FOURIER TRANSFORM :

→ Transformation is the process in which one domain is converted to other domain such that signal analysis becomes easy.

→ For any non-periodic signal, As $T \to \infty$ & $w_0 \to 0$, the discrete spectrum of fourier series is converted to continuous spectrum in fourier transform.

$$TC_n = \int_{-T/2}^{T/2} x(t)\, e^{-jnw_0 t}\, dt.$$

Now $\left.\begin{array}{l} nw_0 \to w \\ T \to \infty \end{array}\right\}$ F.S $\xrightarrow{\text{to}}$ F.T.

$$\lim_{T\to\infty} TC_n = \int_{-\infty}^{+\infty} x(t)\, e^{-jwt}\, dt = X(w).$$

fourier transform →

$$\therefore \mathcal{F}.T\,[x(t)] = X(w) = \int_{-\infty}^{+\infty} x(t)\, \underbrace{e^{-jwt}}_{\text{Kernel.}}\, dt$$

Inverse fourier transform →

$$\therefore \mathcal{F}.T^{-1}\,[X(w)] = x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(w)\, e^{+jwt}\, dw$$

→ $\boxed{w = 2\pi f}$

$dw = 2\pi\, df$

$\frac{dw}{2\pi} = df$

$\boxed{2\pi\, \delta(w) = \delta(f)}$

$2\pi\delta(2\pi f) = 2\pi \cdot \frac{1}{2\pi} \delta(f) = \delta(f)$

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt \;; \quad x(t) = \int_{-\infty}^{+\infty} X(f)\, e^{+j2\pi f}\, df.$$

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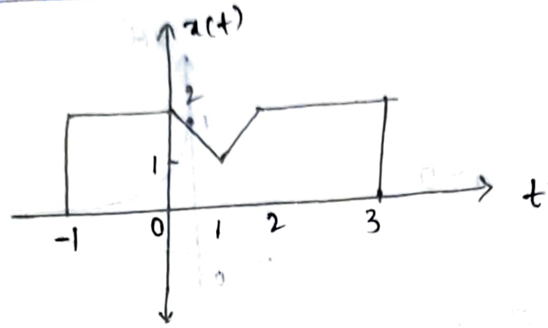
4.1.1. $x(t) \longrightarrow$ voltage $\rightarrow$ volts

$$X(f) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j2\pi ft}\, dt$$

Units of $X(f)$ = (volt · sec) (or) (volt / Hz)

4.1.2



(a) $X(0)$ = Area under the curve = (Rectangular area) − (Δ^{le} area)

$$X(0) = (4 \times 2) - \left(\frac{1}{2} \cdot 2 \cdot 1\right) = 7.$$

(b) $$\int_{-\infty}^{+\infty} X(\omega)\, d\omega = 2\pi x(0) = 2\pi \times 2 = 4\pi.$$

Area under one domain corresponds to observing the other domain at origin.

4.1.3. $$x(t) = \begin{cases} e^{-t} ; & t \geq 0 \\ 0 ; & t < 0 \end{cases}$$

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega)\, d\omega = x(0) = e^{-0} = e^{0} = 1$$

Convergence of fourier transform :-

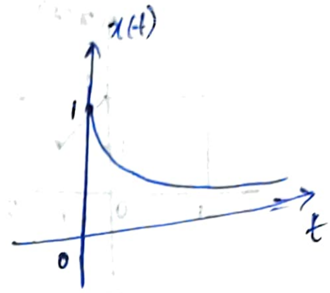
1. F.T is defined for stable & energy signals.

2. F.T power signals is defined as approximation to energy signals.

3. Fourier transform is not defined for neither stable nor square integrable signals. i.e., it is not possible for neither energy nor power signals (or) neither stable nor square integral signals.

Fourier transform of standard signals:

1. Decaying exponential:



$$x_1(t) = e^{-at}\,u(t) \;;\; a>0$$

$$X_1(\omega) = \int_0^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$X_1(\omega) = \int_0^{\infty} e^{-(a+j\omega)t}\, dt = \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty}$$

$$X_1(\omega) = \frac{1}{a+j\omega}$$

$$\boxed{\therefore\; e^{-at}\,u(t) \xrightarrow{F\cdot T} \frac{1}{(a+j\omega)}}$$

