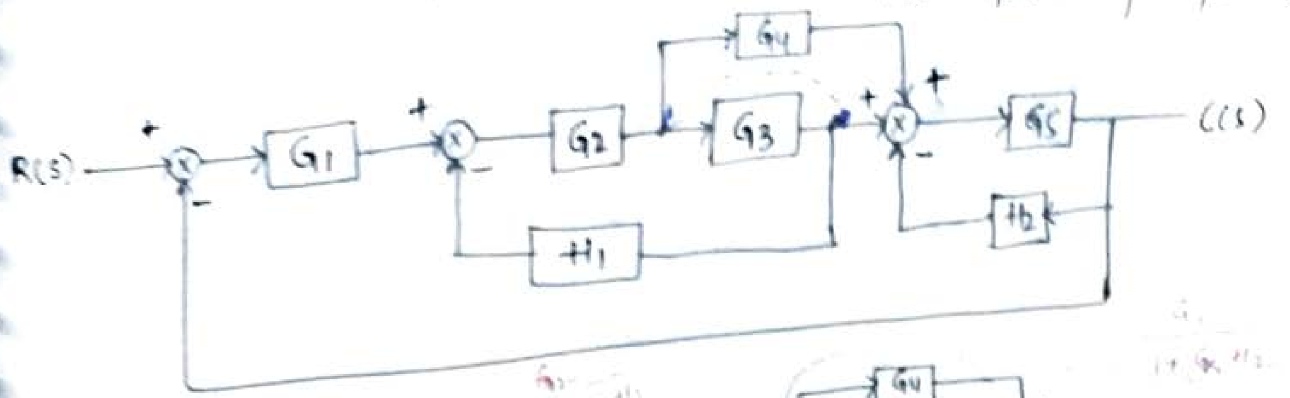




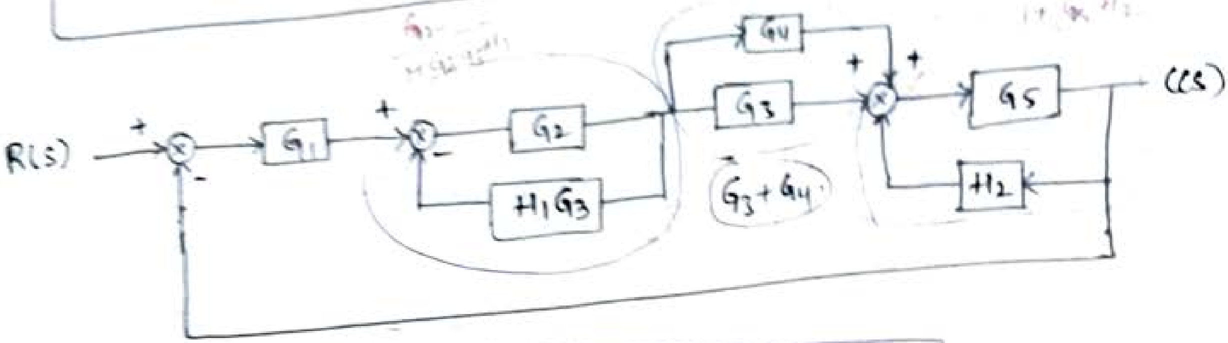
G_3 should be eliminated to satisfy final o/p as that of 1st step.



4\. Find the transfer function to the following systems

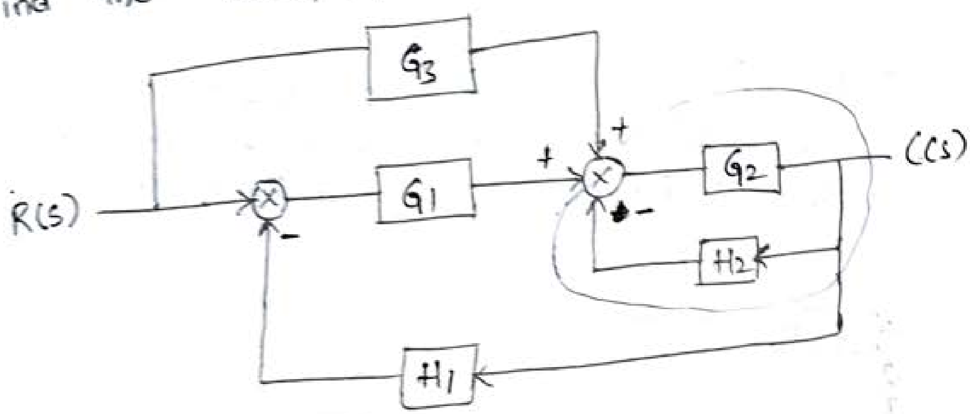


Sol:

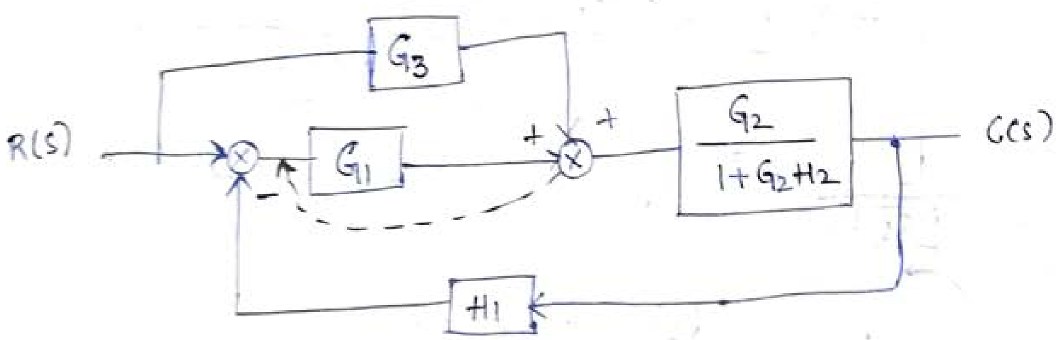


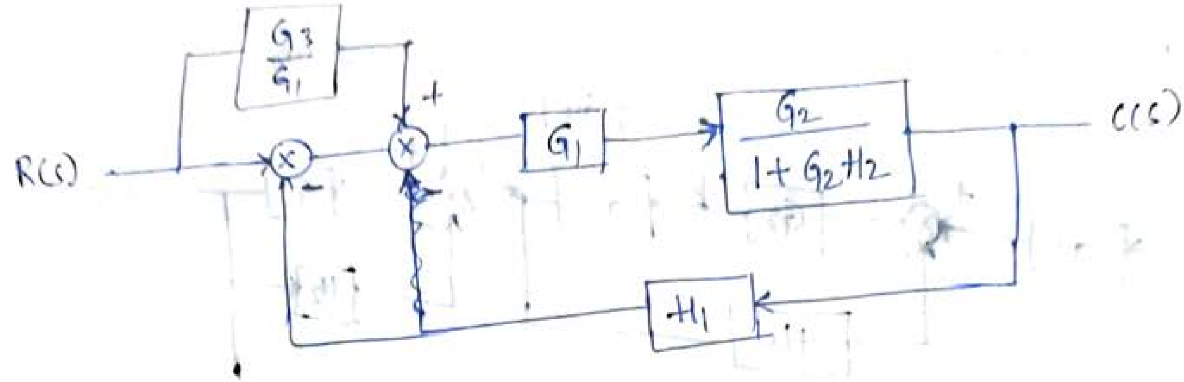
$$\boxed{\frac{C(s)}{R(s)} = \frac{(G_1 G_2 (G_3+G_4) G_5)}{(1+G_5H_2)(1+G_2G_3H_1) + (G_1G_2(G_3+G_4)G_5)}}$$

5\. Find the transfer function of the following system.

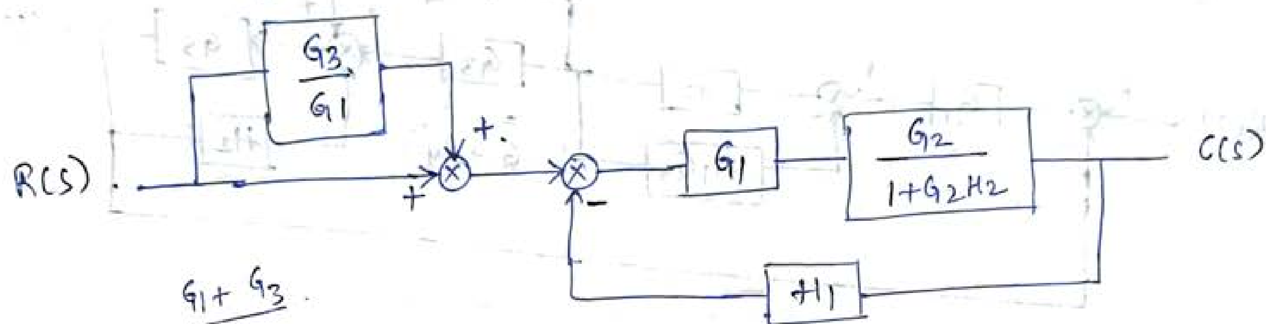


Sol:





Interchange the summing points:

