CHAPTER - 6

DTFT : (Discrete time fourier Transform)

C.T.F.T	D.T.F.T
ω : $-\infty$ to ∞	ω : $-\pi$ to π (0 to 2π
Non periodic Spectrum	Periodic Spectrum.

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Now C.T.F.T $\rightarrow$ $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$.

forward formulae

D.T.F.T $\rightarrow$ $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$.

Now C.T.F.T $\longrightarrow$ $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{+j\omega t} d\omega$

inverse formulae.

D.T.F.T $\rightarrow$ $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$.

Periodicity :- of D.T.F.T :

$$X\left[e^{j(\omega+2\pi K)}\right] = X(e^{j\omega})$$

Proof :- $e^{j(\omega+2\pi K)n} = e^{-j\omega n} \cdot e^{-j2\pi K n} = e^{-j\omega n}$

$\rightarrow 1$

Spectrum repeates for every 2π.

For D.T.F.T

$$x[n] \xleftrightarrow{D.T.F.T} X(e^{j\omega}).$$

Purpose to study D.T.F.T :-

whatever we have done for C.T.F.T ~~we~~. To perform that in computer we use D.T.F.T to descrease the complexity

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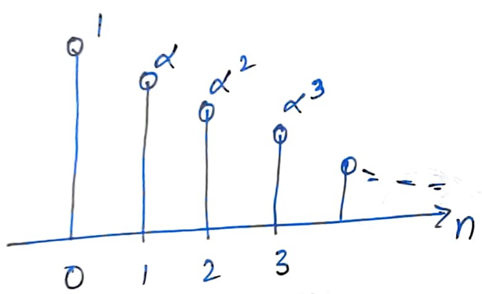
D.T.F.T of standard signal :-

1. $x[n] = \alpha^n u[n]$; $|\alpha| < 1$



for $|\alpha| > 1$ F.T is not defined i.e, why we take $|\alpha| < 1$ ($\because$ decreasing).

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n} = \sum_{n=0}^{\infty} (\alpha e^{-j\omega})^n$$

$$X(e^{j\omega}) = 1 + \alpha e^{-j\omega} + (\alpha e^{-j\omega})^2 + \cdots = \frac{1}{1-\alpha e^{-j\omega}}$$

$$\boxed{\therefore \quad \alpha^n u[n] \xleftrightarrow{D.T.F.T} \frac{1}{1-\alpha e^{-j\omega}}}$$

$$X(e^{j\omega}) = \frac{1}{1-\alpha\cos\omega + j\,\alpha\sin\omega}.$$

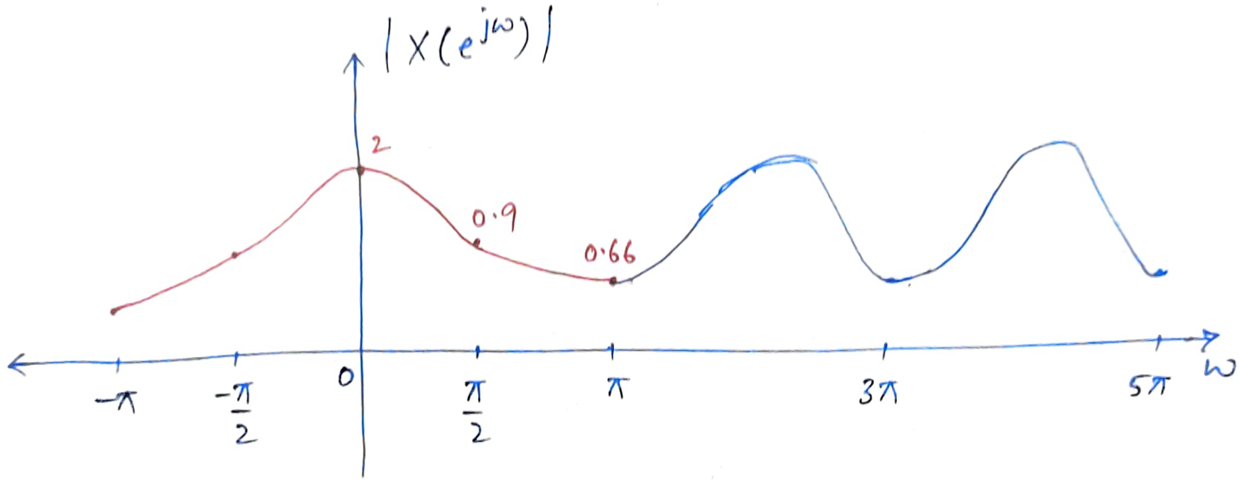
$$|X(e^{j\omega})| = \frac{1}{\sqrt{(1-\alpha\cos\omega)^2 + (\alpha\sin\omega)^2}}$$

