Signals And System (9M - 12M).

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(After 8:30pm)

Syllabus :

1. Analysis — Introduction
   — L.T.I systems

2. Approximation — Fourier Series.

3. Transformation.
   - (i) C.T.F.T
   - (ii) L.T
   - (iii) DTF.T
   - (iv) Z.T
   - (V) D.F.T.

Refrences :-

1. Siagnals And signals - Oppenheim & Nawab.
2. S-S → HayKins & Vanveen.
3. S & S → HSU & Ranjan (TMH).

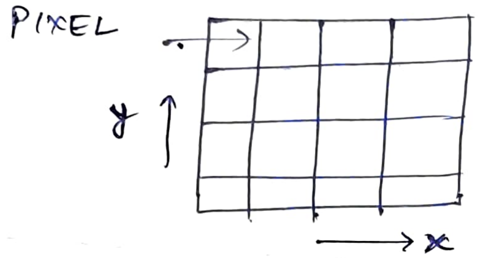
Signal :- signal is an indication about which some amount of information is conveyed.

Characteristics of Signal :

1. For signal varying chances of more than 1 individual variable.

Eg :- Speech → 1 Dimensional (time).

Image → 2D.



$E(x,t) \rightarrow 2D$

T.V. picture — 3D
$I(x,y,t)$

2. Randomness :

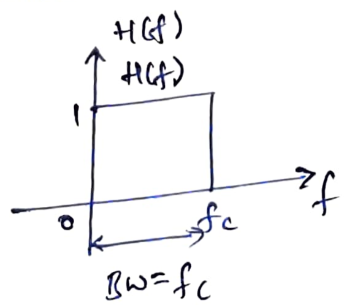
$$I = \log_2 1/P_i$$

$$P_i = \frac{1}{8} \Rightarrow I = 3\text{-bits}$$

$$P_i = \frac{1}{32} \Rightarrow I = 5\text{-bits}.$$

More the randomness more is the signal content.

3. Bandwidth ;



Types of Signals:

1. Continuous signal
2. Discrete signal
3. Digital signal.

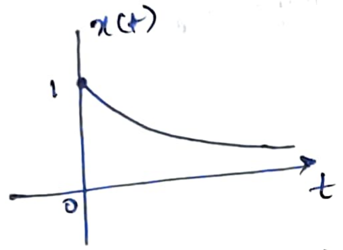
1. Continuous Signal:

It occurs for continuous values of time ($t: -\infty$ to ∞).

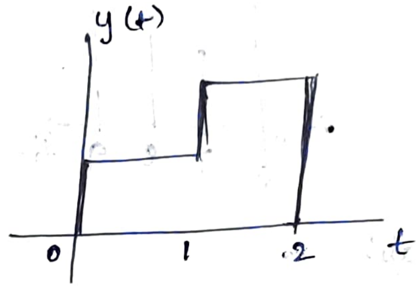
(or)

A signal which is continuous both in time and amplitude is a continuous signal.

$x(t) = e^{-2t} u(t)$



(continuous signals)



at $x = 1, 2$
$y(t)$ is not defined.

(Piecewise / discontinuous signals)

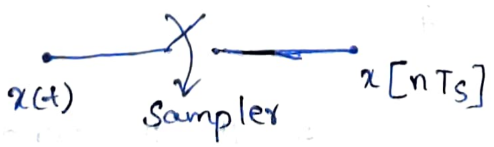
⊛ Continuous signal is also known as analog signal.

Analog means similarity.

2. Discrete Signal:

A signal in which amplitude is continuous and discrete in time (integer values).

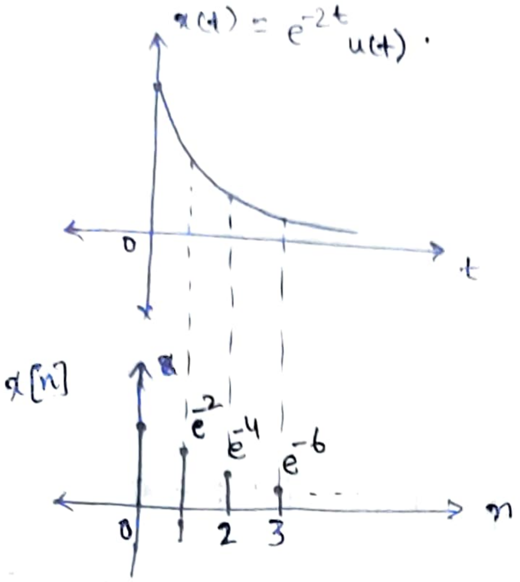
- Concept of multiplexing becomes easy by using discrete signal.



$t = nT_s$

$n = 0, \pm 1; \pm 2 \ldots$

T_s = sampling time.



let $T_s = 1sec$

$x(t) = e^{-2t} u(t)$

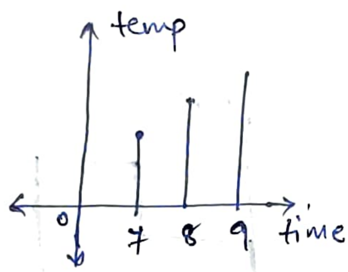
after sampling

$x[nT_s] = e^{-2nT_s} u[nT_s]$

$x[n\cancel{T_s}] = e^{-2n} u[n]$.

→ Every discrete signal is not the sampled version of Continuous signal.

Eg:-



• Sampling is only for multiplexing.

Digital Signal:

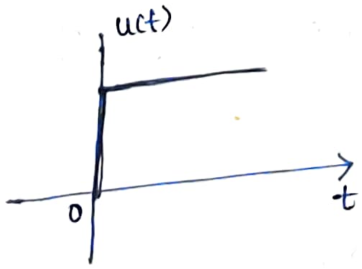
A signal which is discrete both in time and amplitude.

It is discrete in time but quantised in ~~ab~~ amplitude.

→ Quantizer converts the continuous Amplitude to discrete amplitude.

