Fourier Series :

→ Fourier series and fourier transform are the mathematical tools that provide freq. domain representation.

→ Fourier series (F.S) is an approximation process where a non sinusoidal ~~process~~ waveform is converted to sinusoidal waveform such that all the periodic signals are represented in unique form.

→ For a signal to have (F.S) orthogonality is a must condition (two signals $x_1(t)$ and $x_2(t)$ are orthogonal if

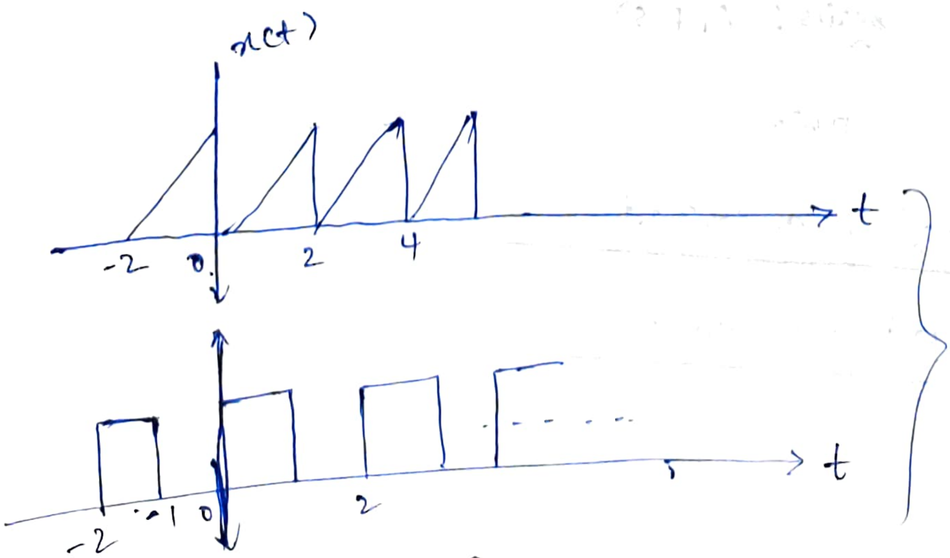
iff $\int_{t_1}^{t_2} x_1(t)\, x_2(t) = 0$; i.e, area under the inner product of the signals is '0'.).

Advantages of Fourier series :

1. We can find the spectral width.
2. We can find steady state response due to periodic inputs easily.

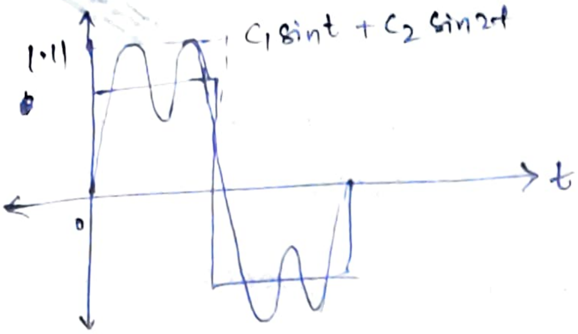
Aim of Fourier series : One period ⇒ many frequencies.

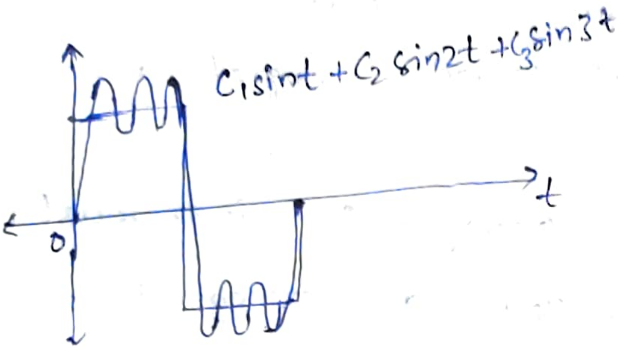
Mathematically $\boxed{\omega_0 \Rightarrow n\omega_0}$