$$|X(e^{j\omega})| = \frac{1}{\sqrt{1-2\alpha\cos\omega+\alpha^2}}$$

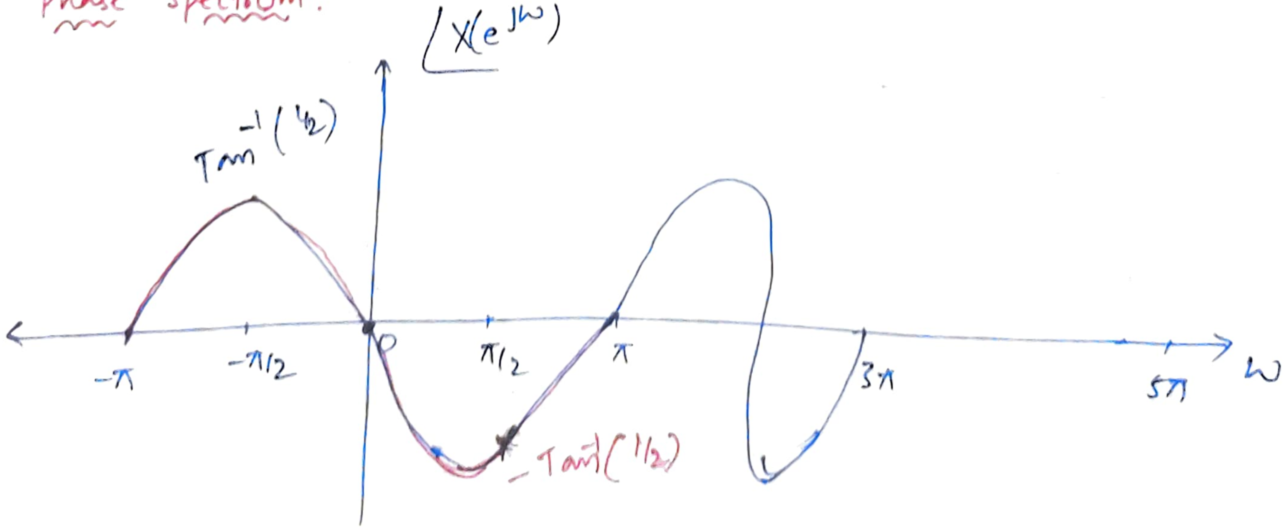
$$\angle X(e^{j\omega}) = -\tan^{-1}\left[\frac{\frac{1}{2}\sin\omega}{1-\frac{1}{2}\cos\omega}\right]$$

Let $\alpha = \frac{1}{2}$ $\Rightarrow |X(e^{j\omega})| = \frac{1}{\sqrt{1.25-\cos\omega}}$

Magnitude Spectrum:



Phase Spectrum:



→ Magnitude spectrum is even function only for DTFT

→ Phase Spectrum is odd function. " " "

Pg - 133.