Standard Signals:

1. Unit step function $u(t)$:



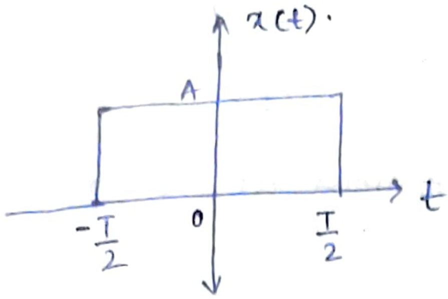
Mathematically $u(0) = \frac{1}{2}$

$$u(t) = \begin{cases} 1 & ; t > 0 \\ 0 & ; t < 0 \end{cases}$$

→ It is bounded in amplitude.

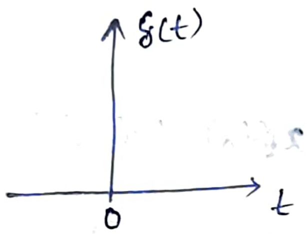
→ It exhibits transient behav

2. Rectangular or Gate function : $\left(A\,\text{rect}(t/T) \text{ or } A\,\Pi(t/T) \right)$



$$x(t) = \begin{cases} A & ; \ -\frac{T}{2} < t < \frac{T}{2} \\ 0 & ; \ \text{elsewhere.} \end{cases}$$

3. Continuous impulse function or Dirac delta function $\delta(t)$:



$$\delta(t) = \begin{cases} \infty & ; \ t = 0 \text{ i.e. } t \to 0 \\ 0 & ; \ t \neq 0. \end{cases}$$

→ It is used as approximation of standard signals.

→ Any signal can be represented as sum of impulse functions.

Properties:

1. $\displaystyle\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = 1$

$\Downarrow$

Area/Strength

2. It is an even function of time

$\delta(-t) = \delta(t)$.

3. Scaling property

$\delta(\alpha t) = \dfrac{1}{|\alpha|} \delta(t)$. $\alpha \to$ Scaling property.

4. **Product of Sampling property**

$$\boxed{x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)}$$

is valid iff $x(t)$ is continuous at $t = t_0$.

$t_0 \rightarrow$ time-shift.

Eg: (i) $\sin t\ \delta\left(t-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\,\delta\left(t-\frac{\pi}{4}\right)$.

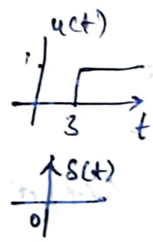
(ii) $t\,\delta(t) = 0\cdot\delta(t) = 0$.

5. **Shifting property:**

$$\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = \begin{cases} x(t_0) & ; t_1 \le t_0 \le t_2 \\ 0 & ; \text{else.} \end{cases}$$

Eg: $$\int_0^{2\pi} (t+\cos\pi t)\,\delta(t-2)\,dt = 2+\cos 2\pi = 3.$$

$\rightarrow$ $$\int_0^{\infty} \cos t \cdot u(t-3)\,\delta(t)\,dt = 0$$

($\because$ 

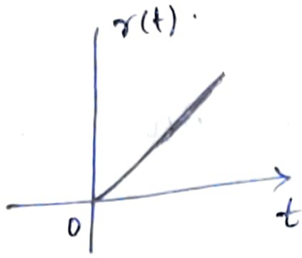
There is no overlapp $u(t)\cdot\delta(t)$)

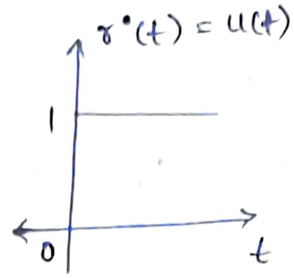
$\rightarrow$ $$\int_{-\infty}^{\infty} x(2-t)\,\delta(3-t)\,dt = \int_{-\infty}^{\infty} x(2-t)\,\delta(t-3)\,dt = x(2-3)$$
$$= x(-1).$$

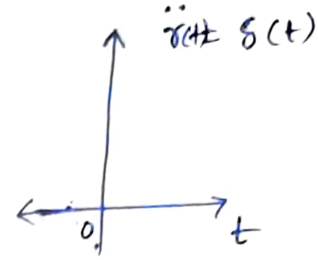
$\rightarrow$ $$\int_0^{\infty} e^{(t-2)}\,\delta(2t-4)\,dt = \int_0^{\infty} e^{t-2}\,\frac{1}{2}\,\delta(t-2)\,dt = \frac{1}{2}e^{0} = \frac{1}{2}$$

Unit Ramp fn

$$r(t) = \begin{cases} t & ; t > 0 \\ 0 & ; t < 0 \end{cases}$$

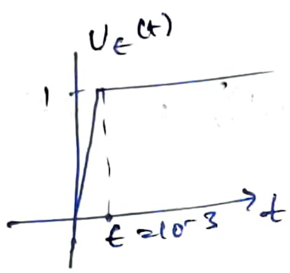


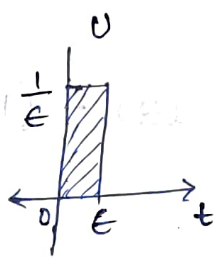




$\delta(t) = \frac{d}{dt} u(t)$; $u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$.

Practical Step function:



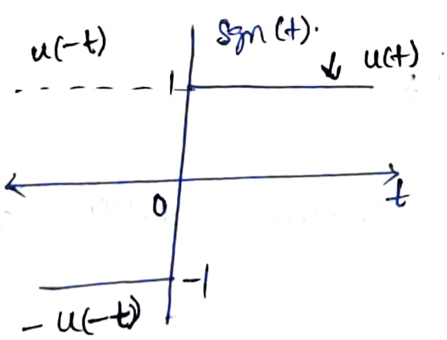


i.e, $\frac{d}{dt} u(t)$ = possible

Singularity functions:- The functions which do not posses higher order derivaties are singularity function.

Eg:- Ramp & Step function

Signum function: (Sgn (t)):

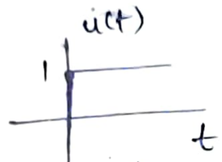


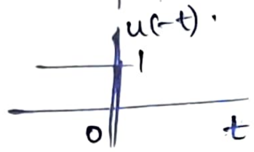
$\rightarrow$ $sgn(t) = u(t) - u(-t)$

$\rightarrow$ $sgn(t) = 2u(t) - 1.$

Proof:

$u(t) + u(-t) = 1.$ (To satisfy this eq'n $u(0) = \frac{1}{2}$).