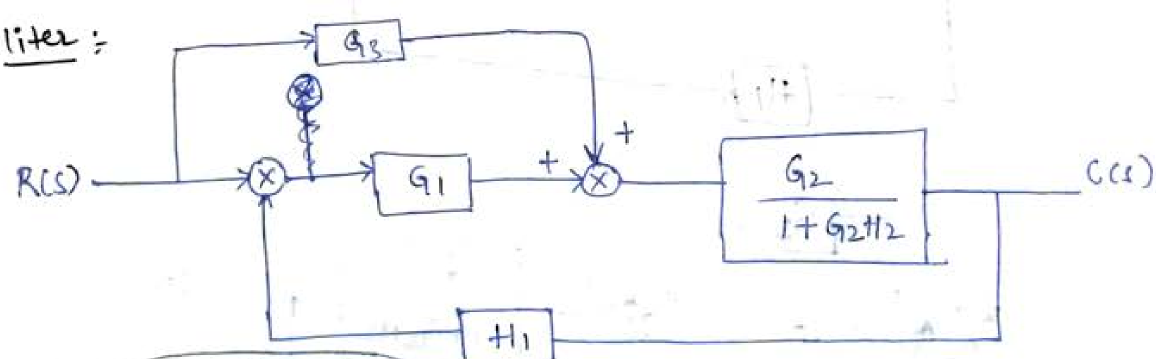


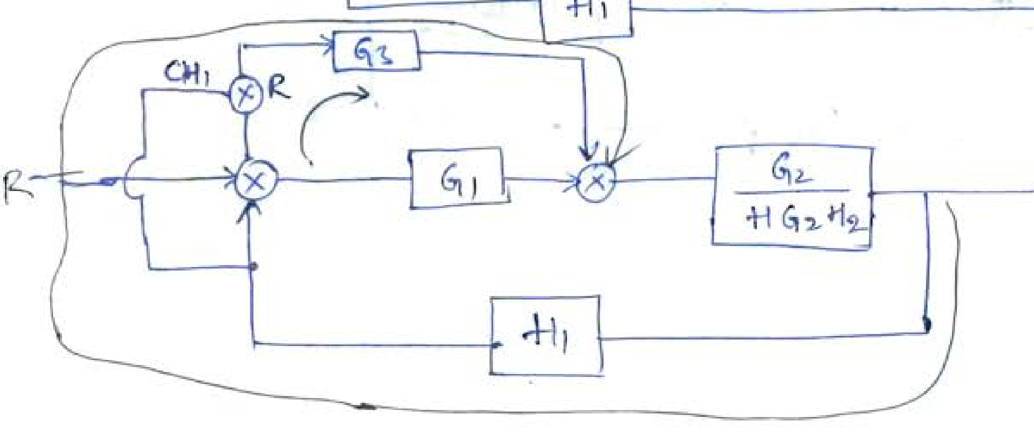
$$\frac{G_1+G_3}{G_1}$$

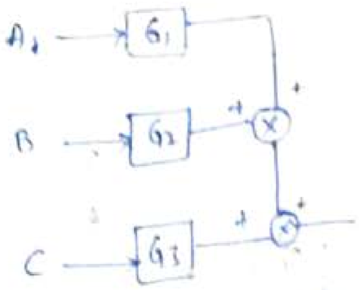
$$\frac{C(s)}{R(s)} = \left(1+\frac{G_3}{G_1}\right) \times \left[\frac{\frac{G_1G_2}{1+G_2H_2}}{1+\frac{G_1G_2H_1}{1+G_2H_2}}\right]$$

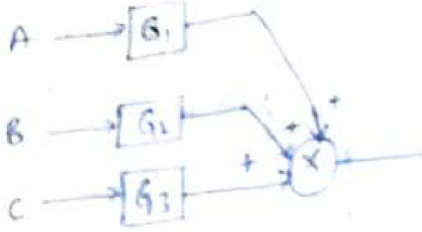
$$\frac{C(s)}{R(s)} = \frac{G_1G_2\left(\frac{G_1+G_3}{G_1}\right)}{(1+G_2H_2)+G_1G_2H_1} = \frac{G_2(G_1+G_3)}{1+G_2H_2+G_1G_2H_1}$$

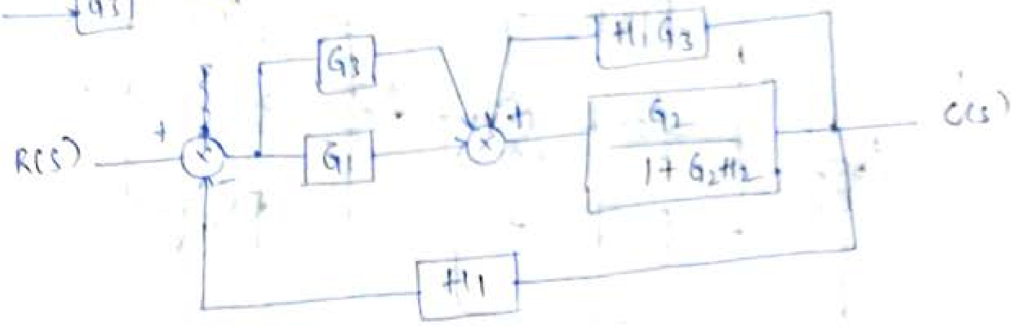
Aliter:







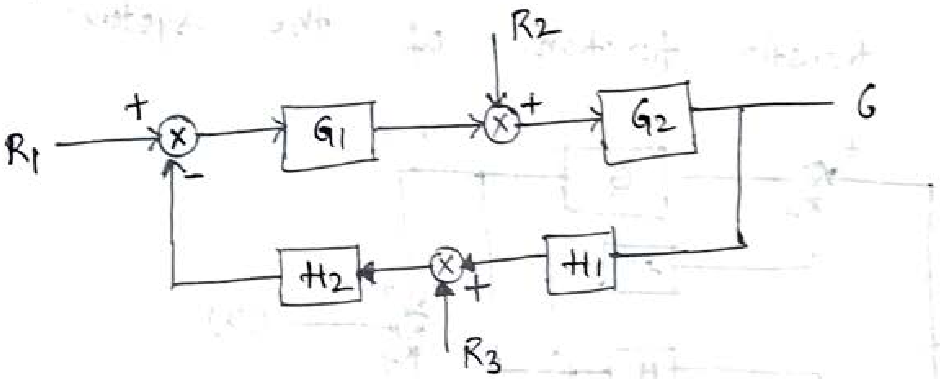




$$\frac{G_2(G_1+G_3)}{1+G_2H_2-G_2G_3H_1+G_2(G_1+G_3)H_1}$$

$$\boxed{\frac{C}{R}=\frac{G_2(G_1+G_3)}{1+G_2H_2+G_1G_2H_1}}$$

6. Find the output due to the multi input to the following system

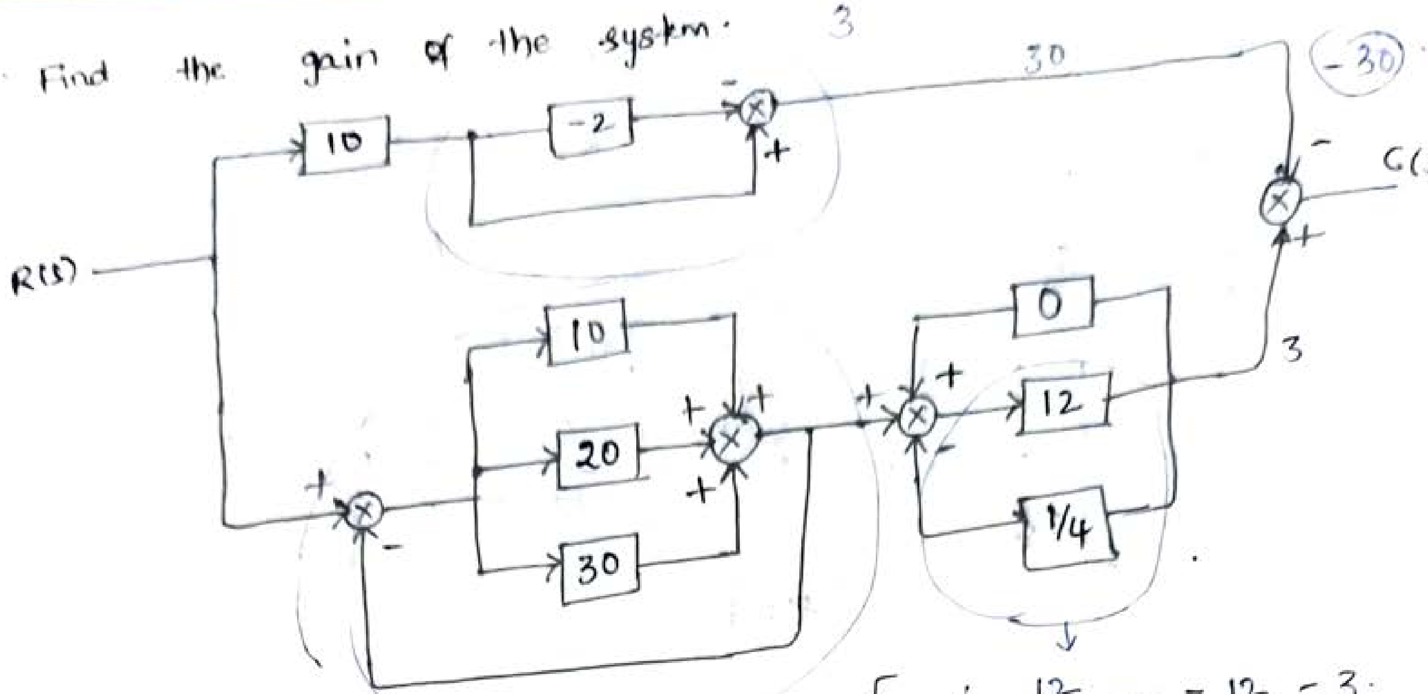


Sol. $C = C_1 + C_2 + C_3$

$$\left.\frac{C_1}{R_1}\right|_{R_2=R_3=0} = \frac{G_1G_2R_1}{1+G_1G_2H_1H_2} \qquad \left.\frac{C_3}{R_3}\right|. = \frac{-G_1G_2R_3}{1+G_1G_2H_1H_2}.$$