6.1.2 (a) $x[n] = \left(\frac{1}{2}\right)^n u[n]$

sol:

$$y[n] = x^2[n] = \left[\frac{1}{4}\right]^n u[n] \xleftrightarrow{F.T} \frac{1}{1 - \frac{1}{4}e^{-j\omega}}$$

$$Y(e^{j0}) = \frac{1}{1 - \frac{1}{4}}$$

$\Downarrow$

Area

$$\sum_{n=-\infty}^{+\infty} y[n] = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

(b) $X(e^{j\omega}) = \cos^3(3\omega)$.

$$X(e^{j\pi}) = \sum_{n=-\infty}^{+\infty} x[n](-1)^n = \cos^3(3\pi) = (-1)^3 = -1$$

(At $\omega = \pi$).

$$\left(\because X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}\right).$$

(c) $n \to$ 0 1 2 3

$$h[n] = \{1, 2, 3, 4\}.$$

sub ($\omega = 0$) D.C gain ; $H(e^{j0}) = \sum_{n=0}^{3} h[n] = 1+2+3+4 = 10.$

sub ($\omega = \pi$) High freq gain ; $H(e^{j\pi}) = \sum_{n=0}^{3} h[n](-1)^n = 1 - 2 + 3 - 4 = -2.$

(At $\omega = \pi$)

6.1.3 (i) we have $x[n-n_0] \longleftrightarrow e^{-j\omega n_0} X(e^{j\omega})$

$$y[n] = \left(\frac{1}{4}\right)^{\underline{n-3}+3} u[\underline{n-3}] \quad \hookrightarrow n_0 = 3$$

$\downarrow$ D.T.F.T

$$Y(e^{j\omega}) = \frac{1}{64}\left[\frac{e^{-j\omega(3)}}{1-\frac{1}{4}e^{-j\omega}}\right]$$

(ii) $x[n] = \delta[6-3n]$.

we have. $\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$.

$$\delta[Kn] = \delta[n].$$

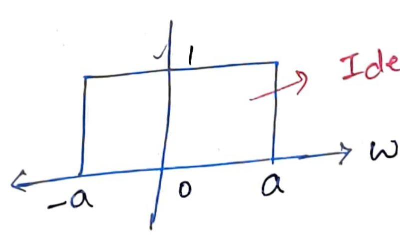
$$x[n] = \delta[-3(n-2)].$$

$$x[n] = \delta[n-2]$$

$\downarrow$ D.T.F.T

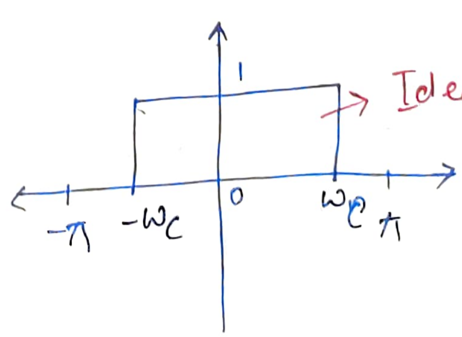
$$X(e^{j\omega}) = e^{-j2\omega}$$

6.1.4 for C.T.F.T $\frac{\sin at}{\pi t} \xleftrightarrow{F.T}$



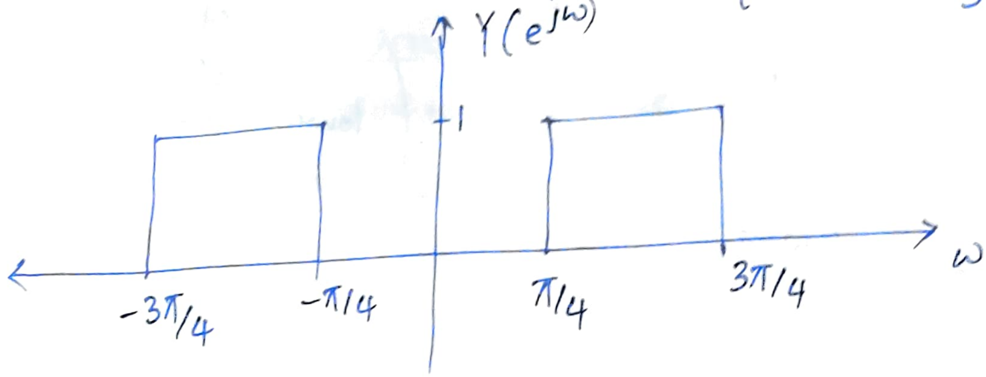
Ideal Analog L.P.F

for D.T.F.T $\frac{\sin \omega_c n}{\pi n} \xleftrightarrow{F.T}$



Ideal digital L.P.F

freq. shift $x[n]\, e^{j w_0 n} \longleftrightarrow X[e^{j(w-w_0)}]$.