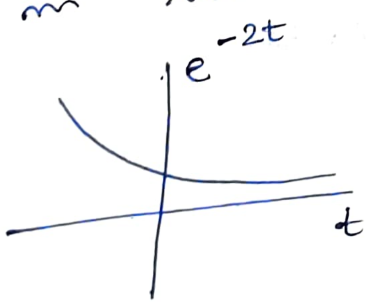
$u(-t) = 1 - u(t).$

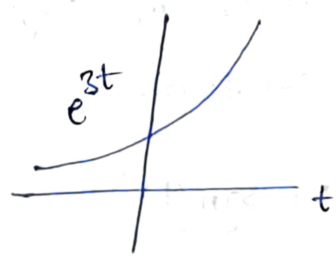
$sgn(t) = u(t) - u(-t) = u(t) - (1 - u(t)) = 2u(t) - 1.$

Exponentials :

Different forms of exponentials:

1. Real exponential ($e^{\sigma t}$):





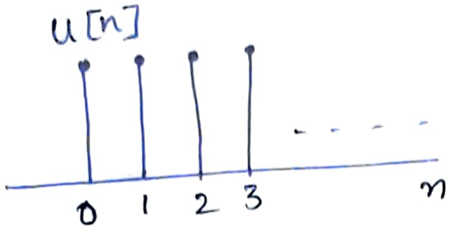
2. Complex sinusoid ($e^{\pm j\omega_0 t}$):

It does not change its nature when fourier transforms is applied.

3. Complex exponential e^{st}, $e^{(\sigma + j\omega)t}$

Predefined Sequences:-

1. Unit Step Sequence



$$u[n] = \begin{cases} 1 & ; \; n \geq 0 \\ 0 & ; \; n < 0 \end{cases}$$

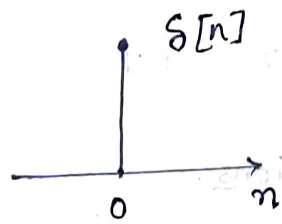
u[n] is defined at $n = 0$.

u(t) is not " " "

u[n] is not sampled version of u(t).

2. Discrete Impulse (or) Kronecker delta :

$$\delta[n] = \begin{cases} 1 & ; \; n = 0 \\ 0 & ; \; n \neq 0. \end{cases}$$



Note:-

1. We have $\delta(t) = \frac{d}{dt} u(t)$.

We use differentiation for continuous.

We use difference in place of differentiation for discrete

$$\nabla x_n = x_n - x_{n-1}.$$

$$\boxed{\delta[n] = u[n] - u[n-1]}$$

2. We have $u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$.

Integration for continuous $\Leftrightarrow$ Summation for discrete.

$\int \rightarrow \Sigma$

$t \rightarrow n$

$\tau \rightarrow K$

$$\boxed{u[n] = \sum_{K=-\infty}^{n} \delta[K]}$$

put $n - K = m$.

$$u[n] = \sum_{m=\infty}^{0} \delta[n-m] = \sum_{m=0}^{\infty} \delta[n-m].$$

$$\rightarrow \boxed{\delta[Kn] = \delta[n]}.$$

$$\delta[n] = \begin{cases} 1 & ; n=0 \\ 0 & ; n \neq 0 \end{cases} \qquad \delta[4n] = \begin{cases} 1 & ; 4n=0 \\ 0 & ; n \neq 0 \end{cases}$$

Transformations:

1. Time scaling $x(at) / x[mn]$.
2. Time-shift $x(t-t_0) / x[n-n_0]$.
3. Time-reversal $x(-t) / x[-n]$.
4. Amplitude scaling $K x(t) / K x[n]$.

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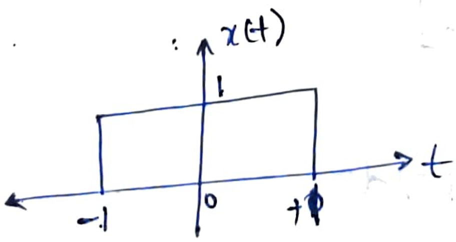
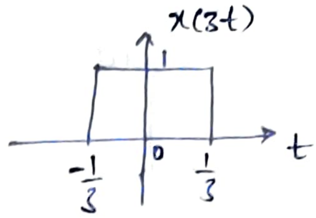
1. Time scaling $x(at) / x[mn]$:

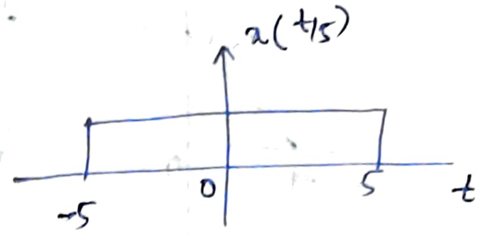


$$x(t) = \begin{cases} 1 & ; -1 < t < 1. \\ & \end{cases}$$

$$x(3t) = \begin{cases} 1 \;; & -1 < 3t < 1 \\ & -\frac{1}{3} < t < \frac{1}{3} \end{cases}$$



$$x(t/5) = \begin{cases} 1 \;; & -1 < \frac{t}{5} < 1 \\ & -5 < t < 5 \end{cases}$$

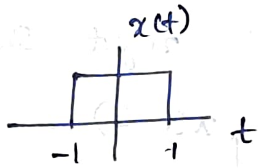


In $x(at)$ if $a > 1$ compression of $x(t)$

if $a < 1$ ~~compression~~ expansion. of $x(t)$

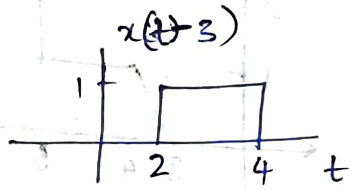
2. Time-shift $x(t - t_0)$ / $x[n - n_0]$:

$$x(t) = \begin{cases} 1 \,; -1 < t < 1 \end{cases}$$



(i) $x(t-3) = \begin{cases} 1 \,; & -1 < t-3 < 1 \\ & 2 < t < 4 \end{cases}$

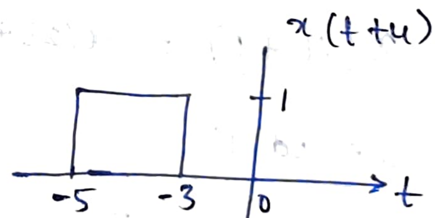
$t_0 = 3$



$t_0 > 0$ (Right - shift) ; (time-delay).

