CHAPTER - 4

TRANSIENTS

Lecture - 1 : Introduction To Transients

1. Resistor is static element
2. Inductor & Capacitors are dynamic elements. (memory elements i.e, memory can be stored).
3. Transient analysis doesn't exist for the resistive elements.
4. For applying transient analysis we should have atleast one memory element (i.e, capacitor (or) inductor).

Standard discharging eq'n : TYPE - I

$$\boxed{y(t) = y_0 e^{-t/\tau}}$$

where τ = time constant.

$$y(t=0) = y_0 e^{-0} = y_0$$

$$y(t=\tau) = y_0 e^{-1} = 0.368\, y_0$$

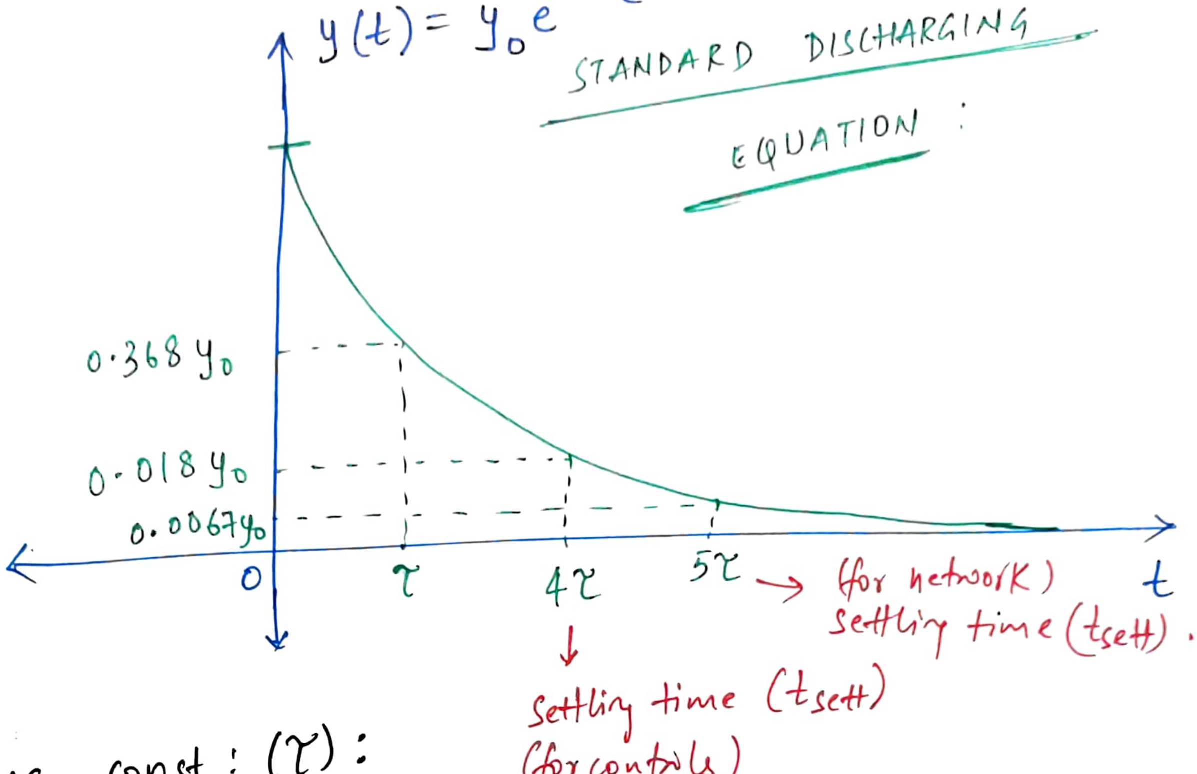
$$y(t=2\tau) = y_0 e^{-2} = 0.135\, y_0$$

$$y(t=3\tau) = y_0 e^{-3} = 0.048\, y_0$$

$$y(t=4\tau) = y_0 e^{-4} = 0.018\, y_0$$

$$y(t=5\tau) = y_0 e^{-5} = 0.0067\, y_0$$

$$y(t=\infty) = y_0 e^{-\infty} = 0$$



STANDARD DISCHARGING EQUATION :

Time const : (τ) :

The time at which the response will be 36.8% of the maximum value (OR) initial value. " This definition is valid for only standard discharging ~~eqn.~~ case. "

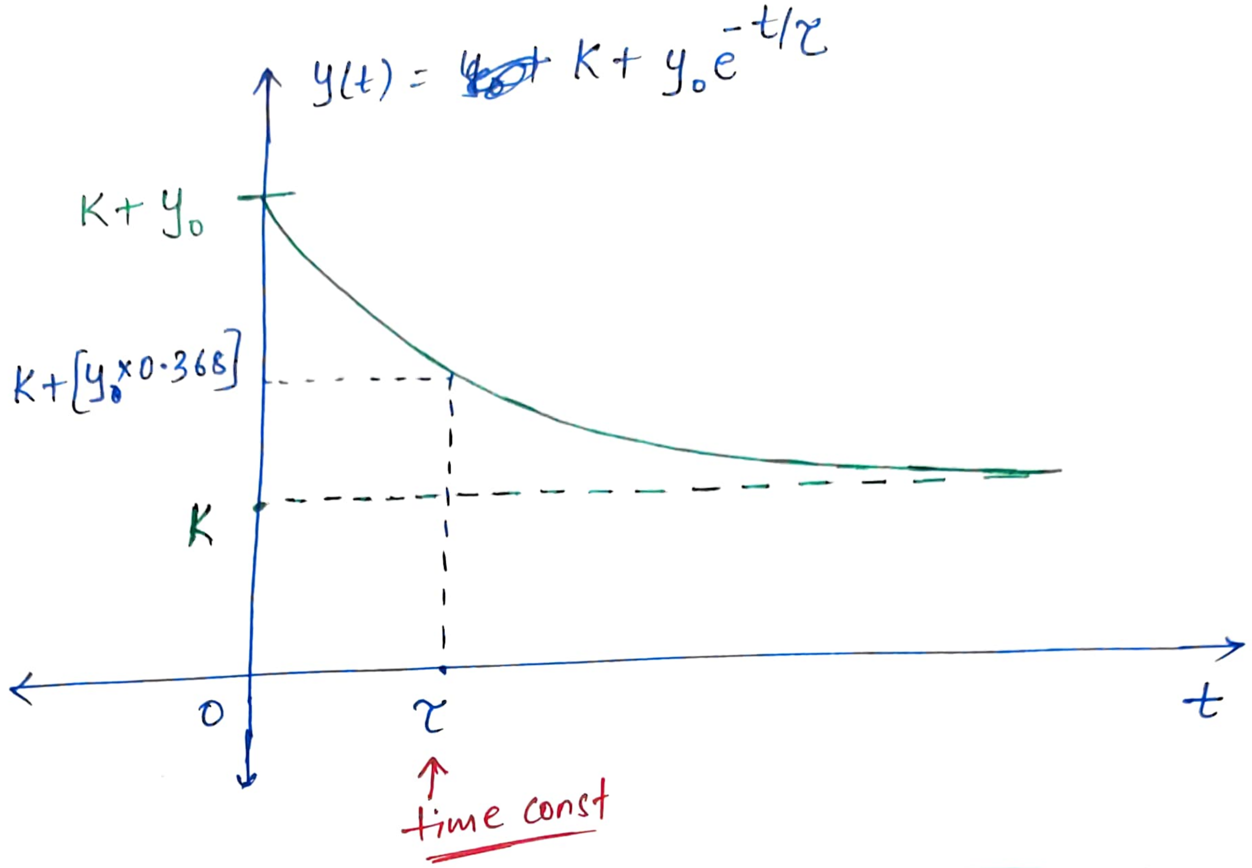
For : $\boxed{y(t) = K + y_0 e^{-t/\tau}}$:- **TYPE - II**

$$y(t=0) = K + y_0 e^{-0} = K + y_0$$

$$y(t=\tau) = K + y_0 e^{-1} = K + 0.368\, y_0$$

$$y(t=\infty) = K + y_0 e^{-\infty} = K$$

This analysis is for the generalised discharging case. ~~Here the~~ If $K=0$ then it is standard discharging case.