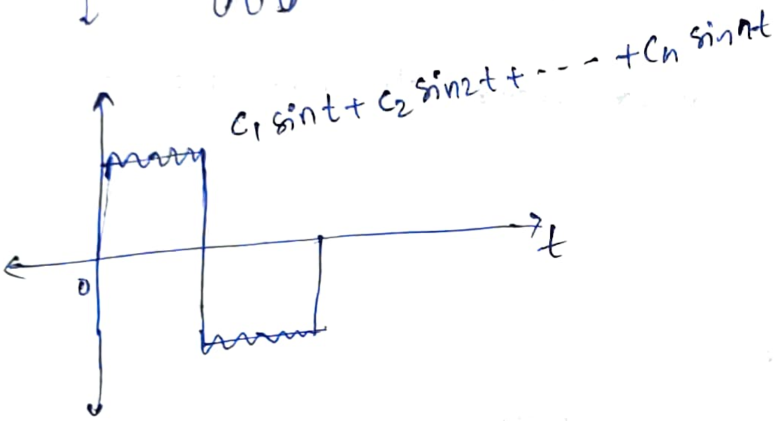


$\sin \omega_0 t$

$\cos \omega_0 t$.







→ Four classes given by fourier.

Frequency \ Time	Continuous	Discrete
Continuous	C.T.F.T	D.T.F.T
Discrete.	C.F.S	DF.T/DFS

Trigonometric fourier series: (T.F.S)

$0,\ \omega_0,\ 2\omega_0,\ \cdots\ n\omega_0,\ \cdots$

$$g(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \cdots$$

$$+ b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \cdots$$

$$\boxed{g(t) = \underbrace{a_0}_{\text{dc comp.}} + \sum_{n=1}^{\infty} \underbrace{\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)}_{\text{ac component}}}$$

Here a_0, a_n, b_n are in freq domain which will convert $g(t)$ (time domain) into freq. domain

Now

→ $\displaystyle\int_0^T \cos n\omega_0 t \, dt = \int_0^T \sin m\omega_0 t \, dt = 0$
$n \neq 0$

→ $\displaystyle\int_0^T \cos m\omega_0 t \cos n\omega_0 t \, dt = \begin{cases} \frac{T}{2} ; & m = n \\ 0 ; & m \neq n \end{cases}$

→ $\displaystyle\int_0^T \sin m\omega_0 t \cdot \sin n\omega_0 t \, dt = \begin{cases} \frac{T}{2} ; & m = n \\ 0 ; & m \neq n . \end{cases}$

→ $\displaystyle\int_0^T \sin m\omega_0 t \cos n\omega_0 t \, dt = 0 \quad \forall\, m \,\&\, n.$

Now.

$$\boxed{a_0 = \text{dc (or) Avg value} = \frac{1}{T}\int_0^T g(t)\, dt = \frac{\text{Area of } g(t) \text{ over a period}}{\text{fundamental period}}}$$

$$\boxed{a_n = \frac{2}{T}\int_0^T g(t) \cos n\omega_0 t \, dt}$$

$$\boxed{b_n = \frac{2}{T}\int_0^T g(t) \sin n\omega_0 t \, dt}$$

Polar form of T.F.S : (P.F.S) :

$$\boxed{g(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos(n\omega_0 t + \theta_n)}$$

Id. By comparing with T.F.S

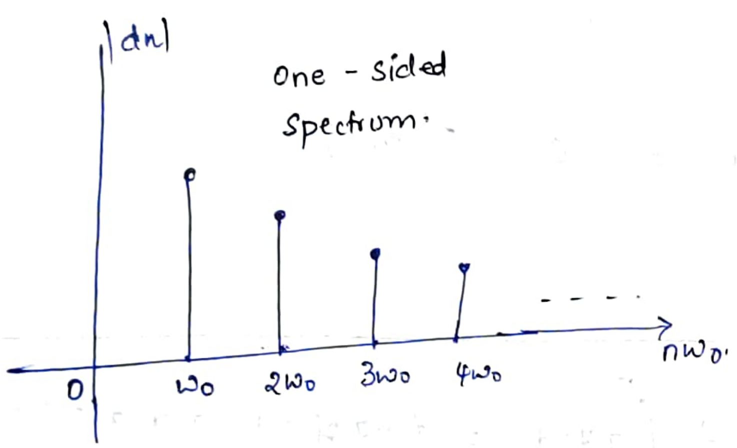
$a_n = d_n \cos\theta_n$; $b_n = -d_n \sin\theta_n$; We get

$$\boxed{|d_n| = \sqrt{a_n^2 + b_n^2}} \longrightarrow \text{Magnitude spectrum}$$

$$\boxed{\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)} \longrightarrow \text{Phase spectrum}$$

Magnitude & phase spectrum together is known as amplitude spectrum.