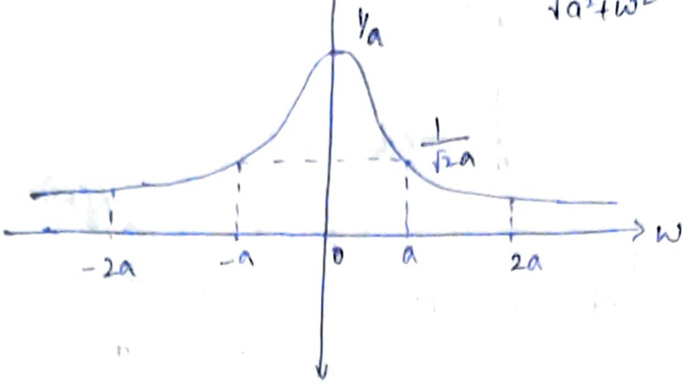
Magnitude Spectrum:-

$$|X_1(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

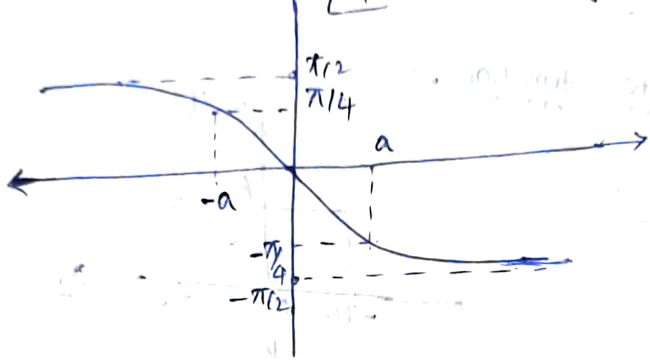
Phase spectrum:

$$\angle X_1(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

$$|X_1(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$



$$\angle X_1(\omega) = -\tan^{-1}(\omega/a)$$



→ F.T of a real valued signal exhibits conjugate symmetry.

Real valued $\xrightarrow{F.T}$ Conjugate Symmetry.

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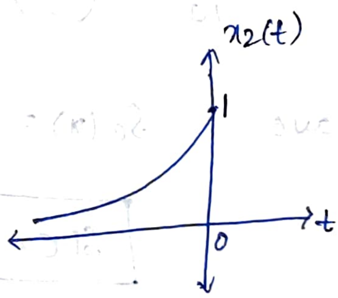
2. <u>Increasing</u> <u>exponential</u> :

$x_2(t) = e^{at}\, u(-t)\,;\ a > 0$



Time reversal

$x(t) \longleftrightarrow X(-\omega)$.

$x_2(t) = x_1(-t)$.

$\downarrow F.T$

$X_2(\omega) = X_1(-\omega) = \dfrac{1}{a-j\omega}$.

By the application of time reversal property phase spectrum changes, magnitude spectrum will remain same.

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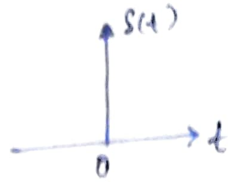
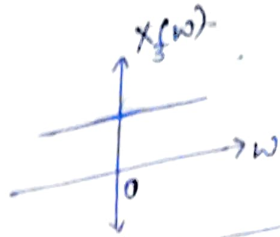
3. Impulse function :

$$x_3(t) = \delta(t)$$

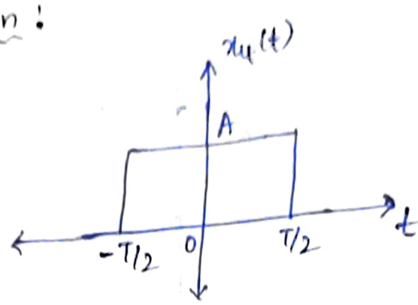


$$X_3(\omega) = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t}\, dt = 1.$$



4. Rectangular (or) Gate function :

$$x_4(t) = A\,\mathrm{rect}\left(t/T\right)$$



$$X_4(\omega) = \int_{-\frac{T}{2}}^{T/2} A e^{-j\omega t}\, dt = A \int_{-T/2}^{T/2} e^{-j\omega t}\, dt = \cancel{\frac{2A\sin}{\omega}\left(\frac{\omega T}{2}\right)}.$$

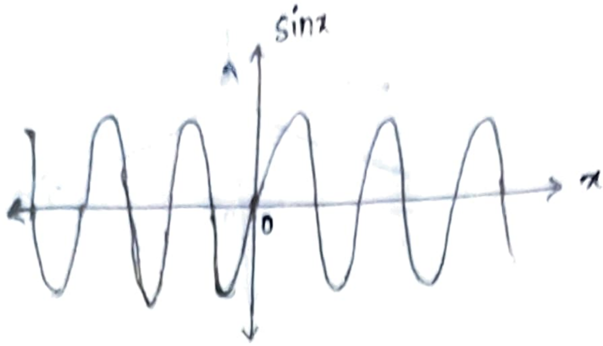
$$X_4(\omega) = \frac{2A}{\omega} \sin\left(\frac{\omega T}{2}\right)$$