NOTE: (VIMP)

"TIME CONSTANT is only valid for stable systems i.e., it is not valid for unstable systems"

TYPE - III: (Standard Charging Equation):

$$\boxed{y(t) = y_0\left[1-e^{-t/\tau}\right]}$$

$$y(t=0) = y_0\left[1-e^{-0}\right] = y_0[1-1] = 0$$

$$y(t=\tau) = y_0\left[1-e^{-1}\right] = 0.632\,y_0$$

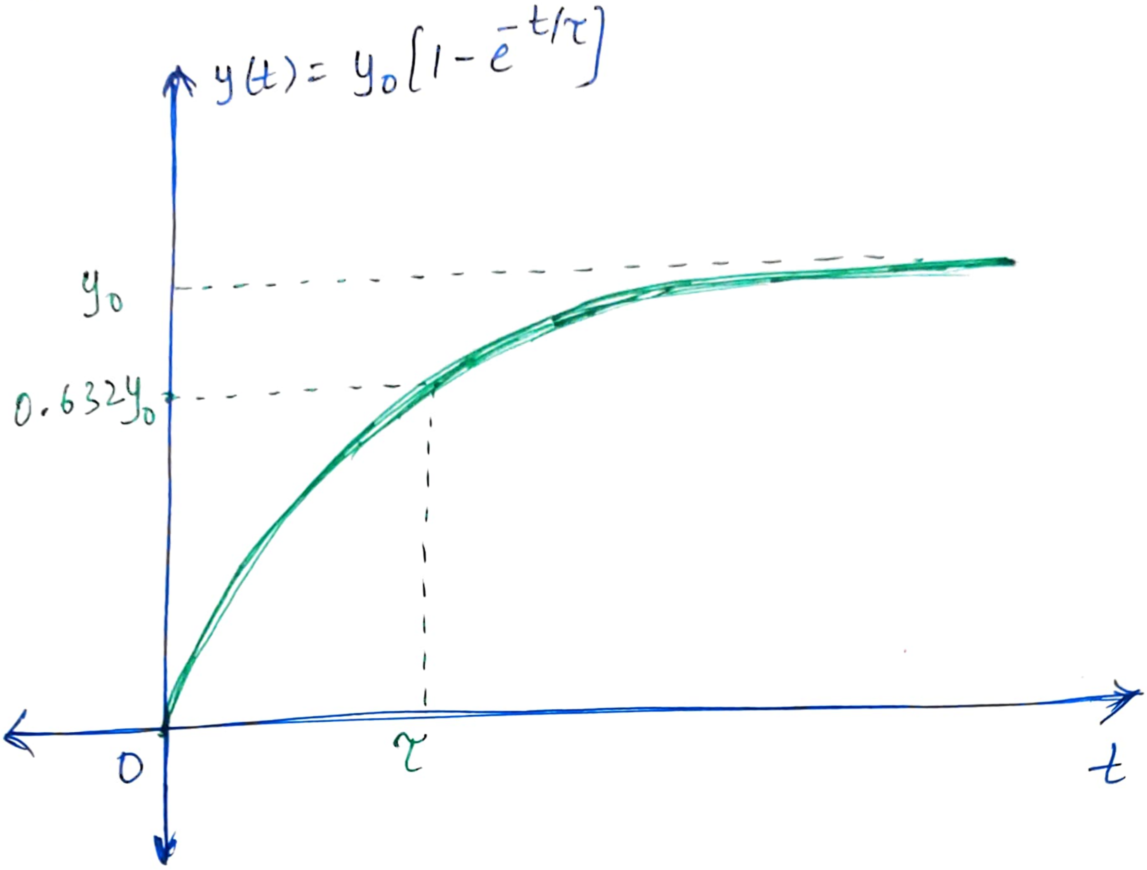
$$y(t=2\tau) = y_0\left[1-e^{-2}\right] = 0.865\,y_0$$

$$y(t=3\tau) = y_0\left[1-e^{-3}\right] = 0.952\,y_0$$

$$y(t=4\tau) = y_0\left[1-e^{-4}\right] = 0.98\,y_0$$

$$y(t=5\tau) = y_0\left[1-e^{-5}\right] = 0.993\,y_0$$

$$y(t=\infty) = y_0[1-e^{-\infty}] = y_0$$



Time Const (τ):

The time at which the response will be ~~0.632y~~ 63.2% of maximum value is called as the time constant. "This definition is valid for only standard charging case".

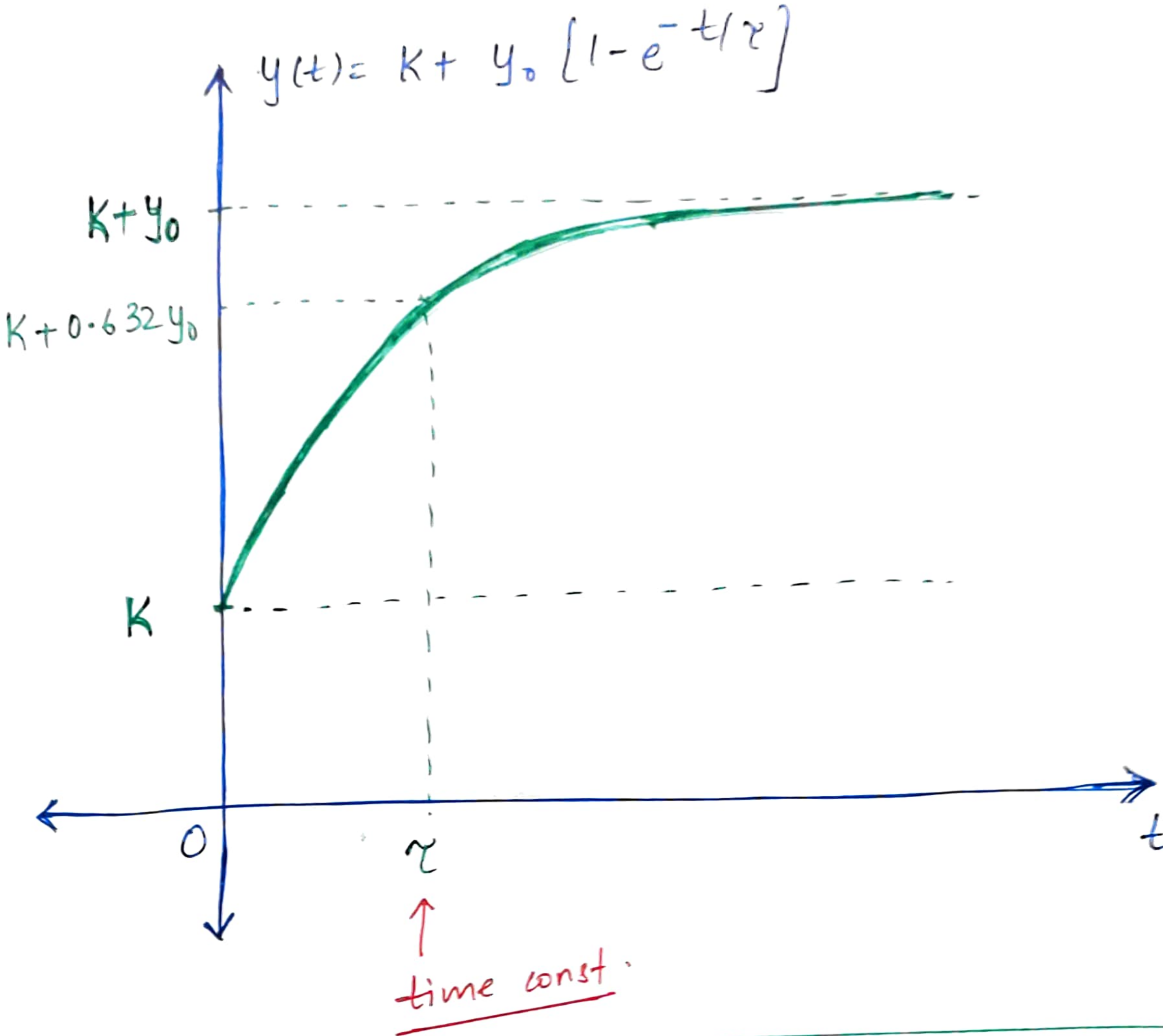
TYPE - IV: Generalised Charging Case:

$$\cancel{y(t) = y_0[1-e^{-t/\tau}]}$$

$$\boxed{y(t) = K + y_0[1-e^{-t/\tau}]}$$

$$y(t=0) = K + 0 = K$$

$$y(t=\infty) = K + y_0 = K + y_0$$



NOTE:

Type I, II, III, IV discussed here are 1st order charging & discharging cases. These are valid for only first order systems.

In transients we have only four type of graphs or equations. They are '2' for charging and the rest '2' for discharging cases.

Discharging Equations/Graphs

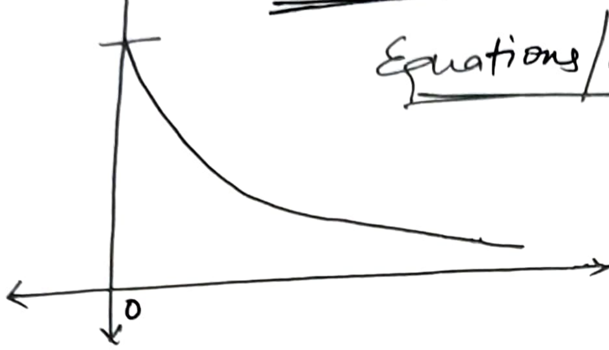


fig-(a)

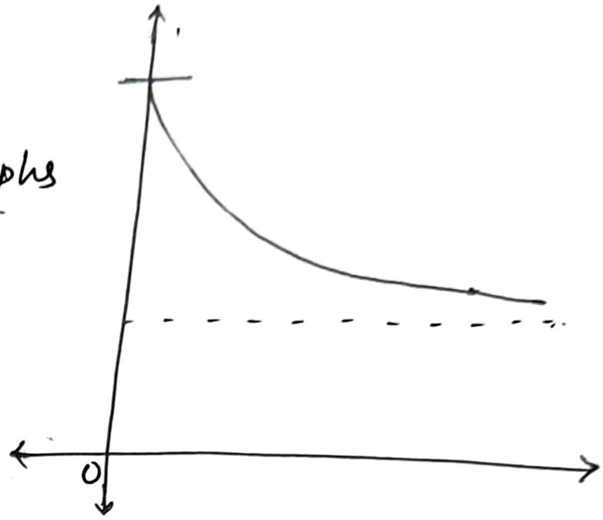


fig-(b)

Charging Graphs / Equations :

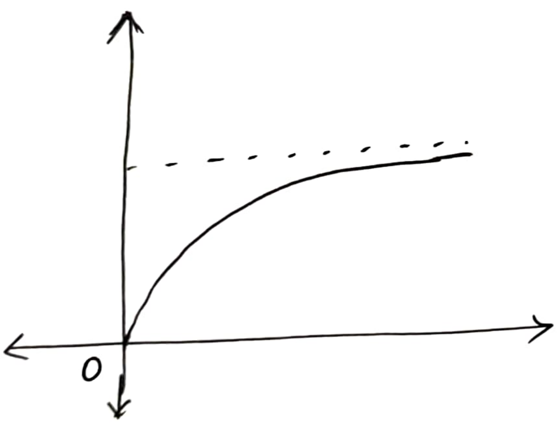


fig-(c)

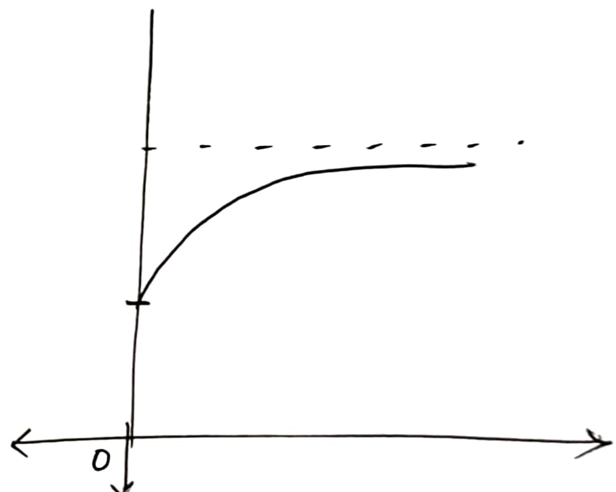


fig-(d).

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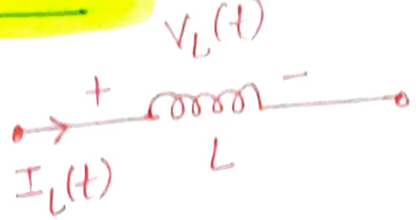
Inductor :



$$V_L(t) = L \frac{d}{dt} I_L(t)$$

$$I_L(t) = \frac{1}{L} \int_{-\infty}^{t} V_L(t)\, dt$$

$$I_L(t) = \frac{1}{L} \int_{-\infty}^{0} V_L(t)\, dt + \frac{1}{L} \int_{0}^{t} V_L(t)\, dt$$

$$I_L(t) = I_L(0^-) + \frac{1}{L} \int_{0}^{t} V_L(t)\, dt.$$

Discharging of Inductor :

Inductor stores the energy in the form of current.

$$\boxed{E_L(t) = \frac{1}{2} L I_L^2(t) \text{ Joules}}$$

In discharging case the inductor energy decreases and Energy at $t = \infty$ will be minimum.

i.e, $E_L(\infty) = \text{min}/0$ when

$E_L(t) \downarrow$ (decreasing)

(ii) In discharging mode of inductor the inductor current is in decreasing fashion and the inductor current at $t \to \infty$ is minimum or '0'.

i.e, $I_L(t) \downarrow$ decreasing when

$$I_L(\infty) = \min / 0 .$$

(iii) So, $\frac{d}{dt} I_L(t) = -ve.$ → VVimp.

$$V_L(t) = L \frac{dI_L(t)}{dt} = -ve$$

i.e, when the inductor is in discharging mode then the inductor current is in decreasing fashion so, inductor voltage is negative.

(iv) At steady state i.e, $t \to 0$ the inductor voltage is 0.