$$\left.\frac{C_2}{R_2}\right|_{R_1=R_3=0} = \frac{G_2R_2}{1+G_1G_2H_1H_2} \qquad C=\frac{G_1G_2R_1+G_2R_2-G_1G_2H_2R_3}{1+G_1G_2H_1H_2}.$$

7. Find the gain of the system.



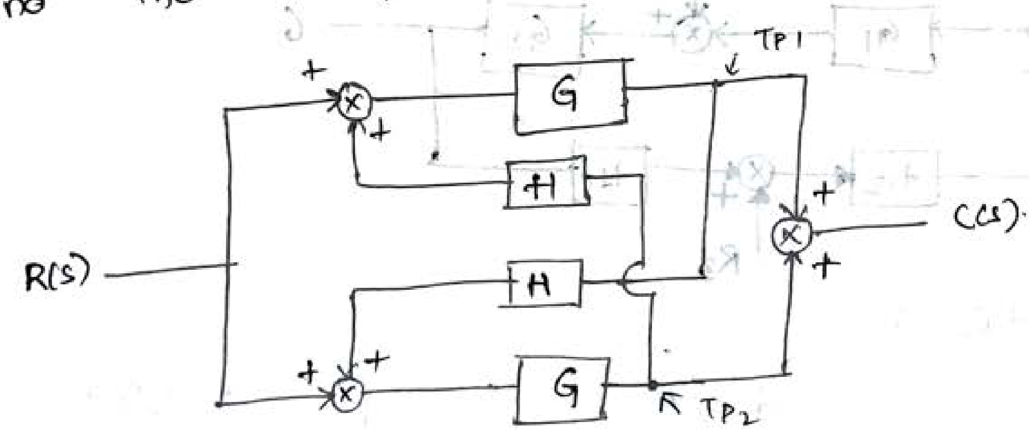
$-[10 \times (+3)]$

$\therefore \frac{12}{1+\frac{12}{3}\times\frac{1}{4}} = \frac{12}{4} = 3$

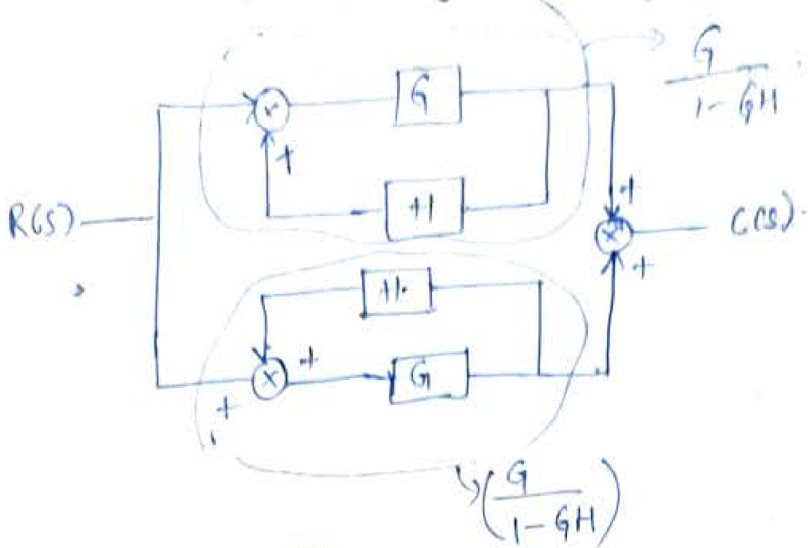
Sol.
$$\frac{C(s)}{R(s)} = \frac{60}{1+60} \simeq 1$$

$$\frac{C(s)}{R(s)} = -27$$

8. Find the transfer function of the system.



Sol. In the above block diagram the gain at take up point 1 is equal to gain at take off point 2 at any instant of time so we can interchange take up points.



$$\boxed{\therefore \frac{C(s)}{R(s)} = \frac{2G}{1-GH}}$$

9. The impulse response of a unity feedback system is $c(t) = -te^{-t} + 2e^{-t}$. The ~~op~~ equivalent open loop transfer function is ?

Ans. The given system response is closed loop system response because feedback is mentioned.

$$C.L.T.F \Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = L\left[\text{Impulse Response}\right]/I_i=0$$

$$\Rightarrow \frac{G}{1+G} = \frac{-1}{(s+1)^2} + \frac{2}{s+1} = \frac{2s+1}{(s+1)^2}$$

$$C.L.T.F = \frac{G}{1+G} = \frac{2s+1}{s^2+2s+1}$$

$$O.L.T.F = G = \frac{2s+1}{s^2+2s+1-(2s+1)} = \frac{2s+1}{s^2}$$

$$\therefore O.L.T.F = \frac{2s+1}{s^2} = G$$

10. Find the open loop dc gain of a unity feedback system

$$\frac{C(s)}{R(s)} = \frac{2s+5}{s^2+5s+20}$$

Sol.

$$C.L.T.F = \frac{G}{1+G} = \frac{2s+5}{s^2+5s+20}$$

$$O.L.T.F = G = \frac{2s+5}{s^2+5s+20-(2s+5)}$$

$$O.L.T.F = G = \frac{2s+5}{s^2+3s+15}$$

for dc-gain $f=0 \Rightarrow s = j\omega = j2\pi f = 0$.

$$O.L.T.F = G = \frac{5}{15} = \frac{1}{3} = 0.33$$

$$\boxed{\therefore \ O.L.T.F = G = 0.33}$$

---

11. The impulse response of the system is $(5 \cdot e^{-5t})$ to produce a response of te^{-5t}. The input must be equal to

Sol.

Impulse Response $g(t) = 5e^{-5t}$

$c(t) = te^{-5t}$

The given system is open loop system

$$\frac{C(s)}{R(s)} = G(s)$$

$$R(s) = \frac{C(s)}{G(s)} = \left[\frac{\frac{1}{(s+5)^2}}{\frac{5}{(s+5)}}\right] \quad \text{(By L.T).}$$

$$R(s) = \frac{0.2}{s+5}$$

Apply inverse laplace transform.

$$\boxed{R(t) = 0.2e^{-5t}}$$

Signal Flow Graphs :

Purpose :- To find the overal transfer function of the system

→ It is a graphical representation of set of linear algebraic equations between input and output.

→ The set of linear algebraic equations represents the systems.

→ The signal flow graph analysis is developed to avoid the mathematical calculations like solving the integro differential equations or linear algebraic equations.

→ The signal flow graph analysis is easier as compared to solving mathematical calculations.

→ S

Construction of signal flow graphs to the linear algebraic eqns :-

1. $y_2 = 10y_1$

2. $y_4 = 2y_1 + 5y_2 + 10y_3$

3. $y_2 = 10y_1$

   $y_3 = 20y_1$

   $y_4 = 30y_1$

The nodes in a signal flow graphs are nothing but variables that are currents and voltage in electrical systems.

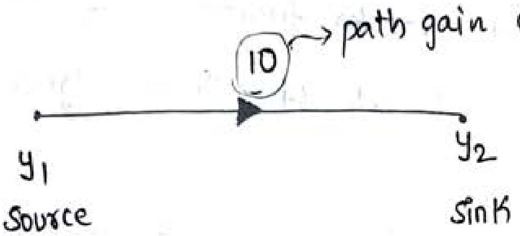
→ The path gains are nothing but impedance or admittance of the system components.

→ The variables lying at the left side is o/p & right side is inputs.

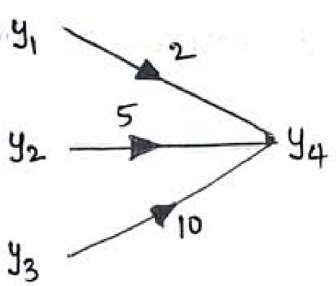
o/p node · i/p node

1. $y_2 = 10y_1$

path gain or transmittance. (unidirectional).



