Z - TRANSFORM:

→ **Purpose:** we want to an convert laplace domain to z domain (engin

→ we want to make complex domain to polar domain

→ For storage purpose we go for z transform.

i.e, we we shift register i.e,

$$\longrightarrow \boxed{z^{-1}} \longrightarrow$$

Invention of z - Transform is done in the yr 1965.

It is invented by Raggazini & Jadeh.

→ generalisation of D.T.F.T.

→ discrete counter part of L.T.

$$x(t) = e^{st} \Rightarrow y(t) = e^{st} H(s).$$

$$x[n] = z^{n} \Rightarrow y[n] = z^{n} H(z).$$

$$z = re^{j\omega}.$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad x[n] \xleftrightarrow{z \cdot T} X(z)$$

$$X(re^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] r^{-n} e^{-j\omega n}$$

Application of Z.T:

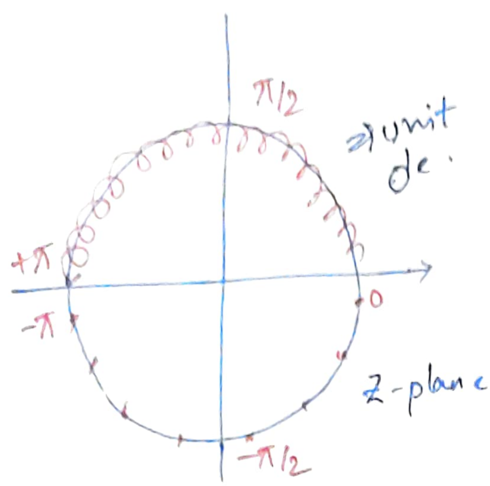
→ Sampled data controls → Jury's, Lyapunov's → methods

→ D.F. Design (Digital filter)

z - transform

$X(z) = F.T\{x[n] r^{-n}\}$

$r = 1 \Rightarrow Z.T = D.T.F.T$

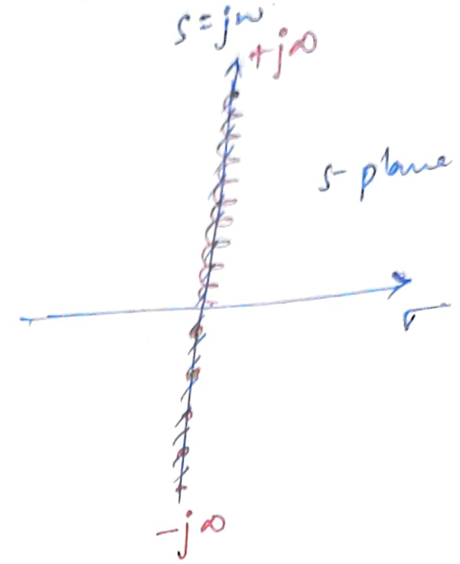


(Range : -π to π)

laplace transform

$X(s) = F.T\{x(t)\, e^{-\sigma t}\}$

$\sigma = 0 \Rightarrow L.T = C.T.F.T$



→ positive part of jw axis corresponds to upper part of the (w varies from 0 to π)

→ negative part of jw axis corresponds to lower half of the circle (w varies from -π to 0)

→ $\boxed{z = e^{sT_s}}$ → reln b/w L.T. & Z.T.

$x(t) = e^{st}$

$x[nT_s] = e^{snT_s}$

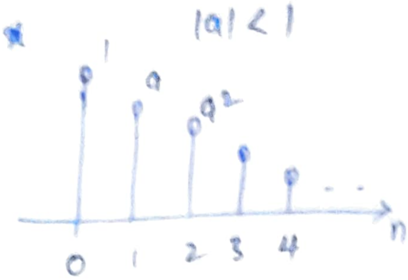
$x[n] = z^n$

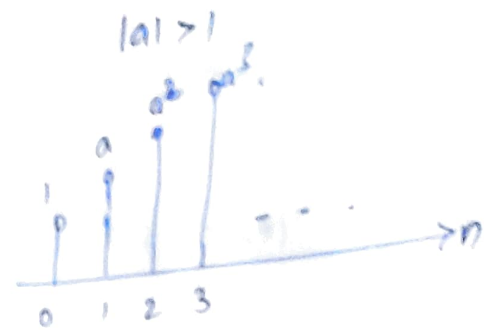
→ Always Z.T is done with reference to laplace transform

→ Always D.T.F.T is done with ref to C.T.F.T.