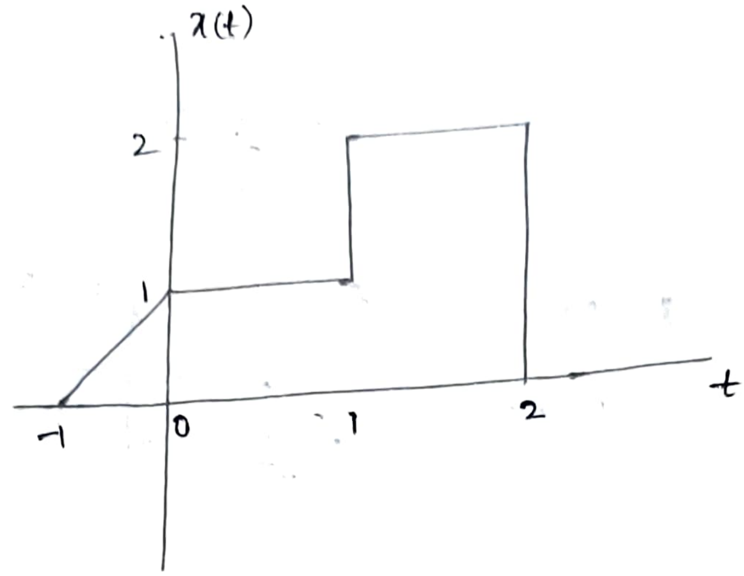
(ii) $x(t+4) = \begin{cases} 1 & -1 < t+4 < 1 \\ & -5 < t < -3 \end{cases}$

$t_0 = -4$.



$t_0 < 0$ (left shift); (time - advance) . This is not really possible.

For the signal $x(t)$ shown in figure draw the following signals



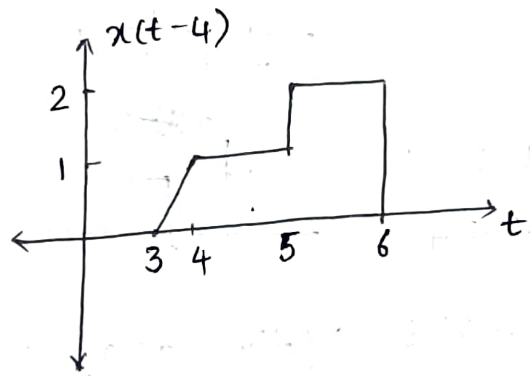
find

(i) $x(t-4)$ (ii) $x(2t+1)$ (iii) $x(5-t)$ (iv) $x(-t-2)$

(v) $[x(t) + x(-t)]\,u(t)$.

(i) $x(t-4)$

Original signal right shifted by '4' units.



(ii) ref. $x(\alpha t + \beta) = x(2t+1)$

sol. Method-1:-

$x\left[\alpha\left(t + \frac{\beta}{\alpha}\right)\right]$ first scaling then shifting.

Method-2:-

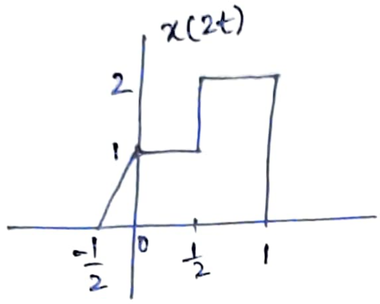
$x(t) \rightarrow x(t+\beta) \xrightarrow{t = \alpha t} x(\alpha t + \beta)$.

$x(2t+1) = x\left[2\left(t+\frac{1}{2}\right)\right]$.

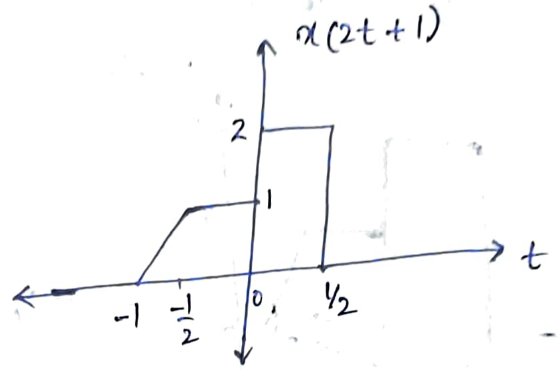
→ time scaling. $x(2t)$

→ shift $x(2t)$ left by "$\frac{-1}{2}$".

→ time scaling $x(2t)$



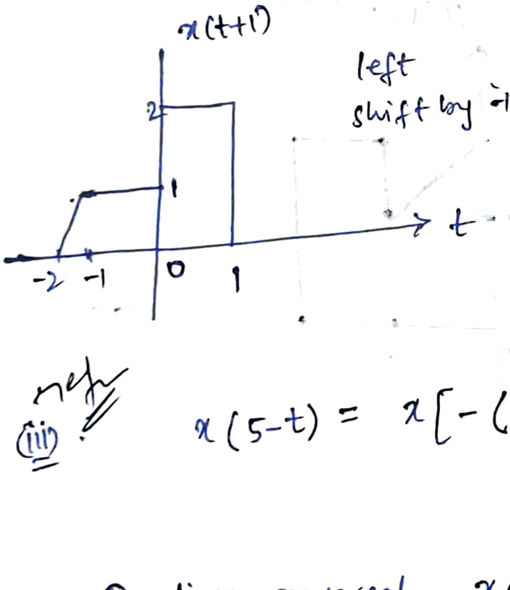
→ Shift $x(2t)$ left by $-1/2$



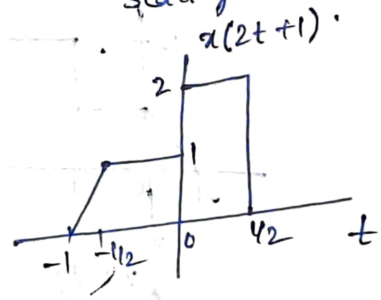
Aliter :- (Method-2)

$$x(t) \rightarrow x(t+1) \xrightarrow{t=2t} x(2t+1).$$

$\downarrow$ $t_0 = -1$



Scaling.