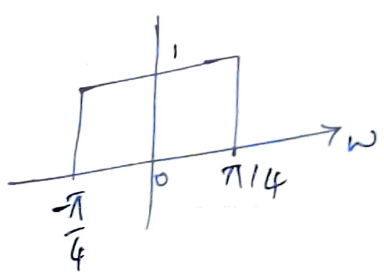


$$\frac{3\pi}{4} - \frac{\pi}{4} = w_c - (-w_c)$$

$$\frac{\pi}{2} = 2w_c$$

$$w_c = \frac{\pi}{4}.$$

Assume $X(e^{jw}) \longleftrightarrow x[n] = \dfrac{\sin \frac{\pi n}{4}}{\pi n}.$



→ obtained by right & left shift by $\pi/4$ units of the above graph.

$$Y(e^{jw}) = X[e^{j(w-\frac{\pi}{2})}] + X[e^{j(w+\pi/2)}].$$

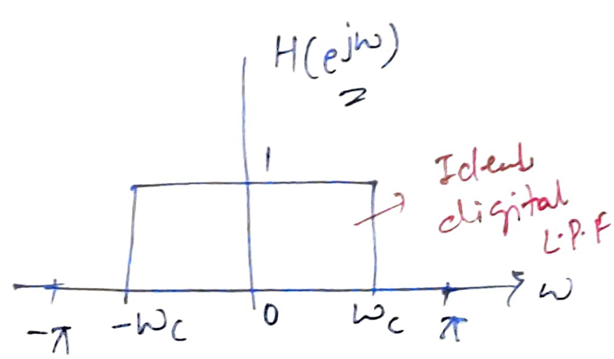
$\big\downarrow$ I.F.T

$$y[n] = x[n]\, e^{j\frac{\pi}{2}n} + x[n]\, e^{j(-\frac{\pi}{2})n}.$$

6.1.5 sol $g(n) = (-1)^n\, h(n)$.

-1 means → $w = \pi$

we have $h[n] = \dfrac{\sin w_c n}{\pi n} \xleftrightarrow{F.T}$



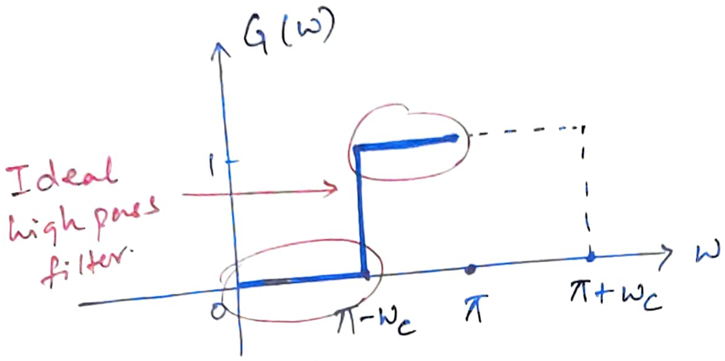
$g[n] = (-1)^n h[n]$.

$= e^{jn\pi} h[n]$

$\Big\downarrow$ F.T

$G(\omega) = H\left[e^{j(\omega-\pi)}\right]$.

(VUIMP
Always lowest freq in discrete DTFT is 0
highest " " " is π.)