Z-transform of standard signals:

1. $x_1[n] = a^n u[n]$





$$X_1(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (a z^{-1})^n$$

$$= 1 + a z^{-1} + (a z^{-1})^2 + \cdots$$

$$X_1(z) = \frac{1}{1 - a z^{-1}} \quad ; \quad |a z^{-1}| < 1$$

$$\left|\frac{a}{z}\right| < 1$$

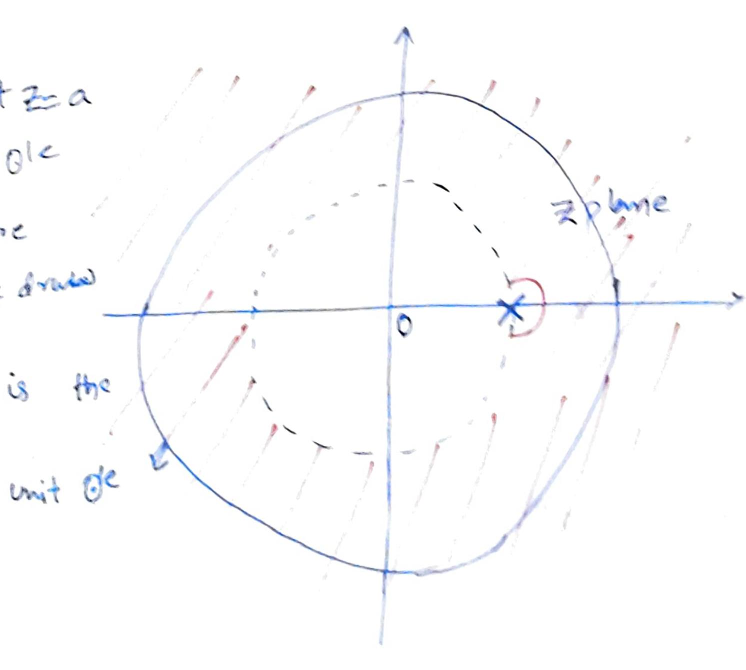
$$\Rightarrow \underbrace{|z| > |a|}_{\text{ROC}}$$

$$\boxed{a^n u[n] \longleftrightarrow \frac{1}{1 - a z^{-1}} \text{ (or) } \frac{z}{z - a} \quad ; \quad |z| > |a|}$$

1 pole at $z = a$
→ one ole

if 2 poles are there we draw two oles

→ Unit ole is the reference.



→ If the given sequence is right sided ROC is outside

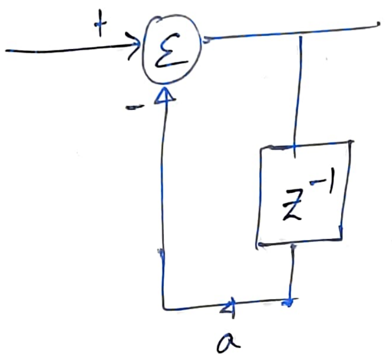
→ " " " " " left sided " " insided.

<u>R.O.C</u>

Right sided → outside

left sided → Inside.

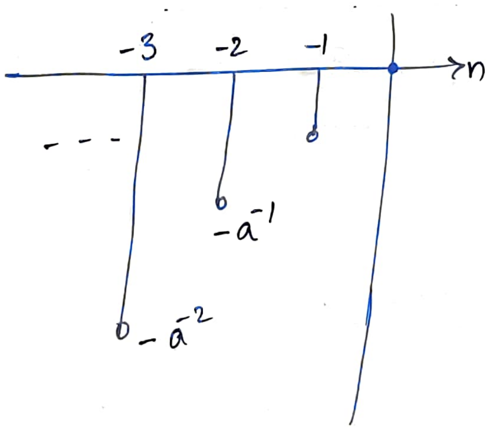
→ To implement or generate $a^n u[n]$ sequence sample we use the following digital ckt