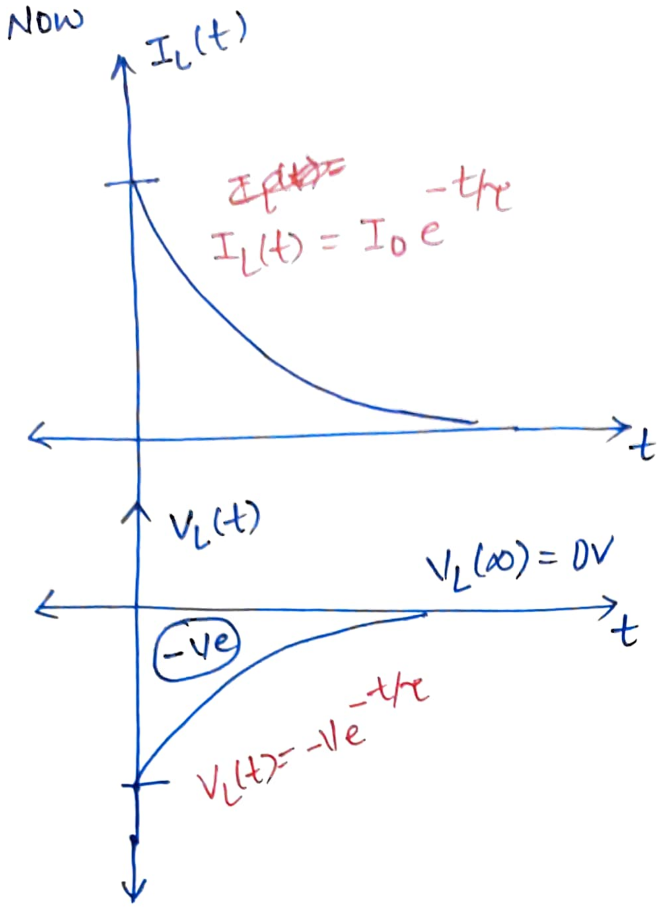
i.e, $I_L(\infty) = \min / 0$

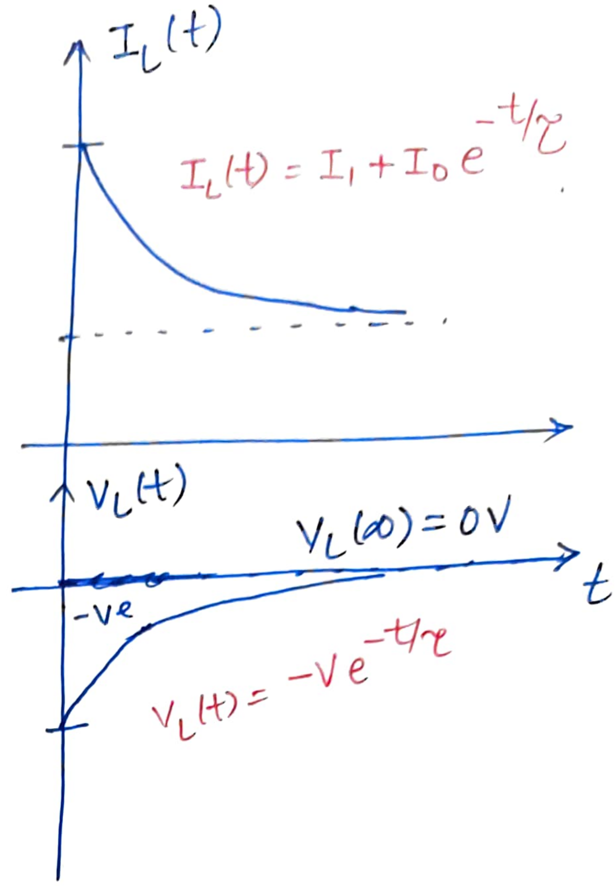
$$V_L(\infty) = L \frac{d}{dt} I_L(\infty) = L \frac{d}{dt}(\text{number or const}) = 0$$

$$\boxed{\therefore \text{At } t \to \infty \text{ i.e, S.S, } V_L(\infty) = 0}$$

↓ i.e, Short circuit (SC)

Now





Note:

Always in discharging mode the current can be

$$\left.\begin{aligned} I_L(t) &= I_0 e^{-t/\tau} \\ I_L(t) &= I_1 + I_0 e^{-t/\tau} \end{aligned}\right\} \text{two forms or graphs.}$$

but the inductor voltage will be only one eqn/graph form

i.e, $\boxed{V_L(t) = -Ve^{-t/\tau} = -ve = \text{negative}}$

Conclusion:

If the inductor current is in discharging mode then the voltage must be negative.

Charging of An Inductor :

$$E_L(t) = \frac{1}{2} L I_L^2(t) \text{ Joules.}$$

(i) If $E_L(t) \uparrow$ (increasing), at $t \to \infty$, $E_L(\infty) = \max$.

(ii) If $I_L(t) \uparrow$ (increasing), at $t \to \infty$, $I_L(\infty) = \max$.

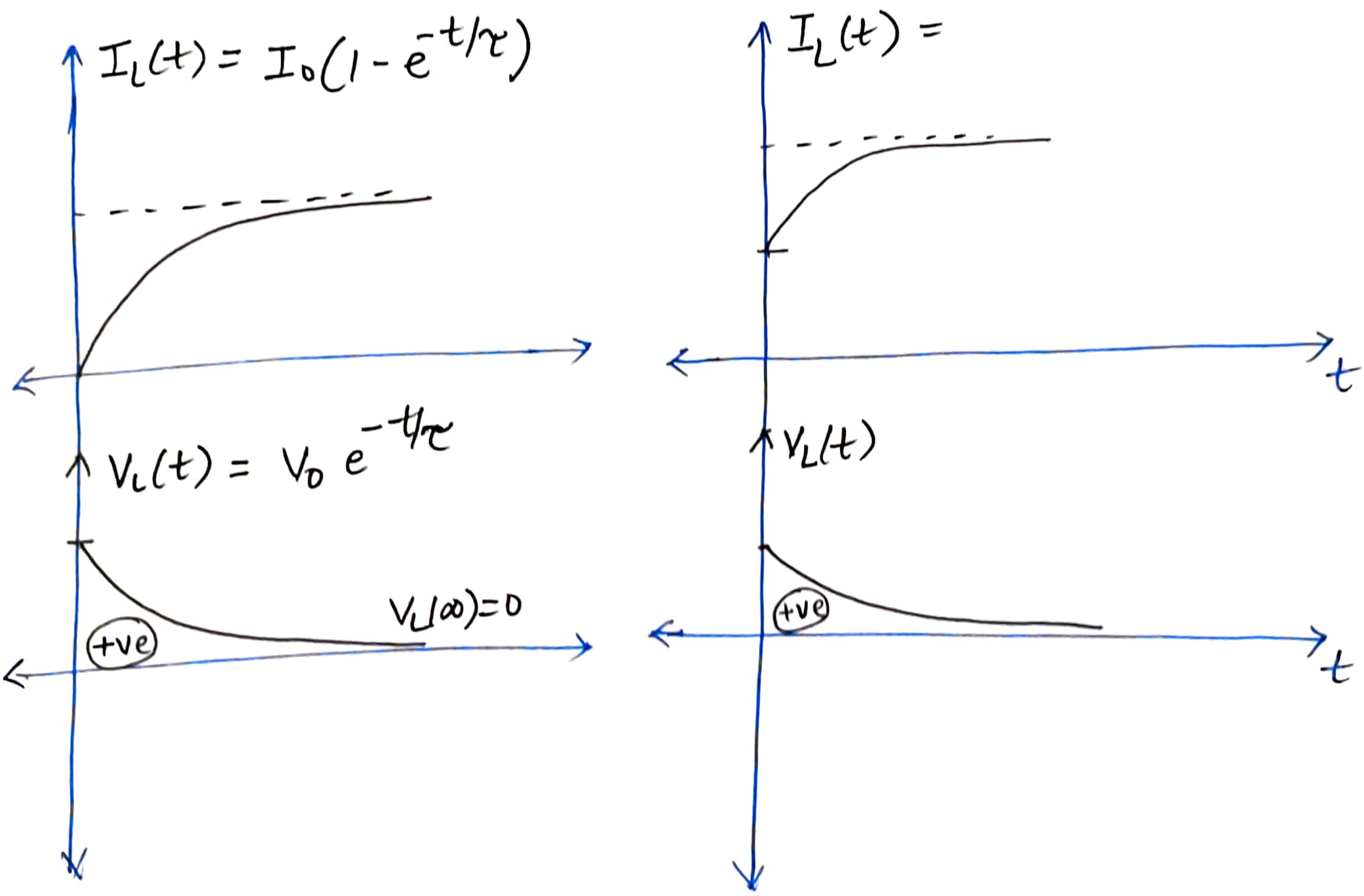
(iii) $\frac{d}{dt} I_L(t) = +ve$ as $I_L(t) \uparrow$ (increasing)

$$V_L(t) = L \frac{d}{dt} I_L(t) = +ve.$$

(iv) $V_L(\infty) = L \frac{d}{dt} I_L(\infty) = L \frac{d}{dt}(\max/\text{const}) = 0$

i.e, At $t \to \infty$, S.S, $V_L(\infty) = 0$

if $I_L(t)$ is increasing.