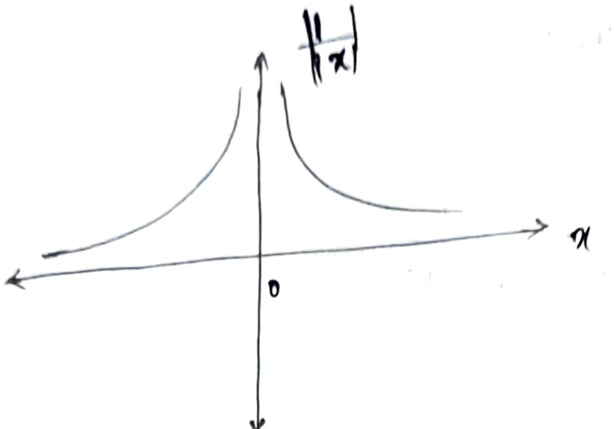
We have $Sa(x) = \frac{\sin x}{x}$; $\operatorname{sinc} x = \frac{\sin \pi x}{\pi x}$.

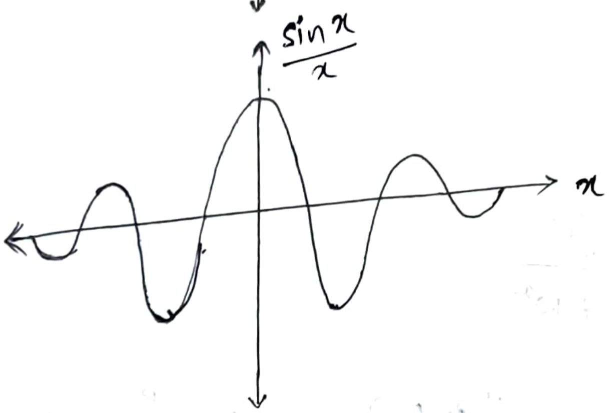
$$\boxed{\operatorname{Sinc} x = Sa(\pi x)}$$

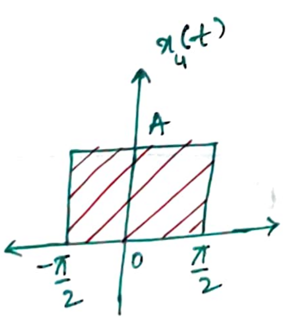
$$X_4(\omega) = \frac{2A \sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right) \cdot \frac{2}{T}} = AT\, Sa\left(\frac{\omega T}{2}\right) \text{ (or) } AT \operatorname{Sinc}\left(\frac{\omega T}{2\pi}\right)$$

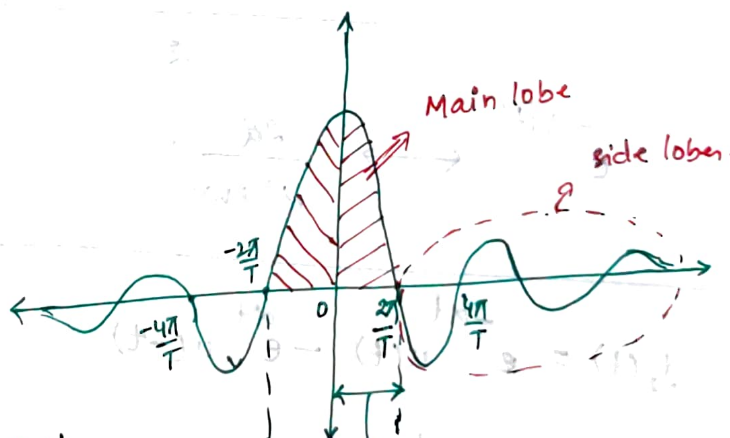
$$\boxed{A\,\mathrm{rect}\left(t/T\right) \xrightarrow{F \cdot T} AT\, Sa\left(\frac{\omega T}{2}\right) \text{ (or) } AT \operatorname{sinc}\left(\frac{\omega T}{2\pi}\right)}$$











null-to-null B.W or zero crossing B.W = $\frac{2\pi}{T}$

Practical Band width = $\frac{2\pi}{T}$

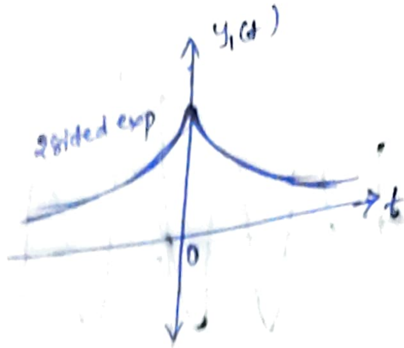
Spectral Bandwidth = $\frac{4\pi}{T}$.