2. $x_2[n] = -a^n u[-n-1] = -a^n \; ; \; n \leq -1$

↓

Signal is left sided.



$$X_2(z) = \sum_{n=-\infty}^{-1} -a^n z^{-n}$$

put $-n = m$.

$$= -\sum_{m=\infty}^{1} a^{-m} \cdot z^m$$

$$= -\sum_{m=1}^{\infty} (a^{-1}z)^m = -\left[(a^{-1}z) + (a^{-1}z)^2 + \cdots\right].$$

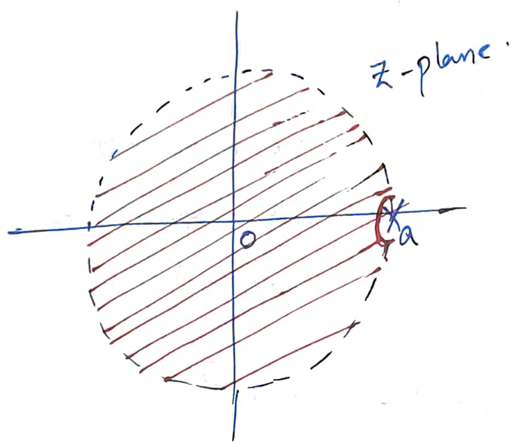
$$= -\frac{a^{-1}z}{1-a^{-1}z} \quad ; \quad |a^{-1}z| < 1$$

$$\Rightarrow |z| < |a|.$$

$$\boxed{\therefore -a^n u[-n-1] \xleftrightarrow{Z.T} \frac{1}{1-az^{-1}} \text{ (or) } \frac{z}{z-a} \quad ; \quad |z| < |a|.}$$

→ from the previous signal we observe that two ~~is~~ different signals but identical transforms with different ROC's.

ROC



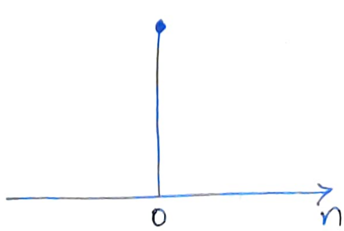
→ when we apply inverse ~~z~~ z- transform

→ if the given sequence ~~use~~ is left sided use $-a^n u[-n-1]$

→ if the given sequence is right sided use $a^n u[n]$.

(*) $x[n] = \delta[n]$

$X(z) = 1$



ROC: entire z-plane

---

* $\delta[n-1] \xleftrightarrow{Z.T} z^{-1}$ (1) $\rightarrow$ positive sided $\delta[n-1]$

ROC: entire z-plane ~~exp~~ except $z=0$; $|z|>0$

for all +ve sided signal $z=0$ will be exception

* $\delta[n+1] \xleftrightarrow{Z.T} z^{-1(-1)} = z$ $\rightarrow$ negative sided $\delta[n+1]$

ROC: entire z-plane except at $z=\infty$ (or) $|z|<\infty$

for all -ve sided signal $z=\infty$ will be exception

* Pg-146.

$$n \rightarrow \quad -2 \quad -1 \quad 0 \quad 1$$

7.1.3 (a). $x_1[n] = \left\{ 1, 2, \underset{\uparrow}{3}, -1 \right\}$

ROC: $0<|z|<\infty$ ($\because$ we have both positive & negative side impulse exception for ROC is $z=0$ & $z=\infty$.

(b) $x_2[n] = (1/2)^n \left[\underbrace{u(n) - u(n-10)}_{0 \le n \le 9} \right]$.

It is a finite length signal.

i.e, ROC is entire z plane except zero.

ROC: $|z|>0$.

(c) $x_3[n] = \left[\left(\frac{1}{2}\right)^n + \left(\frac{3}{4}\right)^n \right] \underbrace{u[n-10]}_{n \geq 9}$.

