DFT:

→ N points (in one domain) $\xleftrightarrow{\text{transformed}}$ N ~~per~~ other points (in other domain)

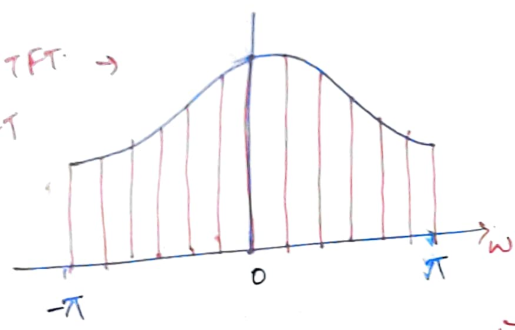
Periodic & Discrete ⟷ discrete & periodic

→ The sampled spectrum of DTFT is DFT.

→ ~~So~~ we will sample the DTFT which is known as DFT.

for DTFT → $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

Sampled DTFT → i.e., DFT



$2\pi \longrightarrow N$ points.

$\omega_k = \frac{2\pi k}{N}$; $k = 0$ to $(N-1)$

$\Delta f = \frac{2\pi}{N}$ = frequency spacing (or) frequency resolution.

$\boxed{\Delta f = \frac{2\pi}{N} = \frac{f_s}{N}}$

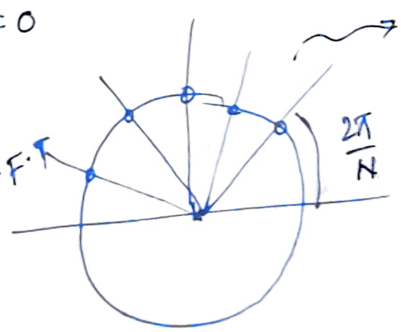
→ Now for DFT.

$$X[e^{j\omega k}] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}\cdot kn} \quad ; \; K = 0 \text{ to } (N-1).$$

(or)

$X[K]$.

If $r=1$
Z.T = D.T.F.T



→ DF.T

Sampled version of D.T.F.T is DFT.

→ Z-transform calculated on the unit circle at equidistant points i.e, an arc length of $\frac{2\pi}{N}$ is called as DFT.

→ ~~ST~~ $\boxed{\Delta f = \frac{2\pi}{N} = \frac{f_s}{N}}$

→ By adding extra zero's (zero padding) better frequency resolut[ion] is observed i.e, as the number of samples are increased the discrete spectrum will appear like conti-nuous spectrum.

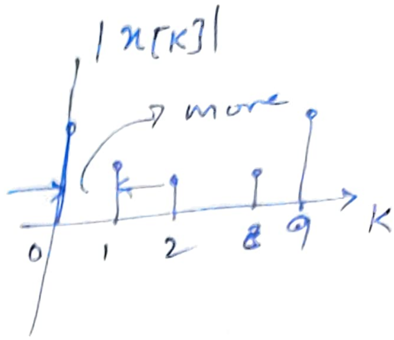
→ $N = 2^m$ pt

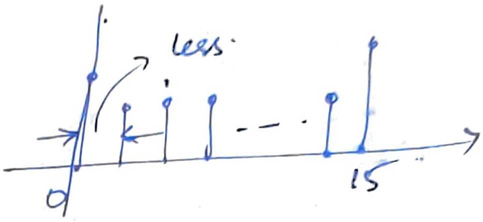
$5pt \Rightarrow 2^3$.

$10pt \Rightarrow 2^4$

Zero padding

(↑ frequency resolution)





→ phase factor $= W_N = e^{-j2\pi/N}$

Properties of phase factor:

1. Periodicity: $W_N^{K+N} = W_N^K$.

Proof: $\left(e^{-j\frac{2\pi}{N}}\right)^{K+N} = e^{-j\frac{2\pi}{N}K} \cdot e^{-j2\pi} = W_N^K$.

$(\because W_4^6 = W_4^2)$.

2. Symmetry: $W_N^{K+\frac{N}{2}} = -W_N^K$

3. $W_N^2 = W_{N/2}$

⇓

$\left(e^{-j2\pi/N}\right)^2 = e^{-j4\pi/N} = e^{-j\frac{2\pi}{N/2}} = W_{N/2}$.

→ Always D.F.T is calculated in terms of W_N but not exponential terms.

→ For a particular function W_N^0 is fixed.

Now for DFT ; $X[K] = \sum_{n=0}^{N-1} x[n]\, W_N^{Kn}$; $K = 0$ to $(N-1)$.

Matrix form:

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W^{2(N-1)} \\ 1 & W_N^3 & W_N^6 & \cdots & W^{3(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix}$$

for $N=2$: $W_2 = e^{-j2\pi/2} = -1$

$$W = \begin{bmatrix} 1 & 1 \\ 1 & W_2^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

ref

for $N=4$: $W_4 = e^{-j2\pi/4} = e^{-j\pi/2} = -j$

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

Since $W_N^{K+N} = W$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

ref