→ A signal cannot be time limited & bandlimited simultaneously.

Exception is Gaussian curve.

$y_1(t) = e^{-\alpha|t|}$

$$= \begin{cases} e^{-\alpha t} & ; t > 0 \\ e^{\alpha t} & ; t < 0 \end{cases}$$



$y_1(t) = e^{-\alpha t} u(t) + e^{\alpha t} u(-t)$

$\downarrow$ F.T

$$Y_1(\omega) = \frac{1}{\alpha + j\omega} + \frac{1}{\alpha - j\omega} = \frac{2\alpha}{\alpha^2 + \omega^2}$$

Aliter:-

Real $\longleftrightarrow$ Even.

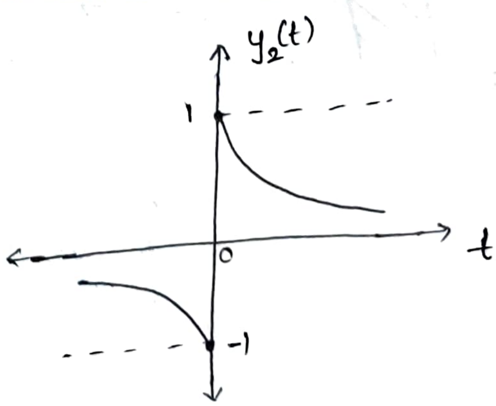
$x(t) = e^{-\alpha t} u(t)$

$$X(\omega) = \frac{1}{\alpha + j\omega} = \frac{\alpha - j\omega}{\alpha^2 + \omega^2}$$

$$x_{even}(t) = \frac{e^{-\alpha t} u(t) + e^{\alpha t} u(-t)}{2} \longleftrightarrow \frac{\alpha}{\alpha^2 + \omega^2}$$

$$e^{-a|t|} \longleftrightarrow \frac{2\alpha}{\alpha^2 + \omega^2}$$

$y_2(t) = e^{-\alpha t} u(t) - e^{\alpha t} u(-t)$



$\downarrow$ F.T

Sol

$$Y_2(\omega) = \frac{-2j\omega}{\alpha^2 + \omega^2}$$

$$\lim_{\alpha \to 0} y_2(t) = u(t) - u(-t) = \text{sgn}(t)$$

$$\lim_{\alpha \to 0} Y_2(\omega) = \frac{-2j\omega}{\omega^2} = \frac{2}{j\omega}$$

Sgn function is power signal.

$$\therefore \boxed{\mathrm{sgn}(t) \xrightarrow{F.T} \frac{2}{j\omega}}$$

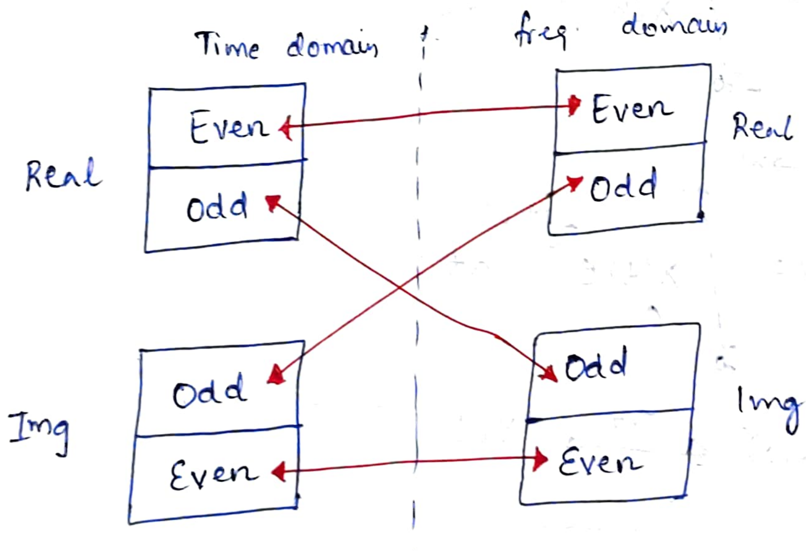
→ Real & Even (time domain) $\xrightarrow{F.T}$ Real & Even (freq domain).