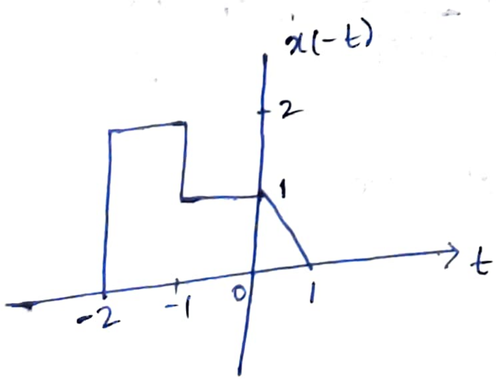


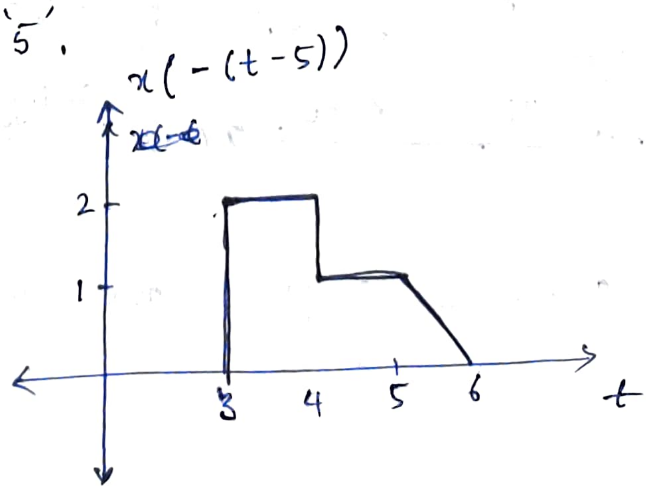
~~(iii)~~ ref

(iii) $x(5-t) = x[-(t-5)]$

$\downarrow$ $t_0 = 5$

- time reversal $x(-t)$
- shift $x(-t)$ right by '5'.





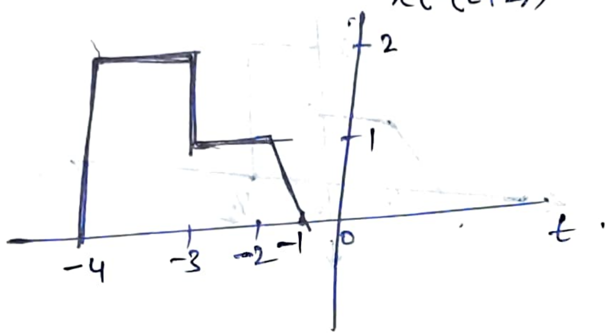
(iv) $x(-t-2) = x[-(t+2)]$.

$t_0 = -2$.

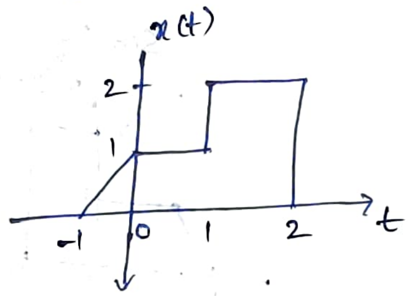
$\rightarrow x(-t)$

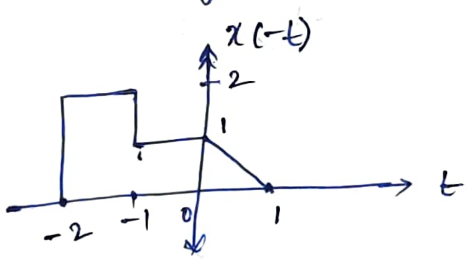
$\rightarrow x(-t-2) = x(-(t+2))$

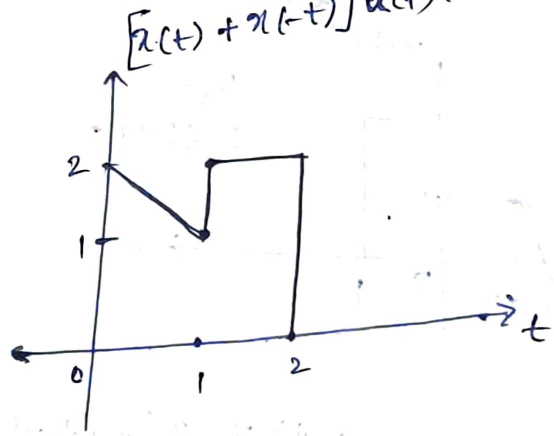
left shift by 2 for $x(-t)$.



(v) $[x(t) + x(-t)]\,u(t)$







Similarly $x\left(1-\frac{t}{4}\right) = x\left[-\frac{1}{4}(t-4)\right]$.

Method

$x(t) \xrightarrow{\text{Time reversal}} x(-t) \xrightarrow{\text{Scaling}} x\left(\frac{-t}{4}\right) \xrightarrow[\text{Shift}]{\text{Right}} x\left(1-\frac{t}{4}\right)$.

Scaling & Shifting are not interchangable.

VVIMP.