2. $y_4 = 2y_1 + 5y_2 + 10y_3$

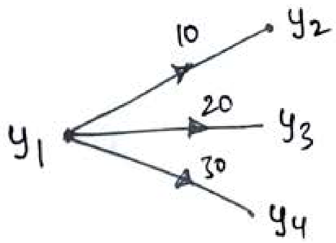


[Additional rule
Many to one rule.]

3. $y_2 = 10y_1$

$y_3 = 20y_1$

$y_4 = 30y_1$



[Transmission rule
One-to-many rule]

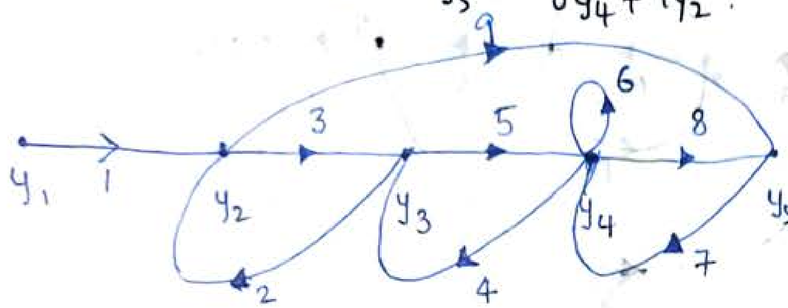
* Draw the signal flow graph to the given set of linear algebraic equations. (a) $y_2 = y_1 + 2y_3$

(b) $y_3 = 3y_2 + 4y_4$

(c) $y_4 = 5y_3 + 6y_4 + 7y_5$

(d) $y_5 = 8y_4 + 9y_2$.

Sol.



(3) Find the no. of forward paths, individual loops, non-touching loops to above signal flow graph.

Sol. Forward path :- It is a path from input to output.

Loop :- Loop is a path it terminates on the same node where it is started.

Non-touching loop: If there is no common node b/w two or more loops then it is called non touching loops.

Input node: The node which has only outgoing branches.

Output node: The node which has only incoming branches.

Chain or link node: The node which has both outgoing and incoming branches.

forw. paths – 2 ; loop – 5 ;

The correct forw. path(or) loop is the one in which each node should be touched only once

While solving 2 non-touching loop using order

$L_1 L_2 \times$ $\quad L_2 L_3 \times$ $\quad L_3 L_4 \times$ $\quad L_4 L_5 \times$; so so =

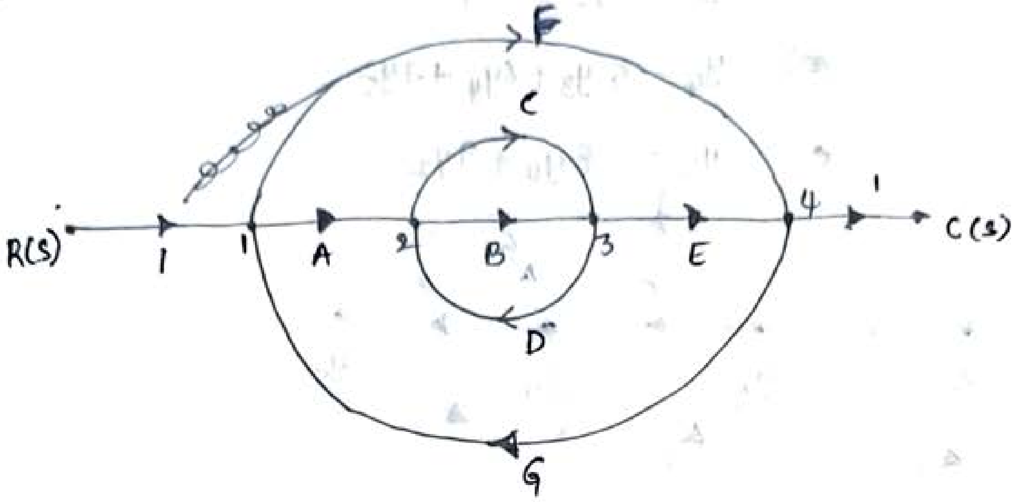
$L_1 L_3 \checkmark$ $\quad L_4 \times$ $\quad L_5 \times$

$L_4 \checkmark$ $\quad L_5 \times$

$L_5 \times$

2-Non touching loops = 2

(*) Repeat the above problem to the given ~~f~~ signal flow graph



Sol

forward paths = 3 ;

Node no.

Loops = L_1 = BD ⇒ 2,3 — 2,3

D → 2 ← 3 L_2 = CD ⇒ 2,3 — 2,3

G ⇒ 1 ← 4 L_3 ⇒ ABEG — 1,2,3,4

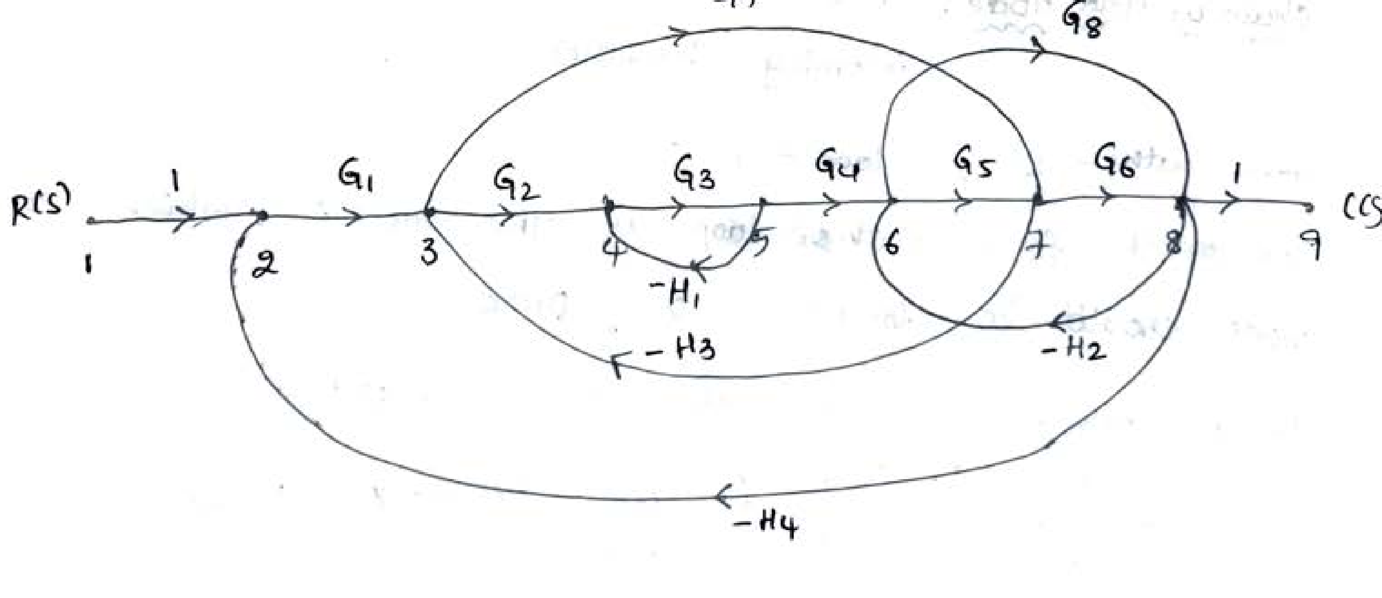
L_4 ⇒ ACEG — 1,2,3,4

L_5 ⇒ AFG — 1,4

2-Non touching loops = $L_1 L_5$, $L_2 L_5$;

(*) Find the number of forward paths, individual loops, 2-non touching loops, 3 non-touching loops.

Sol



No. of forward paths = 3

Node

loops

$H_1 \rightarrow 4 \leftrightarrow 5 \longrightarrow L_1 = 4,5$

$H_3 = 3 \leftarrow 7 = 2$ loops $\longrightarrow L_4 = 3,4,5,6,7)$.

$L_5 = 3,7$.

$H_2 = 6 \leftarrow 8 = \{2$ loops $\}$ $\longrightarrow L_2 = 6,7,8$

$\{= $ No. of form. paths $\}$ $\longrightarrow L_3 = 6,8$.

$H_4 = 8 \leftrightarrow 2 = 3$ loops $\longrightarrow L_6 = 2,3,4,5,6,7,8$

$\longrightarrow L_7 = 2,3,7,8$

$\longrightarrow L_8 = 2,3,4,5,6,8$.

Total loops = 8.

2 - Non touching loops :-

$L_1 L_2$ (4,5,6,7,8)

$L_1 L_3$ (4,5,6,8). 2-Nontouching loops = 5.

$L_1 L_5$ (4,5,3,7)

$L_1 L_7$ (4,5,2,3,7,8)

$L_3 L_5$ (6,8,3,7).