(b). $y[n] = x[n] + (-1)^n x[n]$

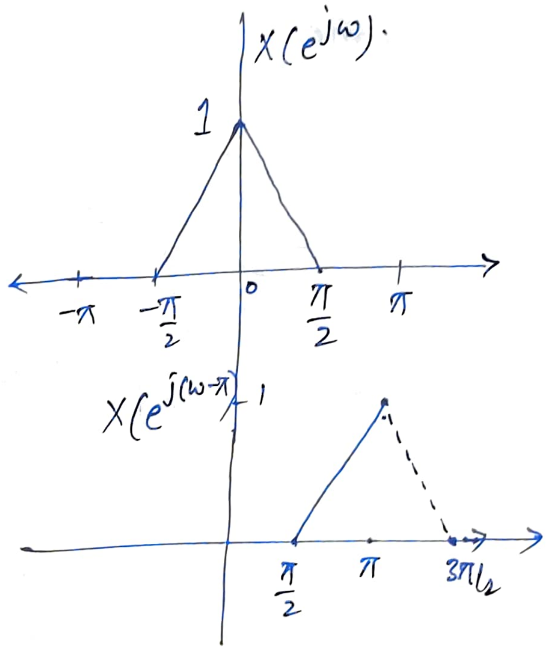
Sol.

$\Big\downarrow$ F.T

$Y(e^{j\omega}) = X\left[e^{j\omega}\right] + X\left[e^{j(\omega-\pi)}\right]$

$Y(e^{j\omega})$ at $\omega = 0$ & π.

$Y(e^{j\omega}) \Rightarrow 1$.



Part - 1

SUMMARY OF SIGNALS AND SYSTEM:

→ In ~~any~~ time domain discrete will be transformed into the periodic in freq domain & vice versa.

Discrete ⟷ Periodic
(D) (P)

→ In ~~any~~ time domain continuous will be transformed into the non-periodic in freq domain & vice versa.

Continuous ⟷ non-periodic
(C) (N.P)

→

	time domain	Freq. Domain
Continuous fourier series. (C.F.S)	Periodic & Continuous.	discrete & Non periodic
Continuous time fourier transform (CTFT)	Continuous & Non-periodic	Non periodic & continuous
Discrete time fourier transform. (DTFT)	discrete & & Non periodic	Periodic & continuous.
Discrete fourier transform (DFT)	periodic & discontinuous.	discontinuous & periodic

(*) → Only for an LTI system.

	I/P	O/P
C·T·F·T	$e^{j\omega t}$	$e^{j\omega t}\underbrace{H(\omega)}_{\text{freq. response}}$
L·T	e^{st}	$e^{st}\underbrace{H(s)}_{\text{transfer function}}$
D·T·F·T	$e^{j\omega_0 n}$	$e^{j\omega_0 n}\underbrace{H(e^{j\omega_0})}_{\text{freq response}}$
Z·T	z^n	$z^n\underbrace{H(z)}_{\text{Transfer function}}$

(*) →

① C·T·F·T

$$\underset{\text{i/p}}{A\cos(\omega_0 t+\theta)} \longrightarrow \boxed{H(\omega) = |H(\omega)|\,\angle H(\omega)} \longrightarrow \underset{\text{o/p}}{y_{ss}(t) = A|H(\omega_0)|\cos(\omega_0 t+\theta+\angle H(\omega_0))}$$

②

$$K\cos(\omega_0 n+\phi) \longrightarrow \boxed{H(\omega) = |H(\omega)|\,\angle H(\omega)} \longrightarrow y_{ss}[n] =$$

$$K\left|H(e^{j\omega_0})\right|\cos\left(\omega_0 n+\phi+\angle H(e^{j\omega_0})\right)$$

→ $y_{ss}(t)$ means steady state o/p.

→ if for L·T ~~if~~ $H(s)$ is given then substitute $s = j\omega$

→ if for Z·T if $H(z)$ is given in question then substitute $z = e^{j\omega}$.

(*) Impulse Response $h(t)$: $\underbrace{\text{I·R}}$ $\xleftrightarrow{\text{C·T·F·T}}$ $\underbrace{H(\omega)}_{\text{freq response}} = \dfrac{Y(\omega)}{X(\omega)}$.

Impulse response $h(t)$ $\xleftrightarrow{\text{L·T}}$ $\underbrace{H(s)}_{\text{Transfer function}} = \dfrac{Y(s)}{X(s)}$.

$h[n]$ Impulse response $\xleftrightarrow{\text{D·T·F·T}}$ $\underbrace{H(e^{j\omega})}_{\text{freq. response}} = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})}$.