$y(t) = x(t-t_0)$

$y(\alpha t) = x(\alpha t - t_0)$

$y(\alpha t) \neq x[\alpha(t-t_0)]$

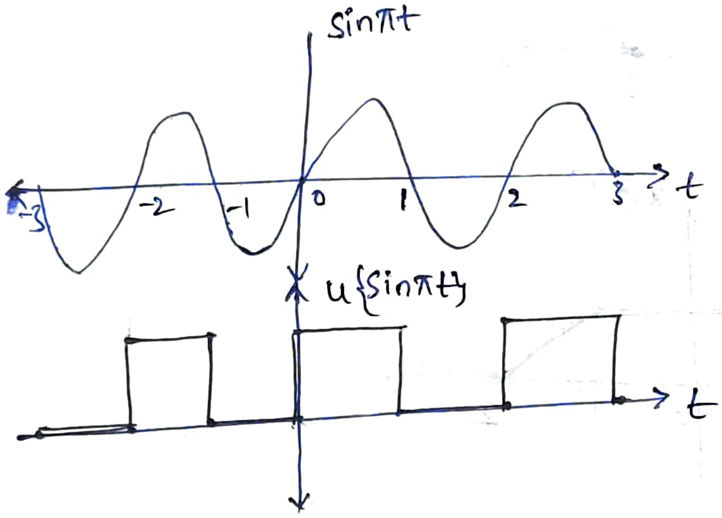
$y(t) = x(\alpha t)$.

$y(t-t_0) = x[\alpha(t-t_0)]$

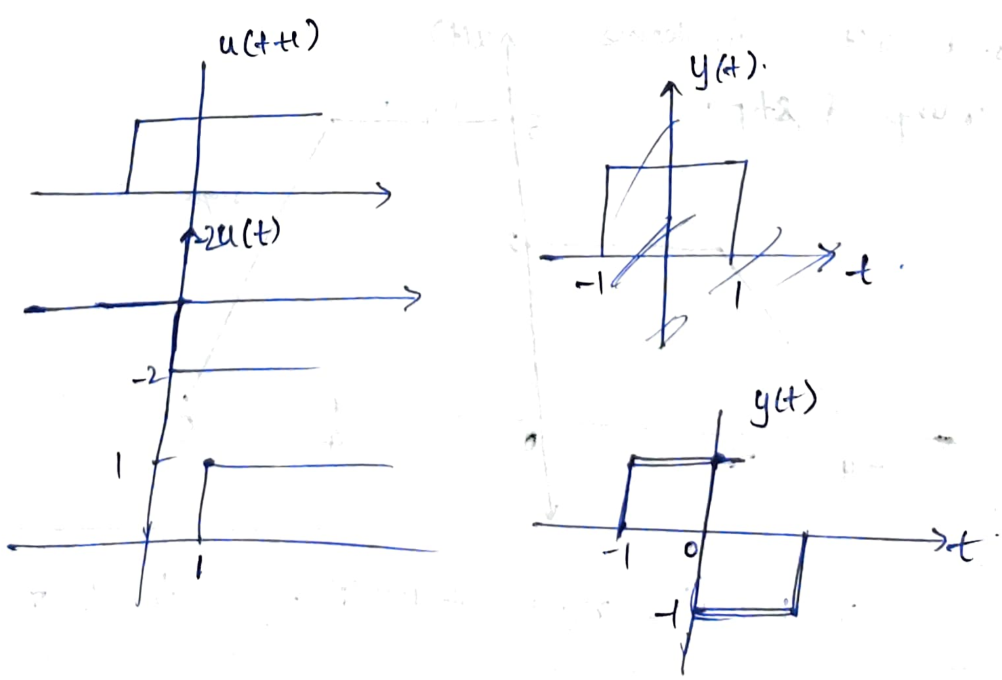
$y(t-t_0) \neq x(\alpha t - t_0)$.

· Draw the following signals:

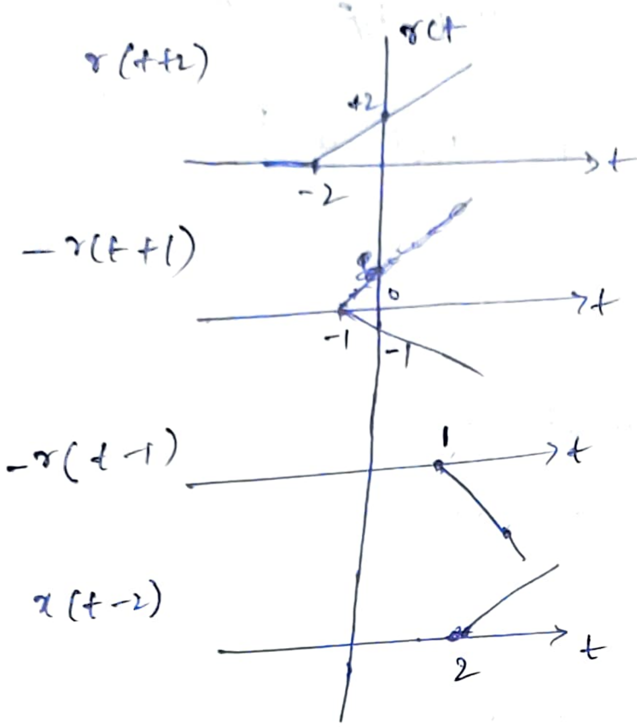
1. $u\{\sin \pi t\} = \begin{cases} 1 & ; \sin \pi t > 0 \\ 0 & ; \sin \pi t < 0 \end{cases}$



2. $y(t) = u(t+1) - 2u(t) + u(t-1)$



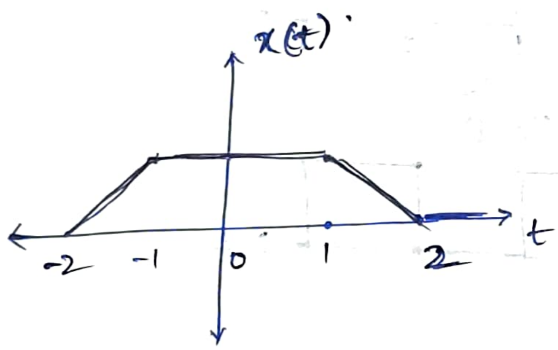
(*) $x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$.



$y = t+2$

$y = t-1$.