3- Non touching loops :-

$L_1 L_3 L_5$ (4,5,6,8)

$L_1 L_5 L_3$ (4,5,3,7)

$L_3 L_5 L_1$ (6,8,3,7)

Mason's Gain Formula:

Purpose:
- To find the overall transfer function of the system.
- To find the ratio of any 2 nodes.
- Overall Transfer function $= \sum_{K=1}^{i} \left(\frac{P_K \Delta_K}{\Delta} \right)$

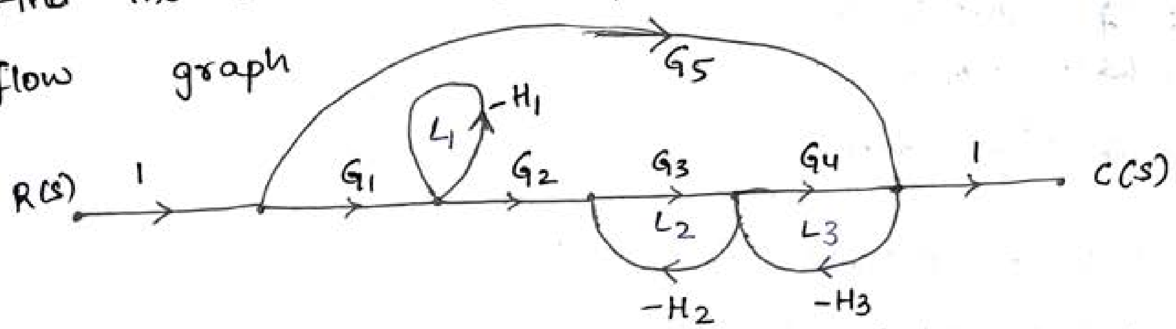
where $P_K = K^{th}$ forward path gain.

$\Delta = 1 - \sum$ (individual loop gains).
$+ \sum$ (Gain products of 2-non touching loop gains)
$- \sum$ (Gain products of 3-non touching loop gains).
$+ \sum$ (Gain products of 4-non touching loop gains).
$- \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

Δ_K is obtained from Δ by removing the loops touching the K^{th} forward path.

Problem:

1. Find the overall transfer function to the given signal flow graph



Sol.

forward paths:

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_5$

Loops:

$L_1 = -H_1$

$L_2 = -G_3 H_2$

$L_3 = -G_4 H_3$

2 non touching loops:

$$L_1 L_2 = G_3 H_1 H_2$$

$$L_1 L_3 = G_4 H_1 H_3$$

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2 + L_1 L_3)$$

$$\Delta = 1 + H_1 + G_3 H_2 + G_4 H_3 + G_3 H_1 H_2 + G_4 H_1 H_3$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1 - (L_1 + L_2) + (L_1 L_2)$$

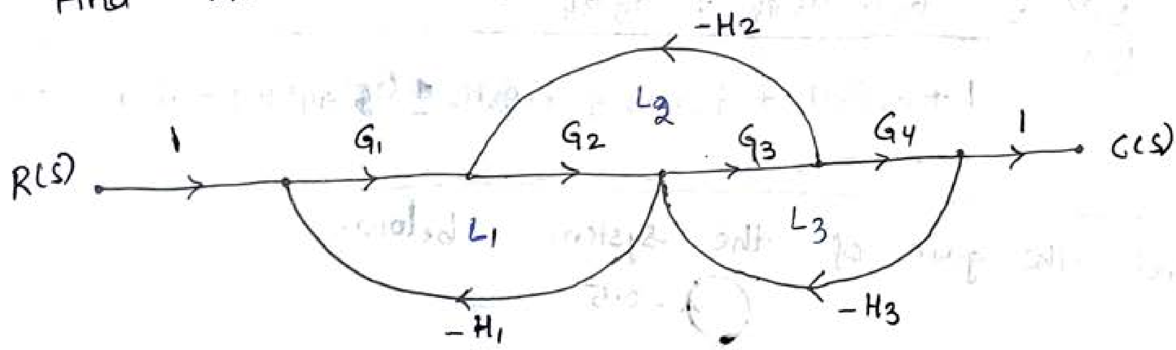
$$\Delta_2 = 1 + H_1 + G_3 H_2 + G_3 H_1 H_2$$

$$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{[G_1 G_2 G_3 G_4] + [G_5 (1 + H_1 + G_3 H_2 + G_3 H_1 H_2)]}{1 + H_1 + G_3 H_2 + G_4 H_3 + G_3 H_1 H_2 + G_4 H_1 H_3}$$

Note: While writing Δ or Δ_K take the opposite sign for odd number of non touching loops & take the same sign for even number of non-touching loops.

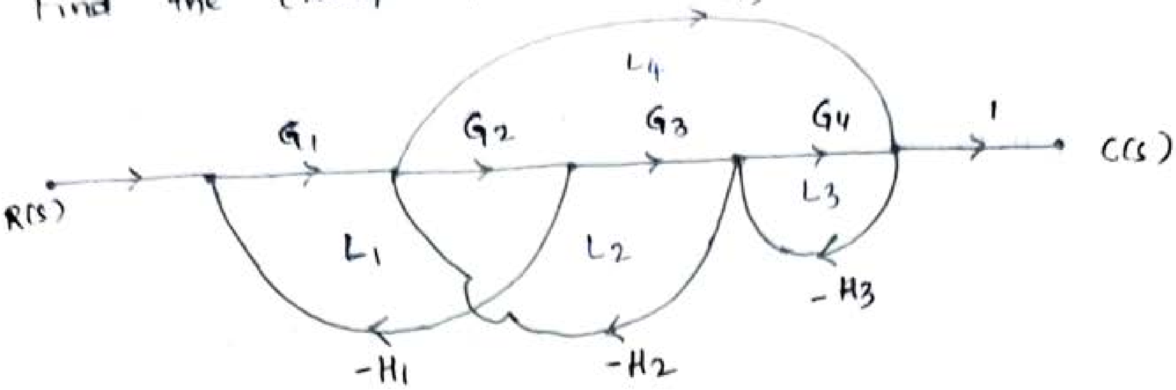
(*) Find the transfer function for:



Sol:

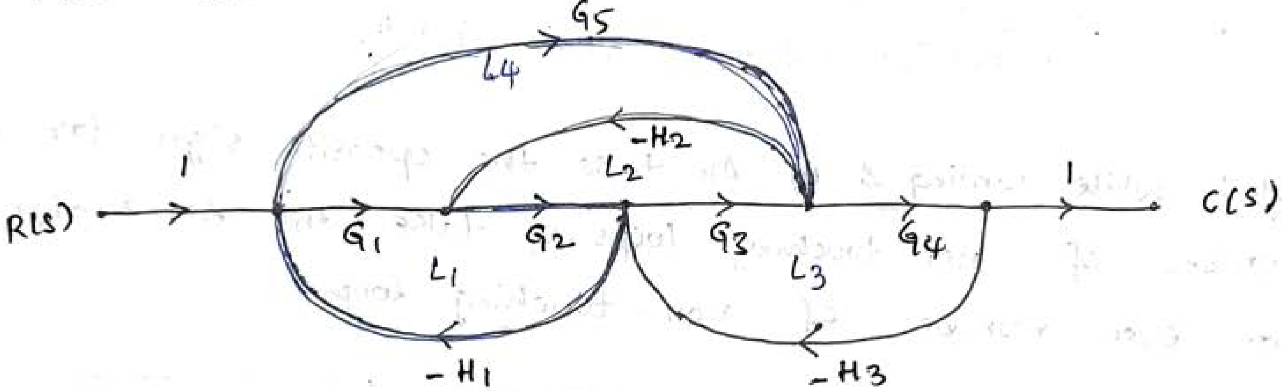
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 (1)}{1 + G_1 G_2 H_1 + G_3 G_4 H_3 + G_2 G_3 H_2}$$

(*) Find the transfer function



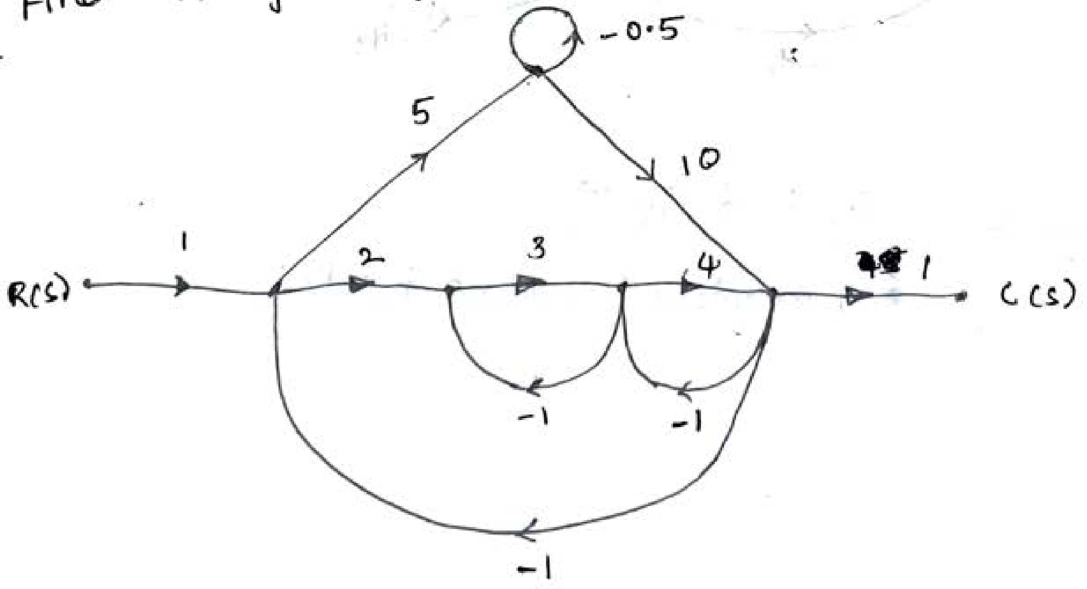
Sol:
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_4 H_3 \pm G_5 H_3 H_2 + G_1 G_2 H_1 G_4 H_3}$$

(*) Find the transfer function.