Impulse response $h[n]$ $\xleftrightarrow{\text{Z·T}}$ $\underbrace{H(z)}_{\text{transfer function}} = \dfrac{Y(z)}{X(z)}$.

$x(t-t_0) \xleftrightarrow{C.T.F.T} e^{-j\omega t_0} X(\omega)$

$\quad$ " $\xleftrightarrow{L.T} e^{-st_0} X(s)$

$x[n-n_0] \xleftrightarrow{D.T.F.T} e^{-j\omega n_0} X(e^{j\omega})$.

$\quad$ " $\xleftrightarrow{Z.T} z^{n_0} X(z)$.

$-jt \xrightarrow{C.T.F.T} \frac{d}{d\omega}$

$-jn \xrightarrow{D.T.F.T} \frac{d}{d\omega}$

$-t \xrightarrow{L.T} \frac{d}{ds}$

$n \xrightarrow{Z.T} -z\frac{d}{dz}$.

END of Summary

i.e, only part - 1

Time Scaling:

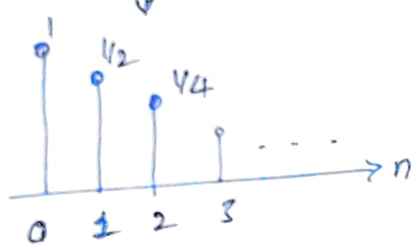
$$x\left[\frac{n}{K}\right] \longleftrightarrow X(e^{j\omega K}).$$

"n" is integer multiple of "K"

Eg: $$Y(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j10\omega}} = X(e^{j10\omega})$$

where $$X(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$\big\downarrow$ I.F.T

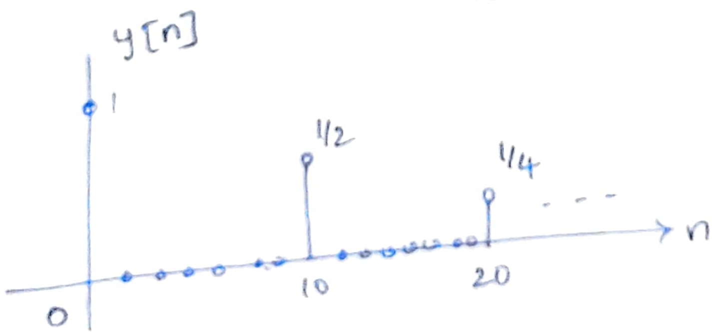


Now $$X(e^{j10\omega}) = \frac{1}{1-\frac{1}{2}e^{-j10\omega}}$$

$\big\downarrow$ I.F.F

$$y[n] = x\left[\frac{n}{10}\right] = \left(\frac{1}{2}\right)^{n/10}$$

where $n = 0, 10, 20 \ldots$



(*) D.T.F.T of $x[3n+2]$ is

Sol. $x\left[3\left(\cancel{n+\frac{2}{3}}\right)\right]$ → not possible.

So;

$$x[n+2] \xleftrightarrow{D.T.F.T} e^{-j\omega(-2)} X(e^{j\omega}).$$

$$x[3n+2] \xleftrightarrow{D.T.F.T} e^{j\frac{\omega}{3}(2)} X(e^{j\omega/3}).$$

Frequency Differentiation:

$$-jn\, x[n] \xleftrightarrow{D.T.F.T} \frac{d}{d\omega} X(e^{j\omega})$$

Eg: (i) $y[n] = n\alpha^n u[n]$

$= n\, x[n]$

F.T

$$Y(e^{j\omega}) = j\frac{d}{d\omega}\left[\frac{1}{1-\alpha e^{-j\omega}}\right] = \frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2}.$$

(*) $\sum_{n=0}^{\infty} n\left(\frac{1}{4}\right)^n = \underline{?}$

Sol Summation is representing area.

Area means spectrum at $\omega=0$.

$$\therefore \sum_{n=0}^{\infty} n\left(\frac{1}{4}\right)^n = \frac{1/4}{\left(1-\frac{1}{4}\right)^2} \quad (\because \text{By using the above formulae}).$$

Area at one domain means taking zero at the other domain