It is infintie length

The two poles are $\frac{1}{2}$ & $3/4$

largest pole = $3/4$

$\therefore$ ROC : $|Z| >$ largest pole

$\therefore$ ROC : $|Z| > 3/4$ except $Z = 0$

2012 gate (d).

Q) $x_4[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^{n} u[n].$

Sol

$$|n| = \begin{cases} n ; & n \geq 0 \\ -n ; & n < 0. \end{cases}$$

$$x_4[n] = -\left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{1}{3}\right)^{|n|}$$

$$\left(\frac{1}{3}\right)^{n} ; \; n \geq 0$$

$$\left(\frac{1}{3}\right)^{-n} ; \; n < 0.$$

$$= -\left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{1}{3}\right)^{n} u[n] + 3^{n} u[-n-1]$$

$|Z| > \frac{1}{2}$ $|Z| < 3$

ROC$_4$: $\frac{1}{2} < |Z| < 3$

(*) $x[n] = 2^{|n|}$

$$= 2^{n} ; \; n \geq 0$$

$$= 2^{-n} ; \; n < 0$$

$$x[n] = 2^{n} u[n] + \left(\frac{1}{2}\right)^{n} u[-n-1].$$

$|Z| > 2$ $|Z| < \frac{1}{2}$

no common ROC.

7.1.2 $x[n] = (-1)^n u[n] + \alpha^n u[-n-n_0]$.

to find 'α' & 'n_0' to get the ROC as $1 < |z| < 2$

Sol: $x[n] = (-1)^n u[n] + \alpha^n u[-n-n_0]$

$|z| > |-1|$

$|z| > 1$

$|z| < |\alpha|$.

$|\alpha| = 2$

$\alpha = \pm 2$. & "n_0" can be any value

Rational Z.T concepts

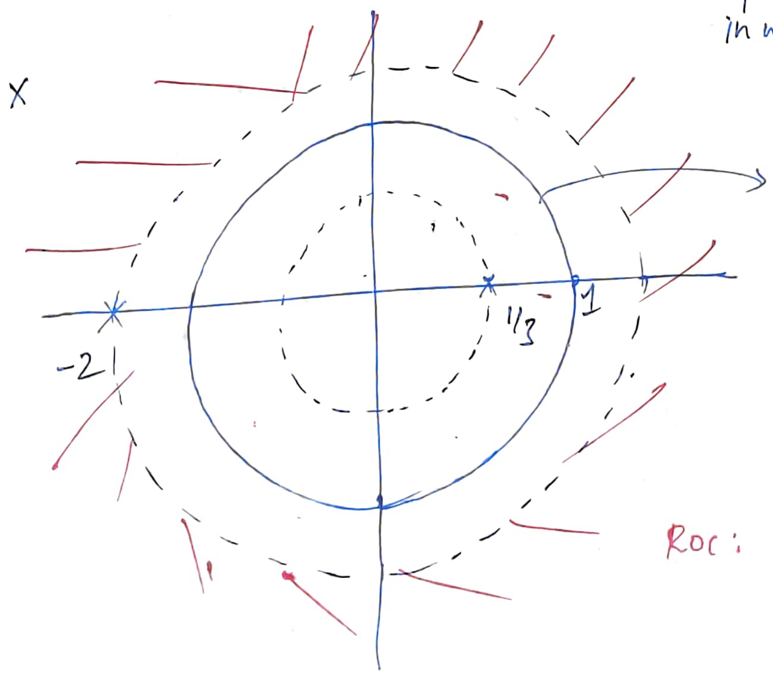
Eg: $X(z) = \dfrac{1}{(1-\frac{1}{3}z^{-1})(1+2z^{-1})} = \dfrac{1/7}{(1-\frac{1}{3}z^{-1})} + \dfrac{6/7}{(1+2z^{-1})}$

refer

Poles at $z = \frac{1}{3}$ & -2. $(a = -2)$.