→ Real & Odd (time domain) $\xrightarrow{F.T}$ Img & Odd (freq domain).

→ Img & Odd (time domain) $\xrightarrow{F.T}$ Real & Odd (freq domain)

→ Img & Even (time domain) $\xrightarrow{F.T}$ Img & Even (freq domain).

→



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④ $$X(\omega) = \frac{\omega^2 + 21}{\omega^2 + 9}.$$

Sol: The above function is proper.

$$X(\omega) = 1 + \frac{12}{\omega^2 + 9}$$

$$= 1 + 2\left[\frac{2(3)}{\omega^2 + 3^2}\right]$$

$$\downarrow \text{I.F.T}$$

$$\boxed{x(t) = \delta(t) + 2e^{-3|t|}}$$

Duality (Similarity) :

$$x(t) \xleftrightarrow{F.T} X(\omega)$$

$$X(t) \xleftrightarrow{F.T} 2\pi x(-\omega).$$

Proof :- $$x(t) = \frac{1}{2\pi}\int X(\omega) e^{j\omega t} d\omega.$$

Replace "t" by "-t".

$$2\pi x(-t) = \int_{-\infty}^{+\infty} X(\omega) e^{-j\omega t} d\omega.$$

$$t \longleftrightarrow \omega$$

$$2\pi x(-\omega) = \int_{-\infty}^{+\infty} X(t) e^{-jt\omega} dt$$

$$\therefore 2\pi x(-\omega) \xleftrightarrow{F.T} X(t).$$

Q. find the F.T of $\frac{1}{5-jt}$

Sol

$$e^{5t} u(-t) \xleftrightarrow{F.T} \frac{1}{5-j\omega}.$$

$$\frac{1}{5-jt} \xleftrightarrow{duality} 2\pi e^{5(-\omega)} u(\omega)$$

(*) $e^{-|t|} \longleftrightarrow \dfrac{2}{\omega^2+1}$

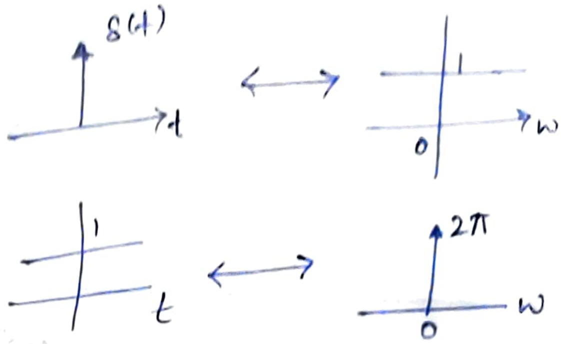
$$\frac{2}{t^2+1} \xleftrightarrow{\text{duality}} 2\pi e^{-|-\omega|}$$

$$= 2\pi e^{-|\omega|}.$$

(*) $\delta(t) \longleftrightarrow 1$

$$1 \xleftrightarrow{\text{duality}} 2\pi\,\delta(-\omega)$$

$$= 2\pi\,\delta(\omega)$$



Impulse in one domain = Constant in other domain

(*) $\text{sign}(t) \xleftrightarrow[\text{dual}]{} \dfrac{2}{j\omega}$

$$\frac{\cancel{2}}{jt} \longleftrightarrow \cancel{2}\pi\,\text{sgn}(-\omega)$$

$$\frac{1}{jt} \longleftrightarrow -\pi\,\text{sgn}(\omega)$$

$$\boxed{\frac{1}{\pi t} \longleftrightarrow -j\,\text{sgn}(\omega)}$$

↓ Impulse response of Hilbert transform.

→ $\text{sgn}(t) = 2u(t) - 1$

$$u(t) = \frac{1+\text{sgn}(t)}{2}$$

$$u(t) \xleftrightarrow{F\cdot T} \frac{2\pi\,\delta(\omega) + \frac{2}{j\omega}}{2}$$

$$\boxed{u(t) \xleftrightarrow{F\cdot T} \frac{1}{j\omega} + \pi\,\delta(\omega).}$$

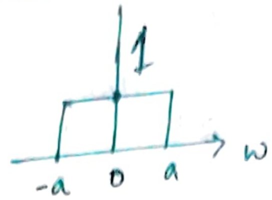
(*) $A=1$, $T=2a$

$$\text{rect}\left(\frac{t}{2a}\right) \xleftrightarrow[\text{Duality}]{F\cdot T} 2a\,Sa\left(\frac{2a\omega}{2}\right)$$

$$2a\,Sa(at) \longleftrightarrow 2\pi\,\text{rect}\left(\frac{-\omega}{2a}\right)$$

$$\frac{a\,\mathrm{sin}\,at}{at} \longleftrightarrow \pi\,\mathrm{rect}\left(\frac{w}{2a}\right)$$

$$\boxed{\frac{\sin at}{\pi t} \xleftrightarrow{F\cdot T}}$$



→ Rect functn in one domain becomes 'Sa' function in other domain.