$x(t) =$

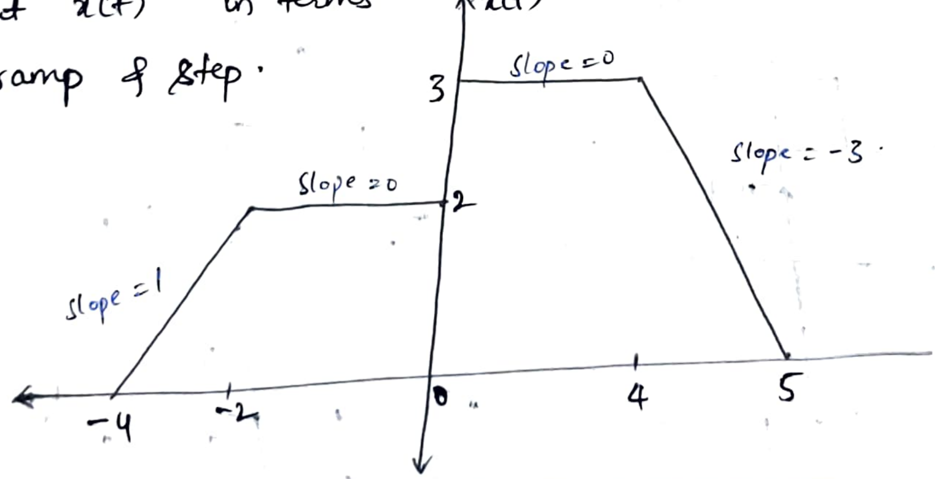


(*) Write the following signal interms of singularity functi

(1) Represent $x(t)$ in terms of ramp & step.

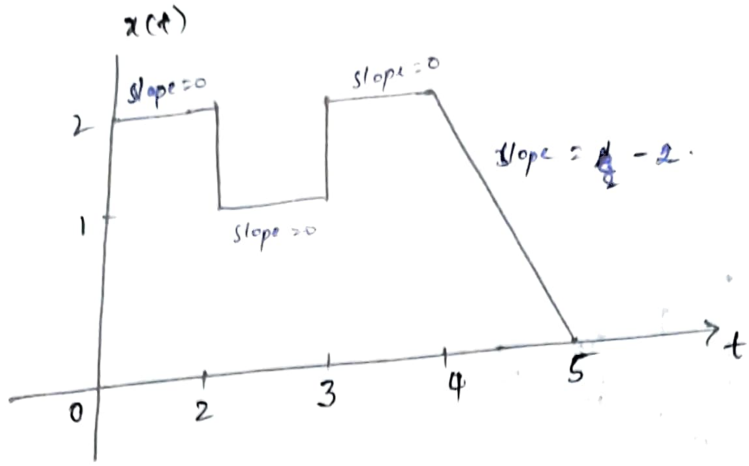
(5,0) (4,3)

(-2,2) (-4,0)



$$\boxed{x(t) = r(t+4) - r(t+2) + 1 \cdot u(t) - 3r(t-4) + 3r(t-5).}$$

2.



· Represt $x(t)$ in terms of $r(t)$ & $u(t)$.

(50) (4,2)

Sol. $x(t) = 2u(t) - u(t-2) + u(t-3) - 2r(t-4) + 2r(t-5)$.

(*) If $x(t) = 0$ for $t < 3$.

then $x(1-t) + x(2-t) = 0$ for $t > -1$

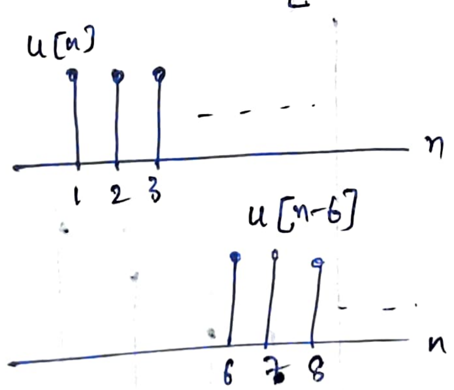
then $x(1-t) \cdot x(2-t) = 0$ for $t > -2$.

-2

-1

(*) Given $x[n] = (6-n)[u[n] - u[n-6]]$. Find (i) $x[n+2]$
(ii) $x[6-n]$ (iii) $x[3n+1]$ (iv) $x\left[\frac{n}{2}\right]$.

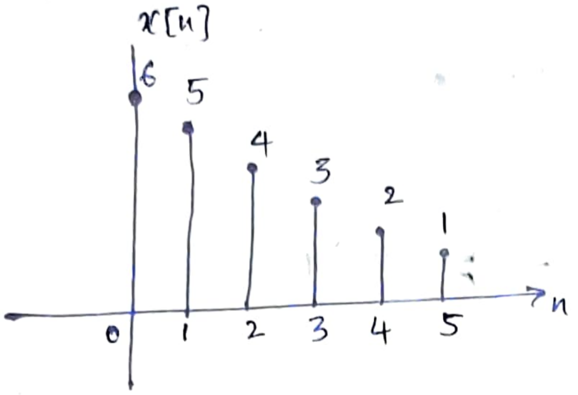
Sol. $x[n] = (6-n)\left[u[n] - u[n-6]\right]$.



$u[n] - u[n-6]$ ~~the~~ represents that the samples are covered upto $0 \le n \le 5$

Now $u[n] - u[n-6] = 1$ $\quad 0 \le n \le 5$.