Case-(i): $x[n]$ is right-sided.

largest pole in magnitude $= |-2| = 2$



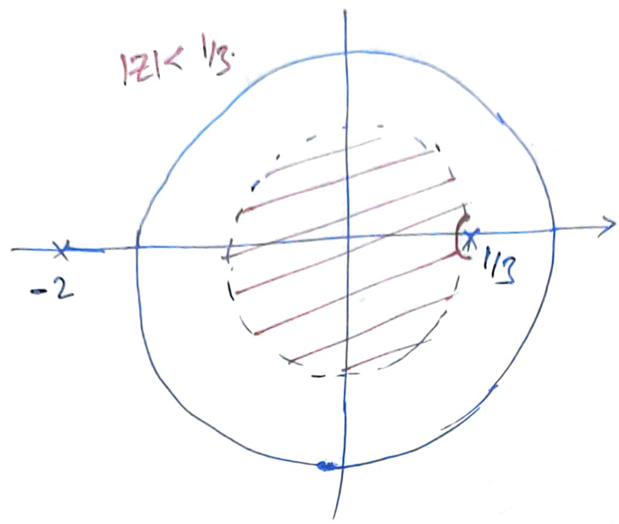
Roc: $|z| > 2$

It is Causal

$$x[n] = \frac{1}{7}\left(\frac{1}{3}\right)^n u[n] + \frac{6}{7}(-2)^n u[n]$$

↓ right sided ↓ right sided.

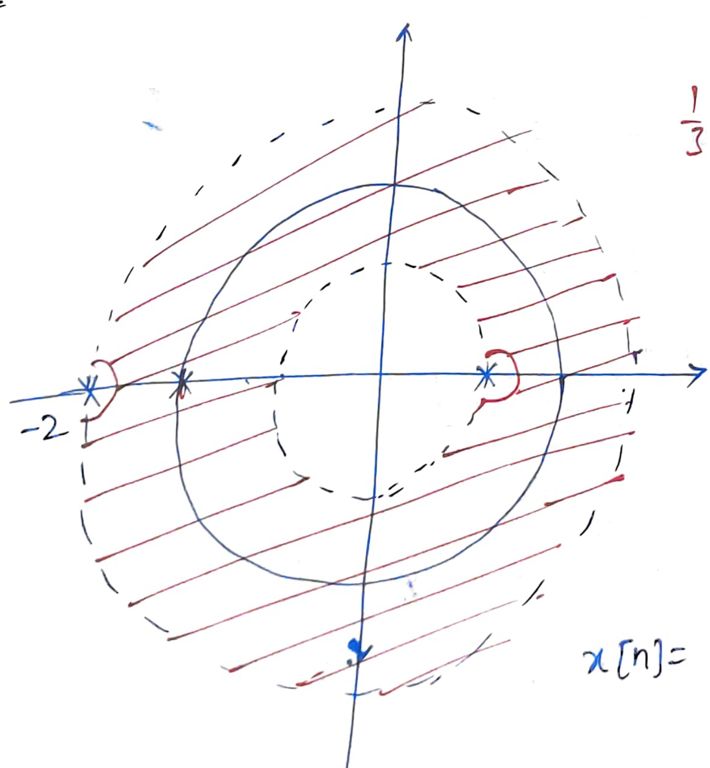
Case-ii. $x[n]$ is left sided.



$$x[n] = \frac{1}{7}\left(\frac{1}{3}\right)^n \left[-u[-n-1]\right] + \frac{6}{7}(-2)^n\left[-u[-n-1]\right].$$

↓ left sided ↓ left sided

Case-3 : $x[n]$ is two sided:



$$x[n] = \frac{1}{7}\left(\frac{1}{3}\right)^n u[n] + \frac{6}{7}(-2)^n\left[-u[-n-1]\right].$$

↓ right sided ↓ left sided

7.2.2 sol Pg. 146.