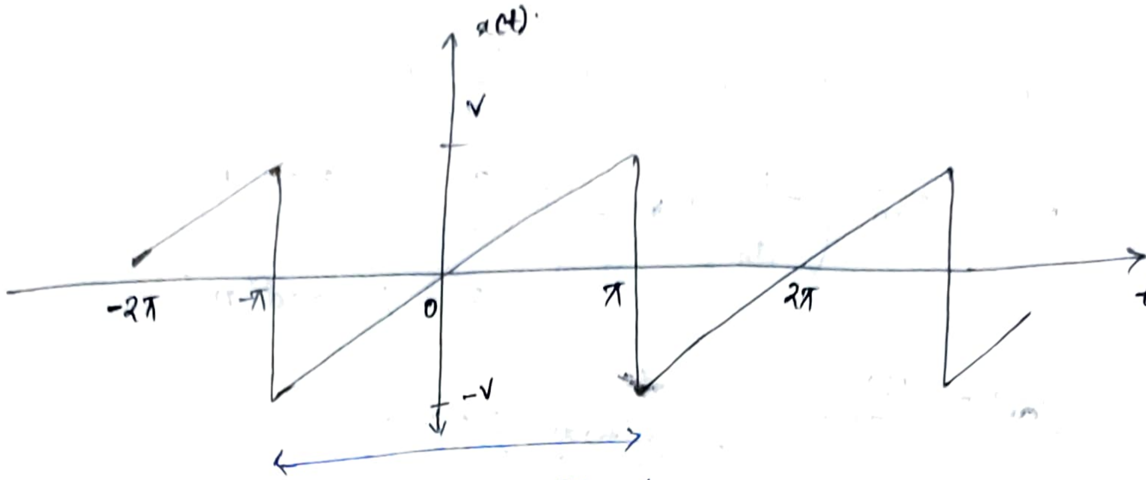
→ Spectrum :



Note :

From the shape of magnitude spectrum we infer that upto what range of frequencies the maximum power is there and fourier series spectrum is discrete.

* Find the T.F.S representation for the periodic signal shown below.



$T = 2\pi \Rightarrow \omega_0 = \frac{2\pi}{T} = 1.$

odd $\Rightarrow a_0 = a_n = 0$

$$x(t) = \left\{ \frac{Vt}{\pi} \;;\; -\pi < t < \pi. \right.$$

$$b_n = \frac{2}{T} \int_0^T \underbrace{x(t)}_{\text{odd}} \underbrace{\sin n\omega_0 t}_{\text{odd}}\, dt = \frac{4}{T} \int_0^{T/2} x(t) \sin \omega_0 t \, dt.$$

$$b_n = \frac{4}{2\pi} \int_0^{\pi} \frac{Vt}{\pi} \sin nt \, dt = \frac{2V}{\pi^2} \int_0^{\pi} t \sin nt \, dt. \quad (\because T = 2\pi).$$

$$b_n = \frac{2V}{\pi^2}. \left[\frac{-t \cos nt}{n} + \frac{\sin nt}{n^2} \right]_0^{\pi}$$

$$\left[\begin{array}{l} \because \cos n\pi = (-1)^n \\ \sin n\pi = 0 \\ e^{\pm jn\pi} = (-1)^n \end{array} \right]$$

$$b_n = \frac{2V}{\pi^2} \left[\frac{-\pi (-1)^n}{n} \right] = \frac{2V}{\pi n} (-1)^{n+1}$$

$$b_n = \begin{cases} \frac{-2V}{\pi n} & ; \; n \text{ is even} \\ \frac{2V}{\pi n} & ; \; n \text{ is odd.} \end{cases}$$