Note:

The expression of inductor is of only two types.

They are:

(i) $V_L(t) = Ve^{-t/\tau}$ (while charging) (i.e, +ve)

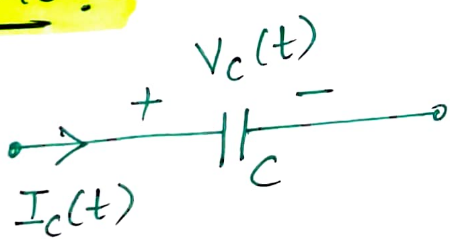
(ii) $V_L(t) = -Ve^{-t/\tau}$ (while discharging) (i.e, −ve)

Inductor voltage cannot be

$\hookrightarrow$ $V_L(t) = V(1-e^{-t/\tau})$ ✗

$V_L(t) = V_1 + V_2 e^{-t/\tau}$ ✗

Capacitance:



$$I_c(t) = C\frac{dV_c(t)}{dt}.$$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} I_c(t)\,dt$$

$$V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0}^{t} I_c(t)\,dt$$

Charging of Capacitor :

Capacitor opposes the change of energy. Here energy is in terms of voltage.

(OR)

Capacitor opposes the change in voltage in it instantaneously.

Now $\quad E_c(t) = \frac{1}{2} C V_c^2(t)$ Joule.

(i) If $E_c(t) \uparrow$ (increases) then $E_c(\infty) = max$.

(ii) If $V_c(t) \uparrow$ (increases) then $V_c(\infty) = max$.

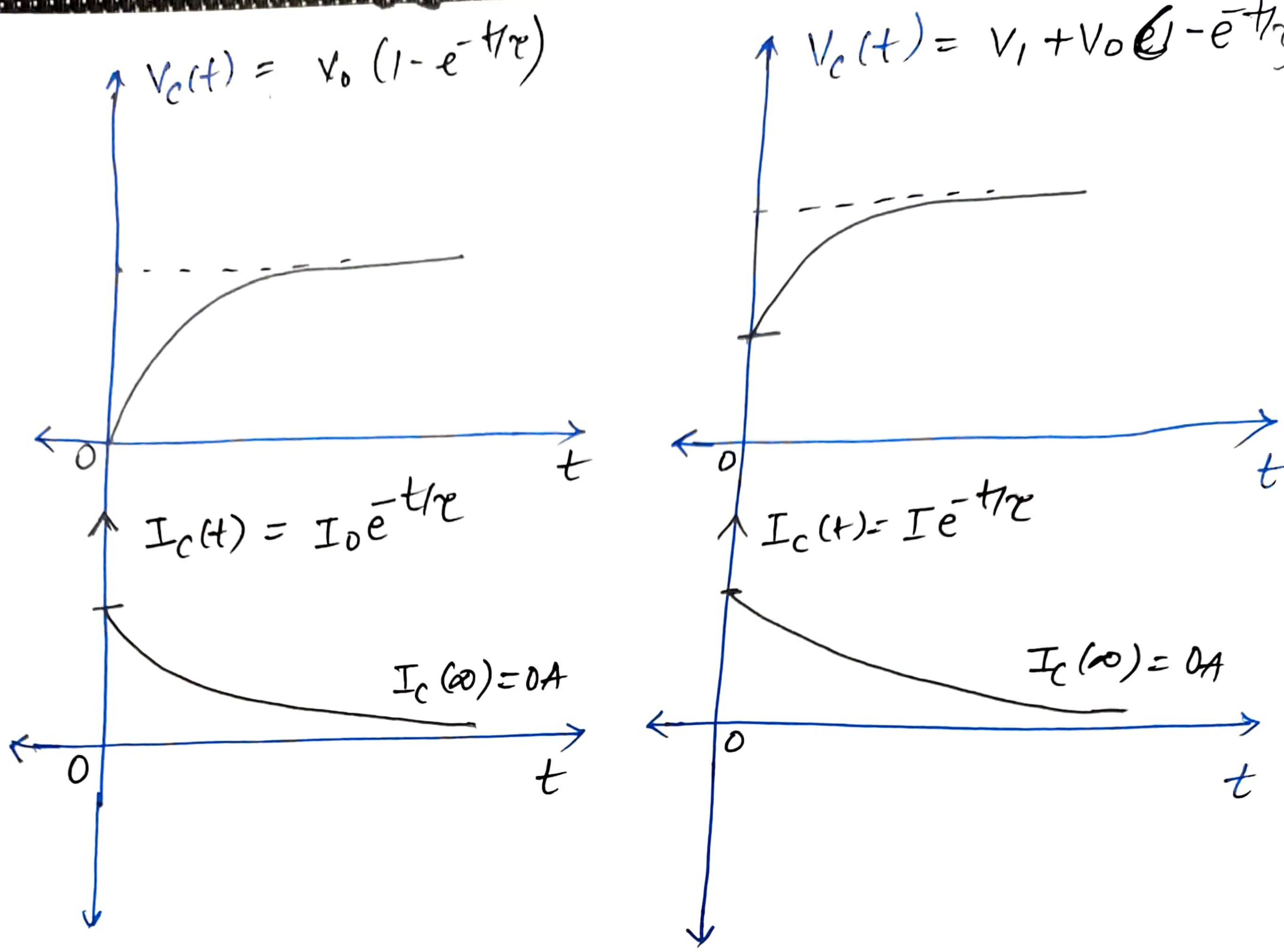
(iii) $\frac{d}{dt} V_c(t) = +ve$

$$I_c(t) = C \frac{d}{dt} V_c(t) = +ve$$

(iv) $I_L(\infty) = C \frac{d}{dt} V_c(\infty) = C \frac{d}{dt} \begin{pmatrix} max \\ or \\ const \end{pmatrix} = C \times 0$.

$I_L(\infty) = 0$ volts. i.e open circuit at steady state.

$\boxed{\text{Capacitor current at S.S is O.C}}$



Discharging of Capacitor :