(DRDO).

$$X(z) = \frac{z^2}{(z-e^{j\pi/2})(z-e^{-j\pi/2})}.$$

$$= \frac{z^2}{(z+j)(z-j)} = \frac{2z^2}{z^2+1} \quad \because X(1)=1$$

right sided ⇒ $|z| > |\pm j|$

ROC: $|z| > 1$

Ans: d

7.2.3 sol

$$y(n) = \left(\frac{-1}{3}\right)^n u[-n-2].$$

We have $x[n-n_0] \xleftrightarrow{Z.T} z^{-n_0}X(z)$.

$$x[n] \xleftrightarrow{Z.T} X(z).$$

Now $x[n] = a^n u[-(n+1)]$.

$$x[n+1] = a^{n+1} u[-(n+2)].$$

$$y(n) = \left(-\frac{1}{3}\right)^n u[-n-2]$$

$$= \left[-\frac{1}{3}\right]^{n+1-1} \underbrace{u[-(n+2)]}_{n_0=-1}$$

Z.T

$$Y(z) = -3\left[\frac{-z^{-(-1)}}{1+\frac{1}{3}z^{-1}}\right]; \quad |z| < \frac{1}{3}.$$

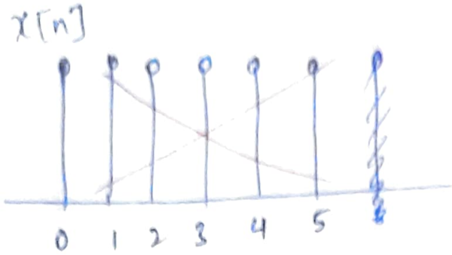
7.2.4 sol

$$x[n] = \begin{cases} 1; & 0 \le n \le 5 \\ 0; & \text{else where} \end{cases}$$

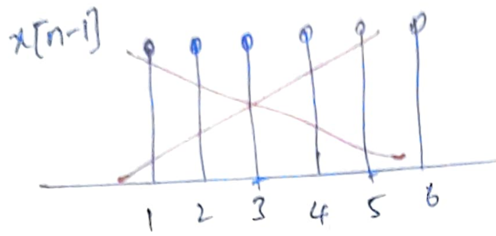
$g(n) = x(n) - x(n-1)$

$x(n) \xleftrightarrow{Z \cdot T} X(z)$

$G(z) = X(z) - X(z) \cdot z^{-(+1)}$



$x[n-n_0] \longleftrightarrow z^{-n_0} X(z)$



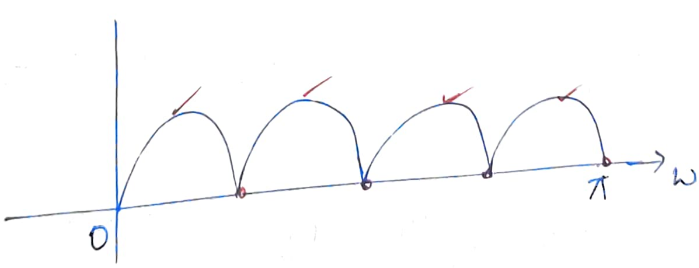
$g[n] = x[n] - x[n-1]$

$$\Big\downarrow Z \cdot T \quad = \delta[n] - \delta[n-6]$$

$G(z) = 1 - z^{-6} \quad ; |z| > 0$

$\Downarrow$

→ The application of this system is comb filter. It is used in T.V which separates colour & black/white component



$X(z) = \frac{z^3 - 2z}{z - 2}$; $x[n]$ is left sided.

Sol $X(z) = \frac{z(z^2 - 2)}{z - 2}$

$z-2\overline{)\,z^3-2z\,}\left(z^2+2z\right.$

z^3-2z^2

$0+2z^2-2z$

$2z^2-4z$

$+2z$

$$X(z) = z^2 + 2z + \frac{2z}{z-2} \qquad \left(\because\ \delta[n-n_0] \xleftrightarrow{Z.T} z^{-n_0}\right).$$

I.Z.T

$$x[n] = \delta[n+2] + 2\delta[n+1] + 2(2)^n\,[-u[-n-1]].$$

left sided.

Method -2 :

$$X(z) = \frac{z^3}{z-2} - \frac{2z}{z-2}$$

$$= z^2 Y(z) - 2Y(z) \qquad ; \quad Y(z) = \frac{z}{z-2}$$

I.Z.T

$$x[n] = y[n+2] - 2y[n] \qquad y[n] = 2^n\left[-u[-n-1]\right].$$

3. Exp multiplication (or) Scaling in z- domain :

For D.T. F.T we have

$$x[n]\,e^{j\omega_0 n} \longleftrightarrow X\left[e^{j(\omega-\omega_0)}\right]$$

for Z.T

Now $a = e^{j\omega_0}$ & $z = e^{j\omega}$

$$\boxed{a^n x[n] \longleftrightarrow X(z/a) \quad ; \quad ROC = |a| R}$$

Application of the above property.