Sol.
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 + G_5 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_3 + G_2 G_3 H_2 \pm G_5 G_4 H_1 H_3 - G_2 G_5 H_1 H_2}$$

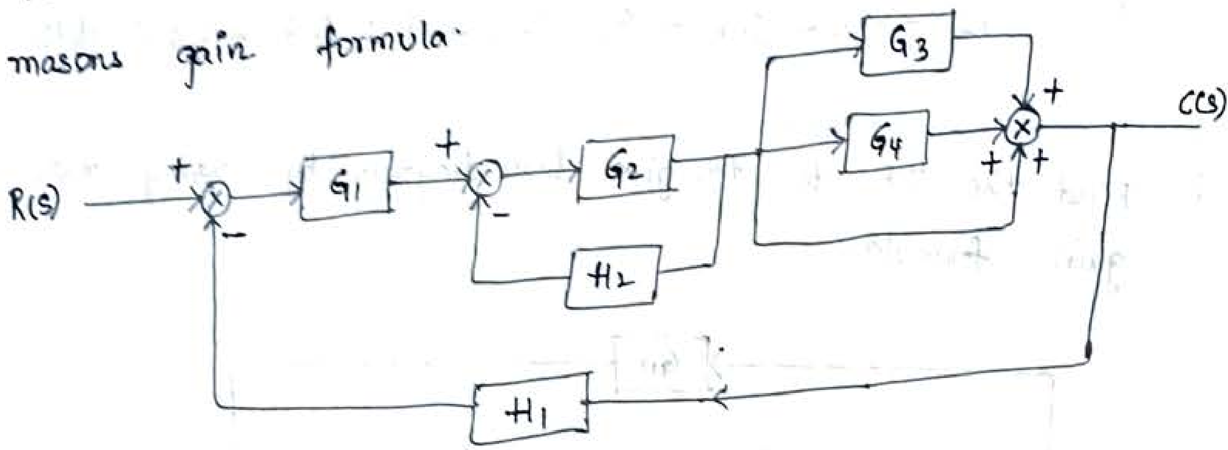
(*) Find the gain of the system below.
VVIMP



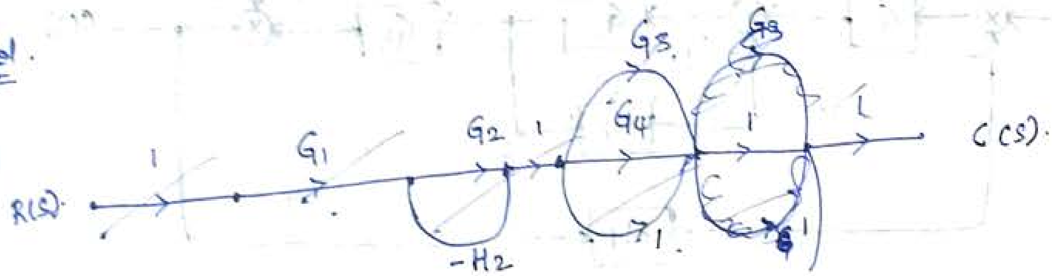
$$\frac{C(s)}{R(s)} = \frac{[(1+0.5)(1\cdot 2\cdot 3\cdot 4)] + [(1\cdot 5\cdot 10)[1+3]]}{1+3+4+(2\cdot 3\cdot 4)+50+0.5+\underbrace{3\times 50}_{L_1L_4}+\underbrace{3\times 0.5}_{L_1L_5}+\underbrace{4\times 0.5}_{L_2L_5}+\underbrace{24\times 0.5}_{L_3L_5}}$$

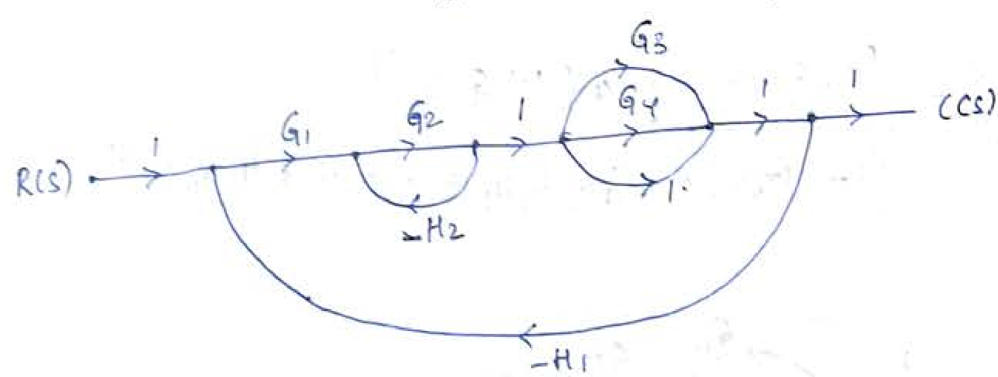
$$\frac{C(s)}{R(s)} = \frac{236}{248} = 0.95$$

(*) Find the T.F to the given block diagram by using the masons gain formula.



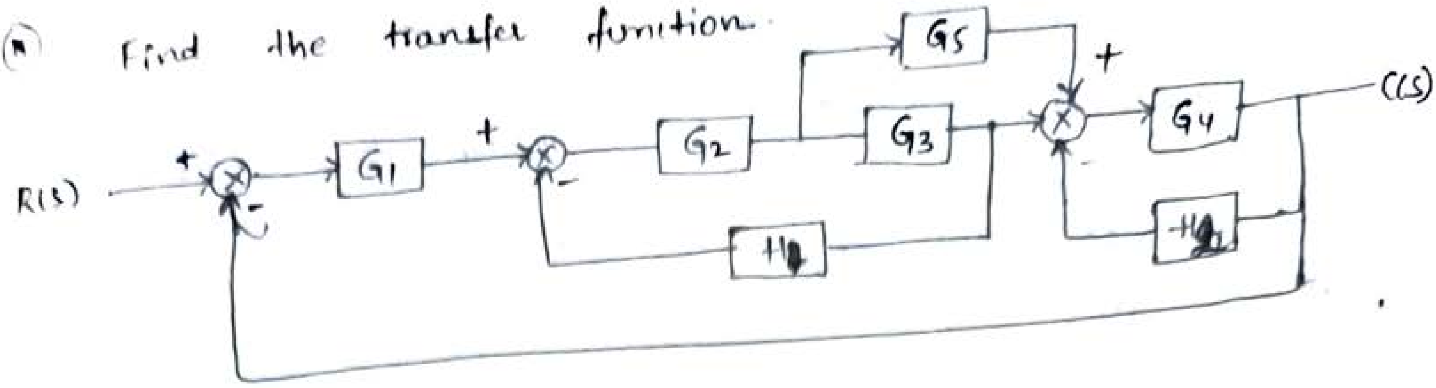
Sol.





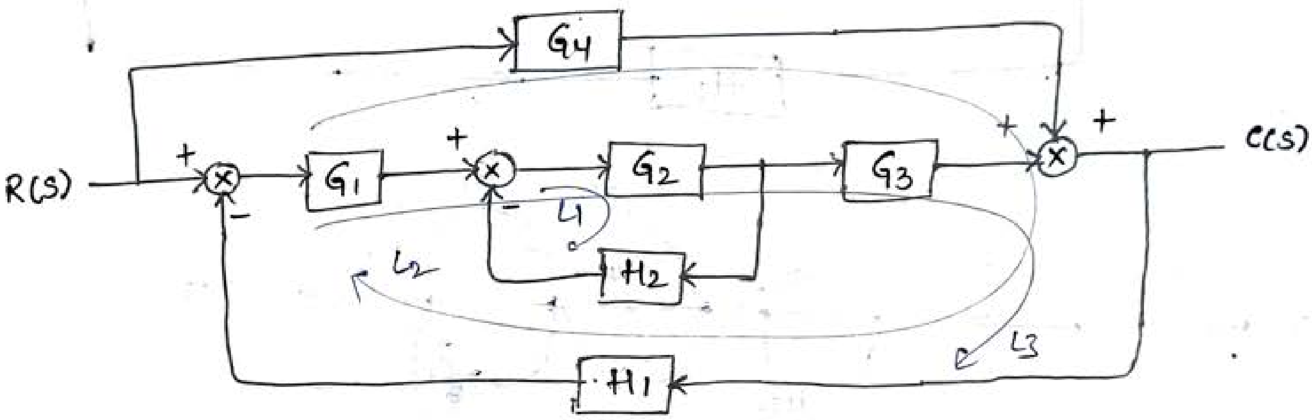
$$\frac{C(s)}{R(s)} = \frac{G_1G_2[1+G_3+G_4]}{1+G_2H_2+G_1G_2(1+G_3+G_4)H_1}$$

(*) Find the transfer function.



$$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_4 + G_1G_2G_5G_4}{1 + G_2G_3H_1 + G_4H_2 + G_1G_2G_3G_4 + G_1G_2G_5G_4 + G_2G_3G_4H_1H_2}$$

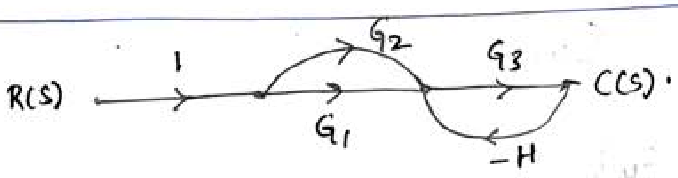
(*) Find the T.F to the given block diagram by using mason's gain formula.