⊛ Find I·F·T of $u(w)$?

Sol

$$u(t) \xleftrightarrow{F\cdot T} \frac{1}{jw} + \pi\delta(w)$$

$$\frac{1}{jt} + \pi\delta(t) \xleftrightarrow{\text{dual}} 2\pi u(-w).$$

Apply time reversal i.e, $(-t \rightarrow -w)$.

$$\left[\frac{1}{j(-t)} + \pi\delta(-t)\right]\frac{1}{2\pi} \xleftrightarrow{F\cdot T} u(w).$$

Time - Scaling :-

$$x(\alpha t) \longleftrightarrow \frac{1}{|\alpha|}X\left(\frac{w}{\alpha}\right)$$

Compression ⟷ Expansion.

⊛ $y_1(t) = A\,\mathrm{rect}\left(\frac{2t}{T}\right)$.

we have

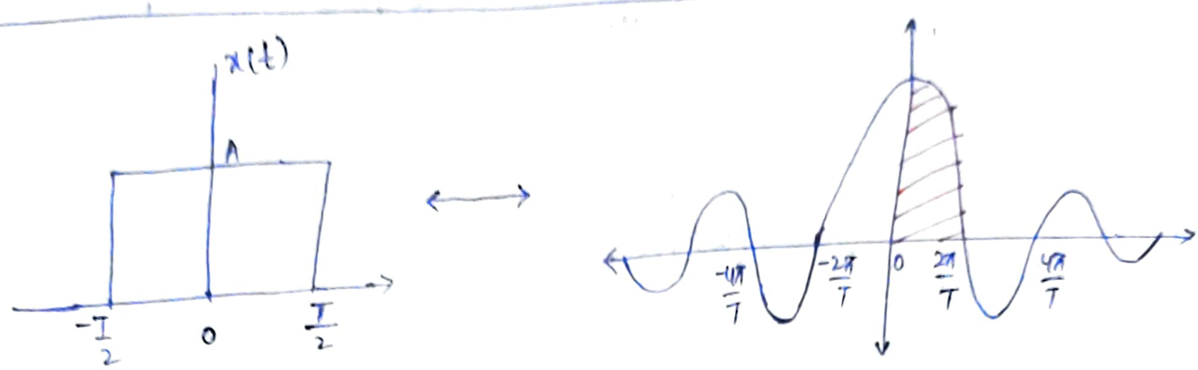
$$\underset{x(t)}{A\,\mathrm{rect}(t/T)} \longleftrightarrow \underset{X(w)}{AT\,Sa\left(\frac{wT}{2}\right)}$$