Eg: $y[n] = \cos\omega_0 n \; u[n]$

$$= \left[\frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}\right] x[n]$$

let $x[n] = u[n]$

$$X(z) = \frac{1}{1 - z^{-1}} \; ; \; |z| > 1.$$

$\downarrow$ Z.T

$$Y(z) = \frac{X\left(z/e^{j\omega_0}\right) + X\left(z/e^{-j\omega_0}\right)}{2}$$

$$Y(z) = \frac{1}{2}\left[\frac{1}{1 - z^{-1}e^{j\omega_0}} + \frac{1}{1 - z^{-1}e^{-j\omega_0}}\right]$$

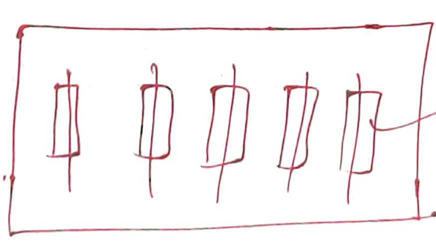
$$Y(z) = \frac{1 - z^{-1}\cos\omega_0}{1 - 2z^{-1}\cos\omega_0 + z^{-2}} \; ; \; |z| > 1 \cdot |e^{\pm j\omega_0}| > 1$$

$\Downarrow$

Digital Resonator.

Application: (Refer practical approach to D.S.P : Ifeactor & Jervis (LPE))

1. It is used as a 2^{nd} order band pass filters in 5-band Graphic Equaliser.



Band pass filter

2. Used as 2^{nd} order band pass filter in Touch tone DTMF.

	[illegible]	[illegible]
697	1	9
770	4	5
	7	8

Goertzel algorithm of $Y(z)$ is the basic formula … Purchange immediately

$\Downarrow$

D.F.T

Analog and Digital S.P (By Ashok Ambardar)

Modern DSP

Echo

$\Downarrow$

reverb filter

Used to reduce the echo

Download free ebooks (standard text books).

(i) esnips.com

(ii) ebook29blogspot.com

(iii) ebookee.com

→ for digital communication refer →(Gaussianwaves.com).

7.2.7 Sol

$$X(z) = \frac{(z+j)(z-j)}{\left(z-\frac{1}{2}\right)}$$

$$y[n] = \left(\frac{1}{2}\right)^n x[n] \Rightarrow Y(z) = X\left(\frac{z}{1/2}\right)$$

$\left(a = \frac{1}{2}\right)$

$$= X(2z) = \frac{4z^2+1}{2z-\frac{1}{2}}$$

Time-reversal:

Reflection in the time domain is inversion in the z-domain.

$$x[-n] \longleftrightarrow X(z^{-1}) \quad ; \quad ROC = \frac{1}{R}$$

Eg:

(*) $y[n] = 3^{n} u[-n]$.

Sol. $y[n] = \left[\frac{1}{3}\right]^{-n} u[-n]$ where $x[n] = \left[\frac{1}{3}\right]^{n} u[n]$.

$= x[-n]$

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} \; ; \; |z| > \frac{1}{3}.$$

$\downarrow$ z.T

$Y(z) = X(z^{-1})$

$$Y(z) = \frac{1}{1-\frac{z}{3}} \; ; \; |z| < \frac{1}{(1/3)}.$$

$; \; |z| < 3$

Differentiation in z-domain:

$$n\,x[n] \longleftrightarrow -z\frac{d}{dz}X(z) \quad ; \quad ROC = R$$

for reference: $-jn \longleftrightarrow \frac{d}{dw}$

$z = e^{jw}$

$dz = je^{jw}\,dw = jz\,dw$.

$$\boxed{dw = \frac{dz}{jz}}$$