Sol

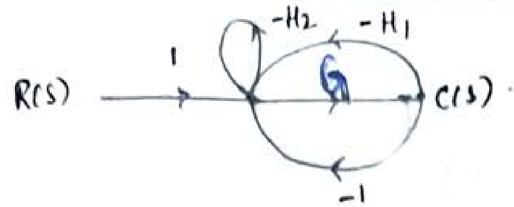
$$\frac{C(S)}{R(S)} = \frac{(G_1G_2G_3) + [G_4(1 + G_2H_2)]}{[1 + G_2H_2 + G_1G_2G_3H_1 + G_4H_1 + G_2H_2G_4H_1]}$$

(*)

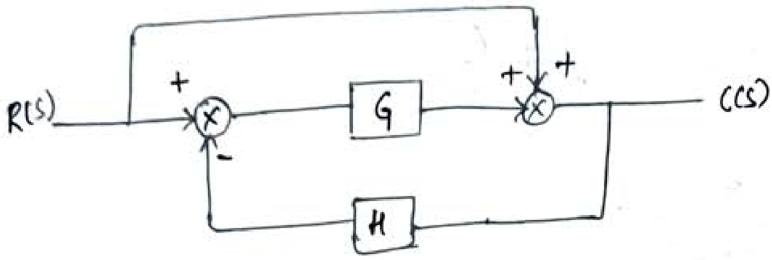


Sol.

$$\frac{C(S)}{R(S)} = \frac{G_1G_3 + G_2G_3}{1 + G_3H.}$$

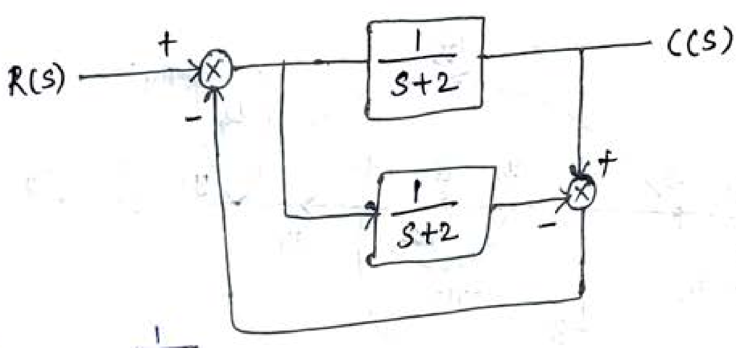


$$\frac{C(s)}{R(s)} = \frac{G}{1+G+GH_1+H_2}$$

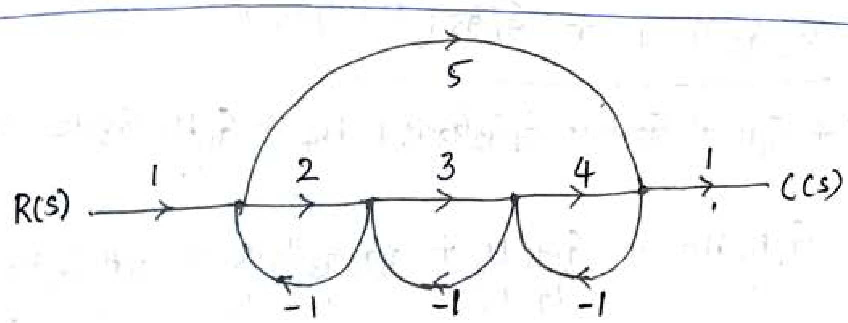


$$\frac{C(s)}{R(s)} = \frac{G+1}{1+GH}$$

Doubt.

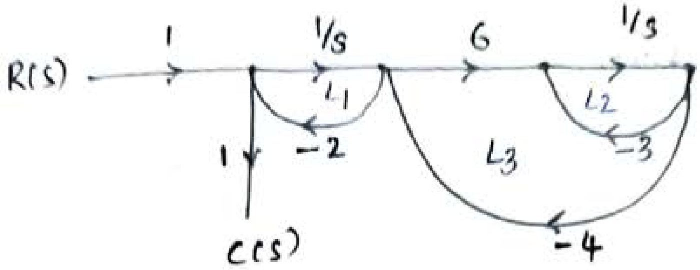


$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}-\frac{1}{s+2}} = \frac{1}{s+2}$$



$$\frac{C(s)}{R(s)} = \frac{24+5(1+3)}{1+2+3+4+5+\underset{4L_3}{8}} = \frac{44}{23}$$

(*)



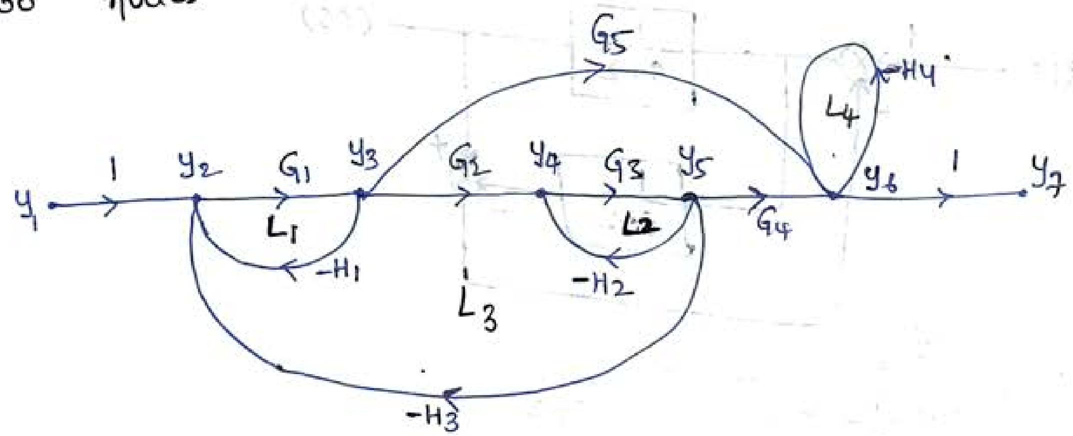
Sol.

$$\frac{C(S)}{R(S)} = \frac{1\left(1+\frac{3}{s}+\frac{24}{s}\right)}{1+\frac{2}{s}+\frac{3}{s}+\frac{24}{s}+\frac{6}{s^2}}$$

$$\frac{C(S)}{R(S)} = \frac{s(s+27)}{s^2+29s+6}.$$

(*) VIMP. Find $\frac{y_6}{y_1}, \frac{y_7}{y_1}, \frac{y_5}{y_1}, \frac{y_2}{y_1}, \frac{y_7}{y_2}, \frac{y_5}{y_3}, \frac{y_5}{y_4}$ ... ratio of any two nodes.

Sol.



$$\frac{y_7}{y_1} = \frac{y_6}{y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{1+G_1H_1+G_3H_2+G_1G_2G_3H_3+H_4+\underset{L_1}{G_1H_1}\,\underset{L_2}{G_3H_2} + \underset{L_1}{G_1H_1}\,\underset{L_4}{H_4}+\underset{L_2}{G_3H_2}\,\underset{L_4}{H_4}+\underset{L_3}{G_1G_2G_3H_3}\,\underset{L_4}{H_4}+\underset{L_1}{G_1H_1}\,\underset{L_2}{G_3H_2}\,\underset{L_4}{H_4}}$$

$$\frac{y_5}{y_1} = \frac{G_1G_2G_3(1+H_4)}{\Delta}$$

Δ → (den is same as above)

$$\frac{y_2}{y_1} = \frac{1(1+G_3H_2+H_4+G_3H_2H_4)}{\Delta}$$

$$\frac{y_7}{y_2} = \frac{y_7/y_1}{y_2/y_1} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{1(1+G_3H_2+H_4+G_3H_2H_4)}.$$

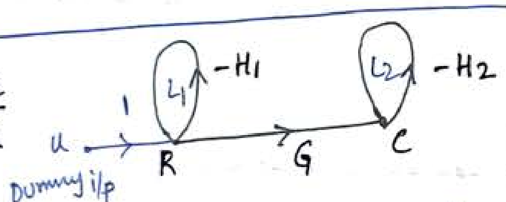
forw path gain

→ Masons gain formula gives the ratio w.r.t input node only but not the middle nodes.

$$\frac{y_5}{y_3} = \frac{y_5/y_1}{y_3/y_1} = \frac{G_1G_2G_3(1+H_4)}{G_1(1+G_3H_2+H_4+G_3H_2H_4)}.$$

$$\frac{y_5}{y_4} = \frac{y_5/y_1}{y_4/y_1} = \frac{G_1G_2G_3(1+H_4)}{G_1G_2(1+H_4)} = G_3.$$

(*) Find $\frac{C}{R}$



Sol: In the above signal flow graph the R is not a i/p node In this case we required to assume a dummy i/p node with the path gain of '1' as shown in fig.