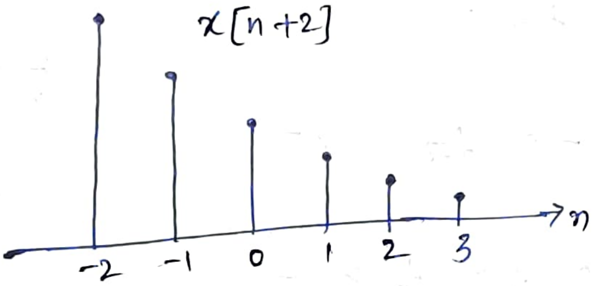
$x[n] = (6-n)(1) \; ; \quad 0 \le n \le 5$



(i) $x[n+2]$

$n_0 = -2$

Sol



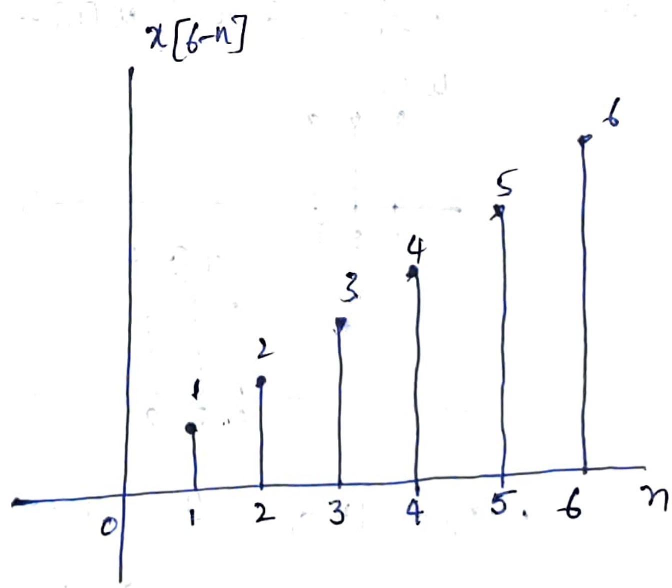
(ii) refl

$x[6-n]$.

Sol. $x[6-n] = x[-(n-6)]$

$\downarrow$

$n_0 = 6$.



$x[n]: \quad 0 \le n \le 5$

$x[6-n]: \quad 0 \le 6-n \le 5$

$-6 \le -n \le -1$

$6 \ge n \ge 1$

$x[6-n] : \quad 1 \le n \le 6$

(iii) $x[3n+1] \neq x\left[3\left(n+\frac{1}{3}\right)\right]$ ✗ because n_0 is not an integer)

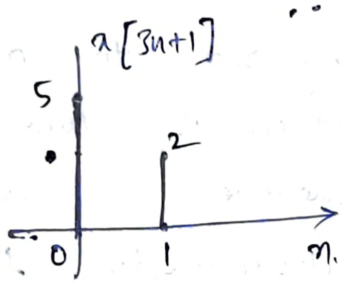
refer $n_0 = -\frac{1}{3}$ So 1st scaling & then shifting is not possible.

Now. $x[3n+1]$: $0 \le 3n+1 \le 5$

$$-1 \le 3n \le 4$$

$$-\frac{1}{3} \le n \le \frac{4}{3}$$

$\therefore n = 0, 1.$



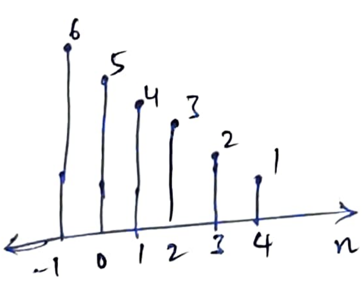
$(\because x[n] = 6-n)$.

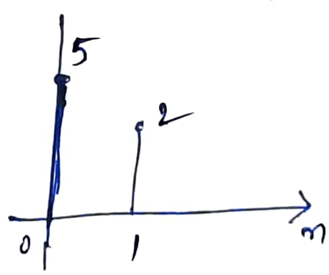
General form of scaling $x[mn]$. If $m>1$ then it is compressed. It is called as decimation or down sampling.

Aliter:

$$x[n] \longrightarrow \underset{n_0=-1}{x[n+1]} \xrightarrow{n \Rightarrow 3n} x[3n+1]$$

$x[n+1]$

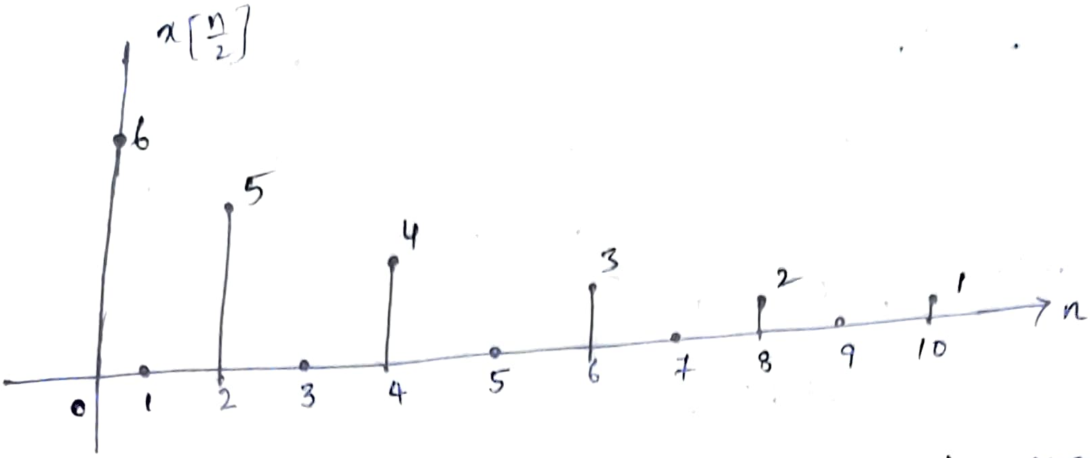




(iv) $x\left[\frac{n}{2}\right]$: $0 \le \frac{n}{2} \le 5$

$$0 \le n \le 10$$

$x\left[\frac{n}{2}\right]$



Here $x[mn]$ if $m<1$ it is $x[n]$ is expanded.

It is called zero interpolation.

Note:- conversion of one sampling rate to another sampling rate is multirate DSP which is done by using decimater and interpolator. Whenever we have to perform these two operations simultaneously first do interpolation then decimation.

(4). $x[n-1]\,\delta[n-3]$

Sol.- Let $x[n-1] = y[n]$

$y[n]\,\delta[n-3]$ $\quad (\because\ z[n]\,\delta[n-n_0] = z[n_0]\,\delta[n-n_0])$.

$= y[3]\,\delta[n-3]$

$= x[2]\,\delta[n-3]$.

$= 4\,\delta[n-3]$