$$E_c(t) = \frac{1}{2} C V_c^2(t) \text{ Joule}$$

(i) If $E_c(t) \downarrow$ (decreases) then $E_c(\infty) = \text{min}$

(ii) If $V_c(t) \downarrow$ (decreases) then $V_c(\infty) = \text{min}$

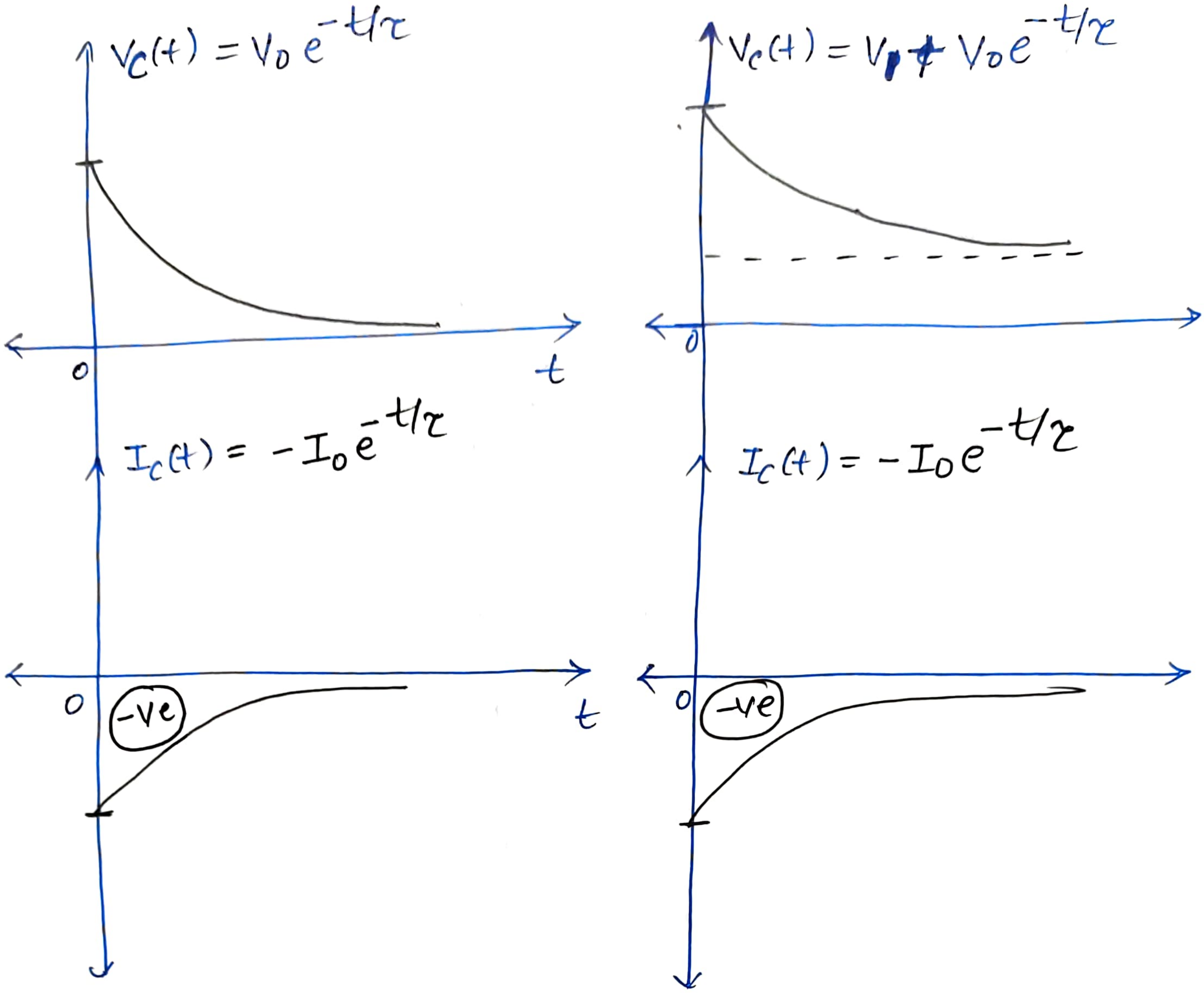
(iii) $\frac{d}{dt} V_c(t) = -ve$

$$I_c(t) = C \frac{d}{dt} V_c(t) = -ve$$

(iv) $I_L(\infty) = C \frac{d}{dt} V_c(\infty) = C \frac{d}{dt}(\text{min/const}) = C \times 0.$

$I_L(\infty) = 0$ volts i.e, open ckt at steady state

$\boxed{\text{Capacitor current at S.S is O.C}}$



NOTE:

The expressions for $I_c(t)$ cannot be

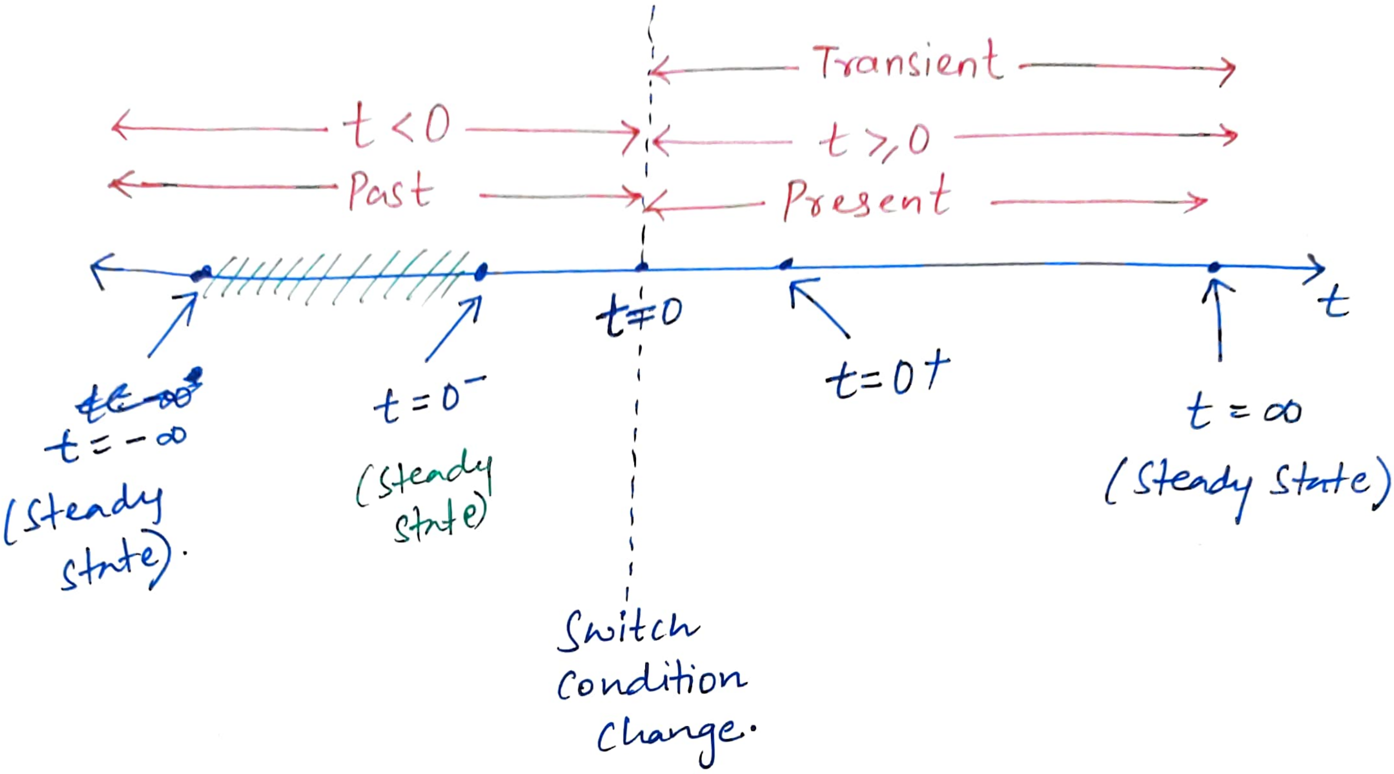
(i) $I_c(t) = I_0(1-e^{-t/\tau})$ ✗ not possible

(ii) $I_c(t) = I_1 + I_0(1-e^{-t/\tau})$ ✗ not possible

at any cost

Lecture - 2

CONCEPT OF 0^-, 0, 0^+ IN TRANSIENT ANALYSIS:



$t = 0^+$:

$t = 0^+$ time represents the momentum time after the switch is operated.

$t = 0^-$:

$t = 0^-$ time represents the momentum time before the switch is operated.

$t = 0$;

$t = 0$ time represents the transition time at which switch is operated.

$t = \infty$:

$t = \infty$ time represents the ~~time~~ steady state time after the switch is operated.

$t = -\infty$:

$t = -\infty$ represents the steady state time before the switch is operated.

Note:

The time $t = 0^-$ & $t < 0$ both represents the steady states. So $t = 0^-$ & $t = -\infty$ are also steady states.

VVVVIMP!

$t = 0^-$ represents the steady state time before the switch is operated.

VVIMP

NOTE:

→ We get steady state for two times in transients i.e, $t = 0^-, \infty^-$ (past s.s)

$t = \infty$ (present s.s).