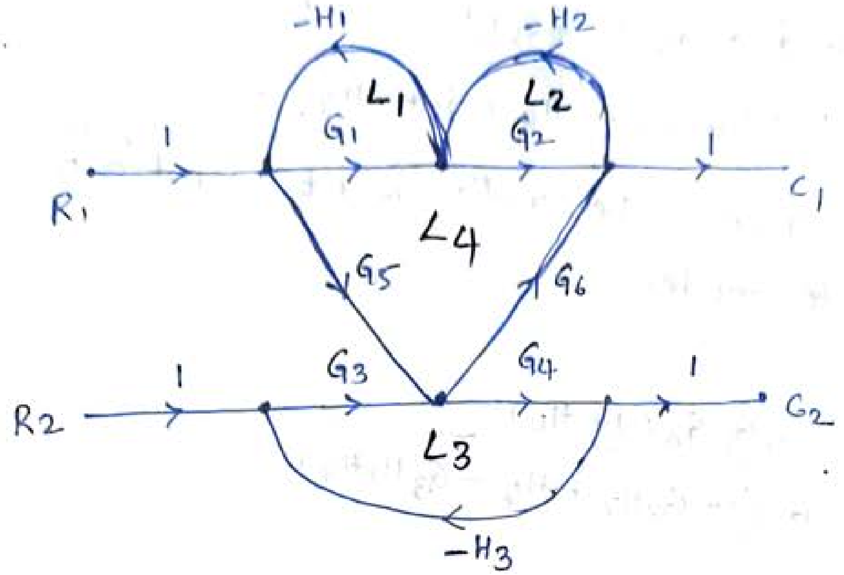
Method-1:
$$\frac{C}{R} = \frac{C/u}{R/u} = \frac{G}{1(1+H_2)}.$$

Method-2: o/p node →
$$C = RG - CH_2$$
$$C(1+H_2) = RG$$
$$\frac{C(s)}{R} = \frac{G}{1+H_2}$$

$$\boxed{\therefore \frac{C(s)}{R(s)} = \frac{G}{1+H_2}}.$$

(*) Find $\frac{C_1}{R_1}$, $\frac{C_1}{R_2}$, $\frac{C_2}{R_1}$, $\frac{C_2}{R_2}$ to the given multi-input multip-output system?

Sol.



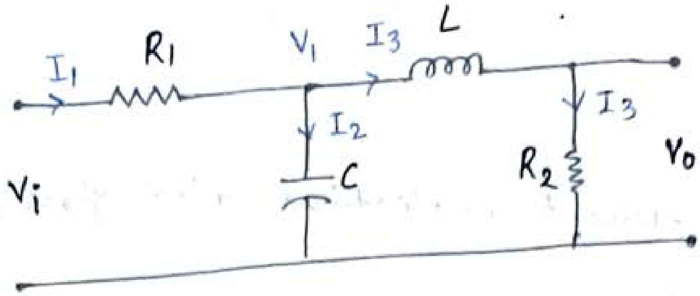
$$\frac{C_1}{R_1}\Big|_{R_2=C_2=0} = \frac{[G_1G_2(1+G_3G_4H_3)]+[G_5G_6]}{1+G_1H_1+G_2H_2+G_3G_4H_3-G_5G_6H_1H_2+\underset{L_1}{G_1H_1}\underset{L_3}{G_3G_4H_3}+\underset{L_2}{G_2H_2}\underset{L_3}{G_3G_4H_3}}$$

$$\frac{C_1}{R_2}\Big|_{R_1=C_2=0} = \frac{G_6G_3(1+G_1H_1)}{\Delta}$$

$$\frac{C_2}{R_1}\Big|_{R_2=C_1=0} = \frac{G_5G_4(1+G_2H_2)}{\Delta}$$

$$\frac{C_2}{R_2}\Big|_{R_1=C_1=0} = \frac{G_3G_4(1+G_1H_1+G_2H_2)}{\Delta}$$

Constuction of signal flow graph to electrical network :



Procedure:

Step-1: Select the branch currents & node voltages

Step-2: Apply the laplace transform to the network variables and elements.

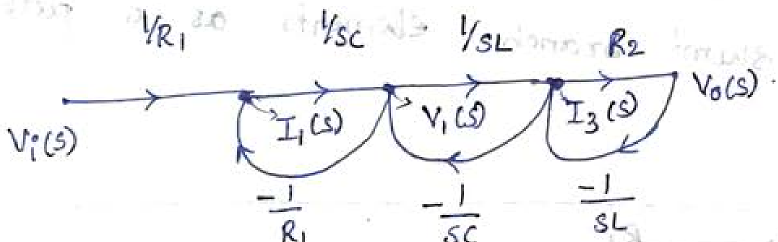
Step-3: Write the equations for unknown currents and unknown voltages.

$$I_1(s) = \frac{V_i(s) - V_1(s)}{R_1} \quad \text{——} \quad ①$$

$$V_1(s) = I_2(s) \cdot \frac{1}{sC} = \frac{I_1(s) - I_3(s)}{sC} \quad \text{——} \quad ②$$

$$I_3(s) = \frac{V_1(s) - V_o(s)}{sL} \quad \text{——} \quad ③$$

$$V_o(s) = I_3(s) R_2 \quad \text{——} \quad ④$$



xx vvimp

Transfer function in electrical n/w	=	Transfer function in block diagram or signal flow graph.

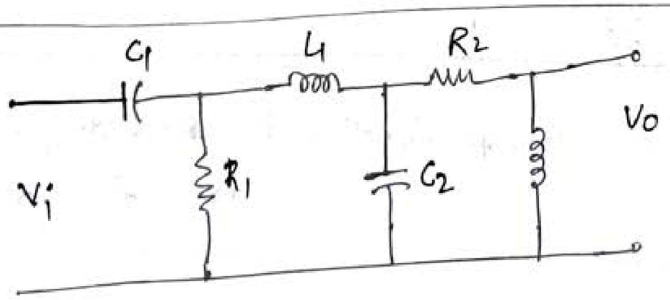
Verify the T/f to the given electrical n/w & given 4 given 4 options. The correct signal flow graph (or) Block diagram if the 1 it should have same transfer function.

Note:

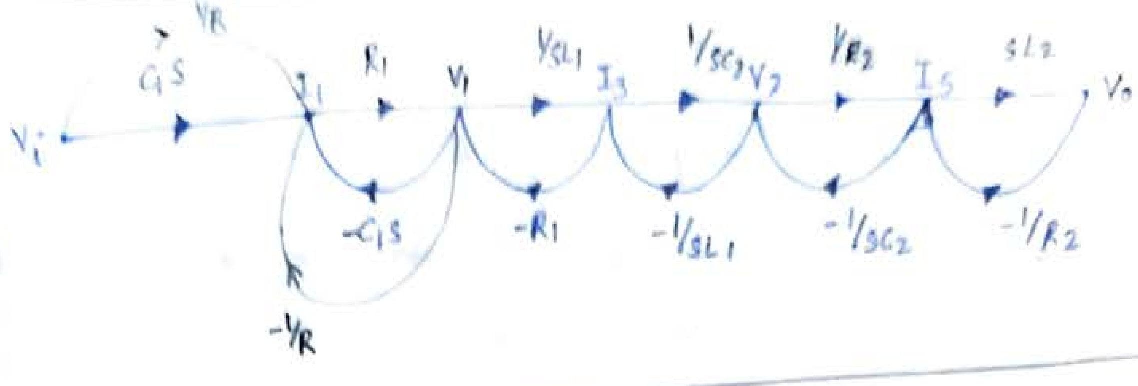
1. ~~No. of n~~ Every element in electrical n/w gives 

8. ~~Tat~~ Procedure to draw directly ~~directly~~ signal flow graph:

1. The nodes in a SFG are nothing but variables along series branch
2. Each element in electrical network gives the one forward path and one negative feedback path except the last element where we take the output.
3. The last element gives only the forward path.
4. Take the inverse of impedance for series branch elements as a path gain and take the same elements as a path impedance for shunt branch elements as a path gain.

(*) 

Convert E.N to S.F.G.



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