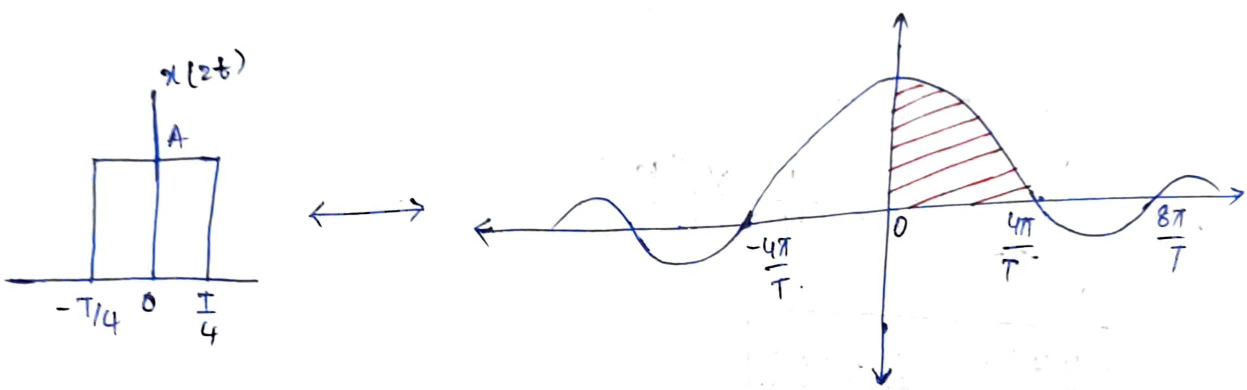
$y_1(t) = x(2t)$

$\alpha = 2$

$$\Big\downarrow \text{F.T}$$

$$Y_1(\omega) = \frac{1}{2}\, X\left(\frac{\omega}{2}\right) = \frac{AT}{2}\, Sa\left(\frac{\frac{\omega}{2}\cdot T}{2}\right) = \frac{AT}{2}\, Sa\left(\frac{\omega T}{4}\right)$$





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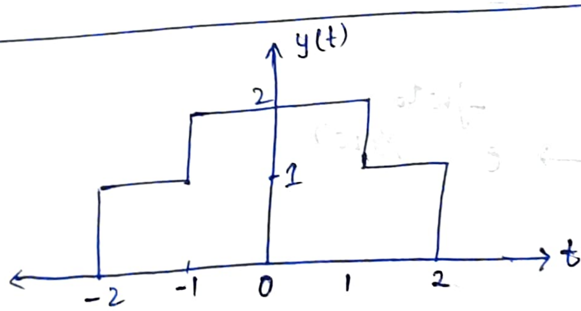
4.2.4.

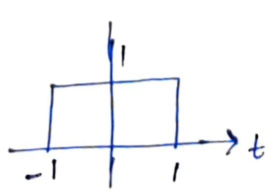
sol.



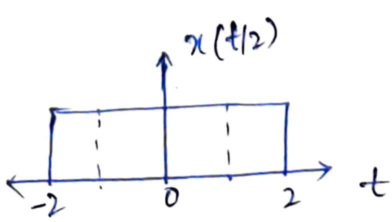
Assume $x(t)$; $X(\omega)$

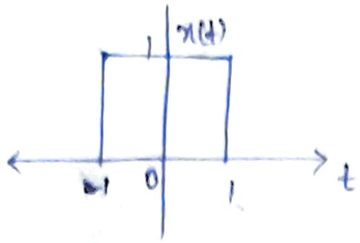
$A=1$

$T=2$



$\xleftrightarrow{\text{F.T}}$ $2\,Sa(\omega)$





$$y(t) = x(t) + x\left(\frac{t}{2}\right)$$

$a = \frac{1}{2}$ $\Big\downarrow$ F.T

$$Y(w) = X(w) + \frac{1}{\left|\frac{1}{2}\right|} X\left(\frac{w}{\frac{1}{2}}\right)$$

$$\boxed{Y(w) = X(w) + 2X(2w)}$$

$$\boxed{\therefore Y(w) = 2\,Sa(w) + 2\left[2\,Sa(2w)\right]}$$

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pg 104.

Q1.

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$$Y(f)\Big|_{f=1} = Y(w)\Big|_{w=2\pi}$$

$$Y(2\pi) = 0$$

P.g-79.

4.2.5.

$$Y(w) = 3X(2w)$$

$a = \frac{1}{2}$

$\Big\downarrow$ I.F.T

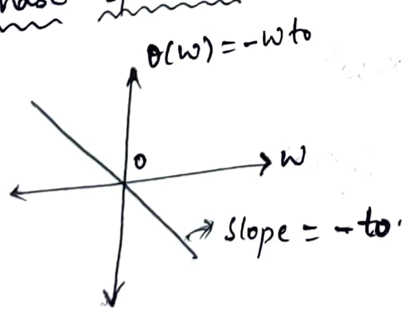
$$= 3\cdot\left(\frac{1}{2}\right)\frac{1}{\left|\frac{1}{2}\right|} X\left[\frac{w}{\frac{1}{2}}\right]$$

$$\boxed{y(t) = \frac{3}{2}\,x\left(\frac{t}{2}\right)}$$

Time shifting:-

$$x(t - t_0) \xleftrightarrow{F.T} e^{-jwt_0} X(w)$$

Phase spectrum:-



(d) $y(t) = e^{2t} u(-t+3)$.

gre

sol

$x(t) = e^{at} \cancel{u(-t)}$

$$y(t) = e^{2(t-3+3)}\, u[-(t-3)] \quad ; \quad t_0 = 3$$

$\downarrow$ F.T

$$\boxed{Y(\omega) = e^{6}\left[\frac{e^{-j\omega(3)}}{2-j\omega}\right]}$$

(*) $z(t) = \text{rect}\left[\frac{t+1}{4}\right] = \text{rec}\left[\frac{1}{4}(t+1)\right]$

Sol. $= x(t+1)$ where $x(t) = \text{rect}(t/4)$.