→ In transients we discuss the charging & discharging of capacitor and inductor i.e, if it is in charging state before the switch is closed, then it (capacitor/inductor) will be in discharging state generally.

NOTE:

→ Generally for inductor, i.e, inductor will be S.C at the steady state.

W.K.T inductor will be in steady state at $t = 0^-$ & $t = \infty$.

i.e, $\left.\begin{aligned} V_L(\infty) &= 0 \text{ volt} \\ V_L(0^-) &= 0 \text{ volt.} \\ V_L(-\infty) &= 0 \text{ volt} \end{aligned}\right\}$ fixed

→ Generally the Capacitor will be O.C at the steady state.

W.K.T capacitor will be in steady state at $t = 0^-$ & $t = \infty$

i.e, $\left.\begin{aligned} I_C(\infty) &= 0 \text{ A} \\ I_C(0^-) &= 0 \text{ A} \\ I_C(-\infty) &= 0 \text{ A.} \end{aligned}\right\}$ fixed.

Lecture - 3 :

Analysis of first order differential equation :

$$A\frac{dy}{dt} + By = C$$

Case - (i) : Consider Input "C" = 0

Method - I

$$A\frac{dy}{dt} + By = 0$$

$$A\frac{dy}{dt} = -By$$

$$\int \frac{dy}{y} = \int -\frac{B}{A}\, dt + K$$

$$\ln y = -\frac{B}{A}t + K$$

$$\ln y = -\frac{B}{A}t + \ln K_1$$

$$\ln y = \ln\left(e^{-\frac{B}{A}t}\right) + \ln K_1$$

$$\ln y = \ln\left[K_1 e^{-B/A\, t}\right]$$

$$y = K_1 e^{-\frac{B}{A}t}$$

$$y(t) = K_1 e^{-B/A\, t}$$

At $t = 0$

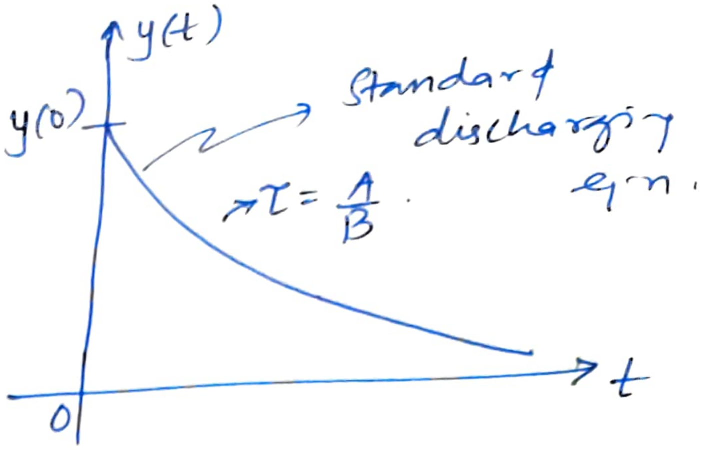
$$y(0) = K_1$$

$$\boxed{\therefore\ y(t) = y(0)\, e^{-B/A\, t}}$$

Comparing with

$$y(t) = y_0 e^{-t/\tau}$$

$$\boxed{\tau = \text{time const} = \frac{A}{B}}$$



$$\boxed{y(t) = \underbrace{y(0)e^{-\frac{B}{A}t}}}$$

is called

- → Complimentary function
- → Natural Response
- → ZIR
- → force free response
- → Discharging eq^n.

Eg: Find the time const of $\frac{di}{dt} + 10i = 0$

soln: $\tau = \frac{A}{B} = \frac{1}{10} = 0.1$ sec.

i.e by comparing with $\frac{di}{dt} + \frac{B}{A} i = 0$.

- **Method-II :**

$$\frac{dy}{dt} + \frac{B}{A} y = 0$$

$$Dy + \frac{B}{A} y = 0$$

$$D + \frac{B}{A} = 0$$

$$D = -\frac{B}{A}.$$

$$\boxed{\text{Complimentary Function (CF)} = y(t) = Ke^{-B/A\,t}}$$

At $t=0$ $K = y(0)$

$$\boxed{\therefore \quad y(t) = y(0)\,e^{-B/A\,t}}$$

Case – (ii) Consider input $C \neq 0$

$$I/p \neq 0 \quad \text{i.e,} \quad C \neq 0$$

$$A\frac{dy}{dt} + By = C$$

$$\frac{dy}{dt} + \frac{B}{A}y = \frac{C}{A}$$

Clearly it is 1^{st} order linear D.E

$$I.F = e^{\int \frac{B}{A} dt} = e^{\frac{Bt}{A}}.$$

$$y \times I.F = \int \frac{C}{A} \times (I.F)\,dt + K$$

$$y \times e^{B/A\,t} = \frac{C}{A}\int e^{B/A\,t}\,dt + K$$

$$y\,e^{B/A\,t} = \frac{C}{A} \times \frac{A}{B}\,e^{B/A\,t} + K$$

$$\boxed{y e^{B/A\, t} = \frac{C}{B} e^{B/A\, t} + K}$$

$$y(t) = \frac{C}{B} + K e^{-B/A\, t}$$

At $t = 0$

$$y(0) = \frac{C}{B} + K$$

$$K = y(0) - \frac{C}{B}.$$

$$y(t) = \frac{C}{B} + \left(y(0) - \frac{C}{B}\right) e^{-B/A\, t}$$

$$\boxed{y(t) = \frac{C}{B}\left[1 - e^{-B/A\, t}\right] + \left[y(0)\, e^{-B/A\, t}\right].}$$

At $t = \infty$

$$y(\infty) = \frac{C}{B} + 0$$

$$y(t) = y(\infty)\left[1 - e^{-B/A\, t}\right] + y(0)\, e^{-B/A\, t}.$$

$$y(t) = y(\infty) - y(\infty)\, e^{-B/A\, t} + y(0)\, e^{-B/A\, t}$$

$$\boxed{y(t) = y(\infty) + \left[y(0) - y(\infty)\right] e^{-B/A\, t}}$$ VVIMP

where τ = time const $= \cancel{B/A}\ \frac{A}{B}$.

$y(t) = f($

We know that

$$y(t) = y(\infty) + [y(0) - y(\infty)] e^{-t/\tau}$$

Let's apply to ~~the~~ current & voltages of inductor & capacitor; resister

$$I_L(t) = I_L(\infty) + [I_L(0) - I_L(\infty)] e^{-t/\tau} \quad ; \quad t \geq 0$$

$$V_L(t) = V_L(\infty) + [V_L(0) - V_L(\infty)] e^{-t/\tau} \quad ; \quad t \geq 0$$

$$I_C(t) = I_C(\infty) + [I_C(0) - I_C(\infty)] e^{-t/\tau} \quad ; \quad t \geq 0$$

$$V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)] e^{-t/\tau} \quad ; \quad t \geq 0$$

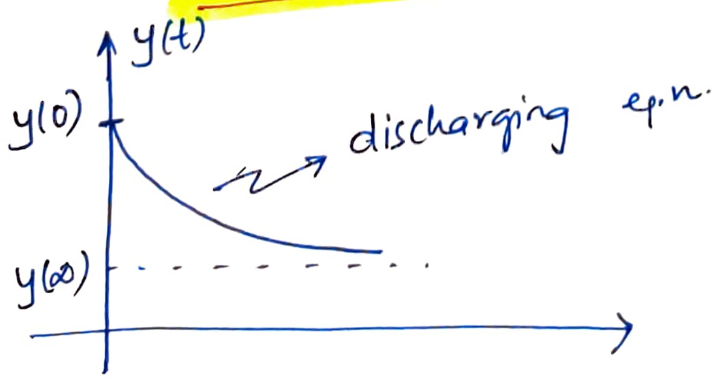
$$V_R(t) = V_R(\infty) + [V_R(0) - V_R(\infty)] e^{-t/\tau} \quad ; \quad t \geq 0$$

$$I_R(t) = I_R(\infty) + [I_R(0) - I_R(\infty)] e^{-t/\tau} \quad ; \quad t \geq 0$$

The above results are valid only for 1st order network. They are not valid for 2nd order networks.