F.T $\downarrow$ $t_0 = -1$. $A = 1$ & $T = 4$.

$$z(\omega) = e^{-j\omega \times (-1)}\, X(\omega).$$

$$X(\omega) = 4\,Sa(2\omega)$$

$$\boxed{z(\omega) = e^{j\omega}\, X(\omega)}$$

$$\boxed{z(\omega) = e^{j\omega}\, 4\,Sa(2\omega)}$$

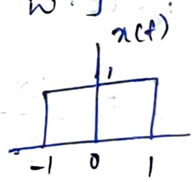
Gate 2004.

(*) Given $H(\omega) = \frac{4\sin 2\omega \cos\omega}{\omega}$; Find $h(0) = ?$

refer

Sol

$$h(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\, d\omega = 4\left[\frac{2\sin\omega}{\omega}\right]\cos^2\omega$$



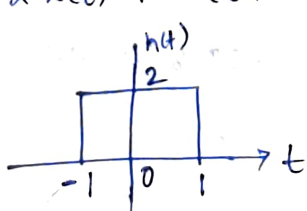
$\longleftrightarrow$ $X(\omega)$.

$$h(0) = 4X(\omega)\left[\frac{1+\cos 2\omega}{2}\right]$$

$$H(\omega) = 2X(\omega) + e^{j2\omega}X(\omega) + e^{-j2\omega}X(\omega).$$

$\downarrow$ I.F.T

$$h(t) = 2x(t) + x(t+2) + x(t-2).$$



At $t = 0$; $h(t) = 2$

Pg 81:

4·2·11.

(b) → $f_1(t) = f(-t)$

$\downarrow$ F.T

$F_1(\omega) = F(-\omega)$

→ $f_2(t) = f(t-1) + f_1(t-1)$

$\downarrow$ F.T

$F_2(\omega) = e^{-j\omega(1)} F(\omega) + e^{-j\omega(1)} F_1(\omega)$

→ $f_3(t) = f(t-1) + f_1(t+1)$

$\downarrow$ F.T

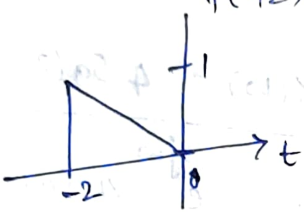
$F_3(\omega) = e^{-j\omega(1)} F(\omega) + e^{-j\omega(-1)} F_1(\omega)$

• $f_4(t) = f\left(\frac{t}{2} - 1\right) + f_1\left(\frac{t}{2} + \frac{1}{2}\right)$.

$\downarrow$ F.T

$F_4(\omega) = e^{-j\omega(\frac{1}{2})} F(\omega) + e^{-j\omega(\frac{-1}{2})} F_1(\omega)$

→ $f(t/2)$



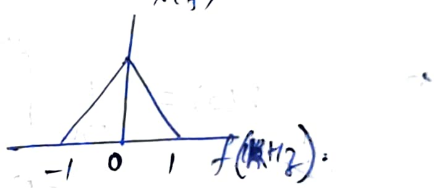
$f_5(t) = 1.5 f\left(\frac{t-2}{2}\right)$.

Modulation :

Frequency Shift (or)

$x(t)\, e^{j\omega_c t} \longleftrightarrow X(\omega - \omega_c)$

$x(f)$



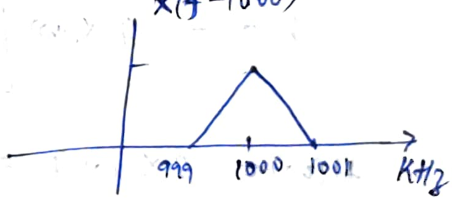
~~x(t) e^{j}~~

Voice ⇒ 3 KHz

$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^3} = 100\,Km$

↘ MHz

$x(f-1000)$



$\cos\omega_c t = \frac{1\cdot e^{j\omega_c t} + 1\cdot e^{-j\omega_c t}}{2}$

$1 \xleftrightarrow{F.T} 2\pi\,\delta(\omega)$

$\cos\omega_c t \xleftrightarrow{F.T} \frac{2\pi\,\delta(\omega-\omega_c) + 2\pi\,\delta(\omega+\omega_c)}{2}$