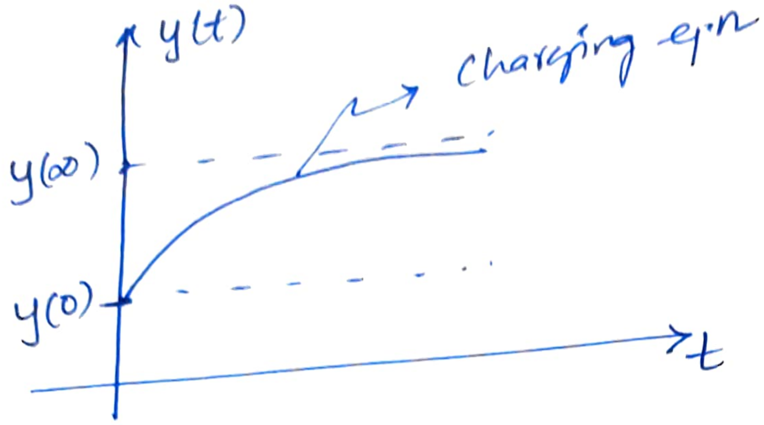
Case (i): Initial Value > final value

i.e. $y(0) > y(\infty)$:



Case – (ii): Initial Value < Final Value.

$$y(0) < y(\infty)$$



Note: If the switching happens at $t = t_0$ then the current & voltages of inductor, capacitor and resistor are as follows

$$I_L(t) = I_L(\infty) + \left[I_L(t_0) - I_L(\infty)\right] e^{-\left(\frac{t-t_0}{\tau}\right)} \; ; \; t \geq t_0$$

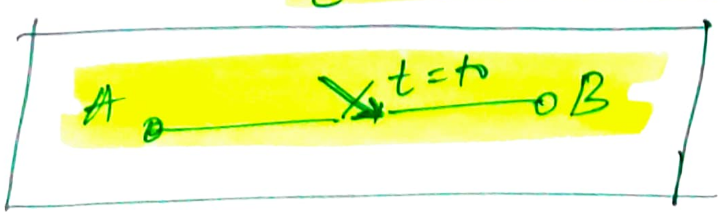
$$V_L(t) = V_L(\infty) + \left[V_L(t_0) - V_L(\infty)\right] e^{-\left(\frac{t-t_0}{\tau}\right)} \; ; \; t \geq t_0$$

$$I_C(t) = I_C(\infty) + \left[I_C(t_0) - I_C(\infty)\right] e^{-\left(\frac{t-t_0}{\tau}\right)} \; ; \; t \geq t_0$$

$$V_C(t) = V_C(\infty) + \left[V_C(t_0) - V_C(\infty)\right] e^{-\left(\frac{t-t_0}{\tau}\right)} \; ; \; t \geq t_0$$

$$V_R(t) = V_R(\infty) + \left[V_R(t_0) - V_R(\infty)\right] e^{-\left(\frac{t-t_0}{\tau}\right)} \; ; \; t \geq t_0$$

$$I_R(t) = I_R(\infty) + \left[I_R(t_0) - I_R(\infty)\right] e^{-\left(\frac{t-t_0}{\tau}\right)} \; ; \; t \geq t_0$$

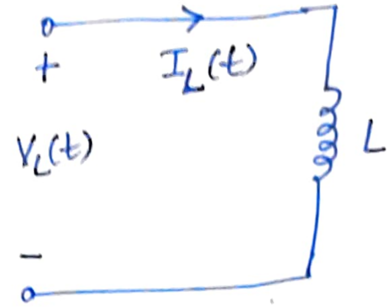


Lecture - 4 :

Transform Domain Of Inductor & Capacitor :

(1)

$$V_L(t) = L \frac{d}{dt} I_L(t)$$



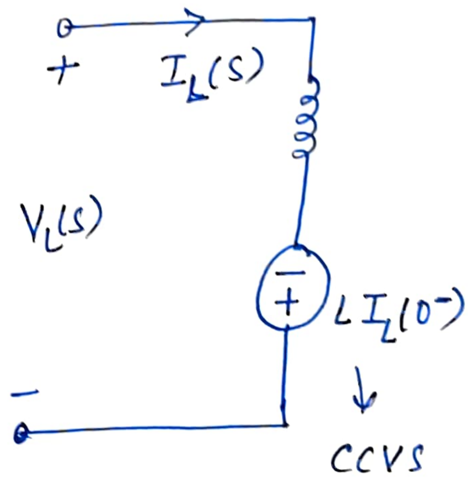
Apply L.T on both sides.

$$V_L(s) = L\left[s I_L(s) - I_L(0^-)\right]$$

$$V_L(s) = L s I_L(s) - L I_L(0^-) \quad \left[\because L\left[\frac{dy}{dx}\right] = s Y(s) - y(0)\right]$$

$$\boxed{V_L(s) = L s I_L(s) - L I_L(0^-)}$$

represents



"Time Domain"

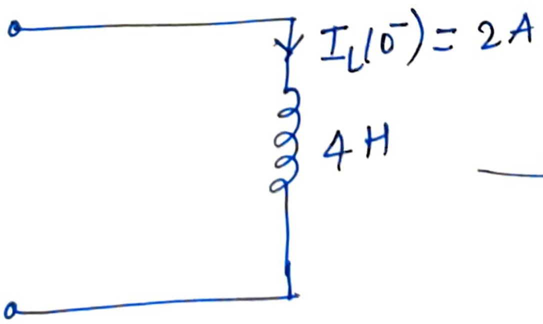
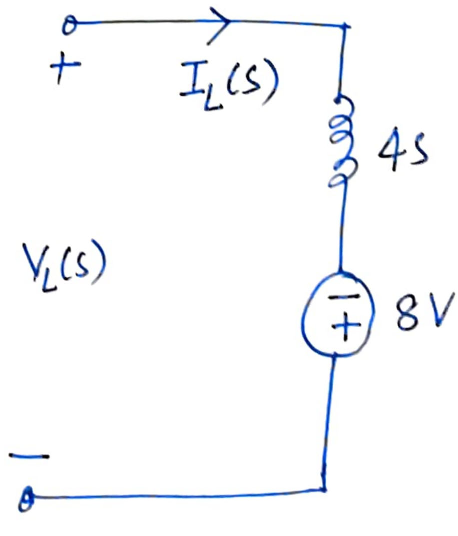


fig Energised Inductor

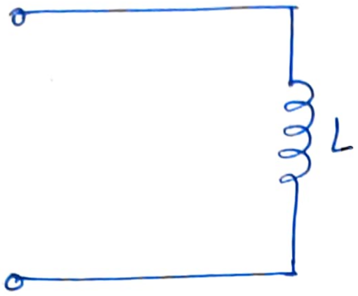
$\xrightarrow{L[\cdot]}$

"'s' domain"

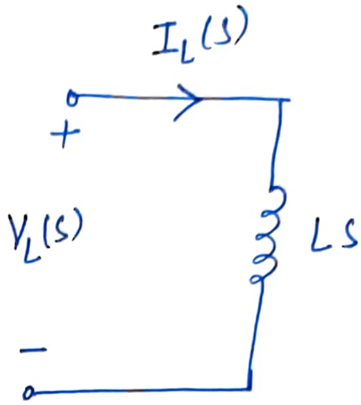


Consider unenergised inductor :

Time Domain | **'S' Domain**



$$\xrightarrow[L[\ ]]{\text{xform}}$$



here $I_L(0^-) = 0\,A$

$E_L(0^-) = 0\,J$

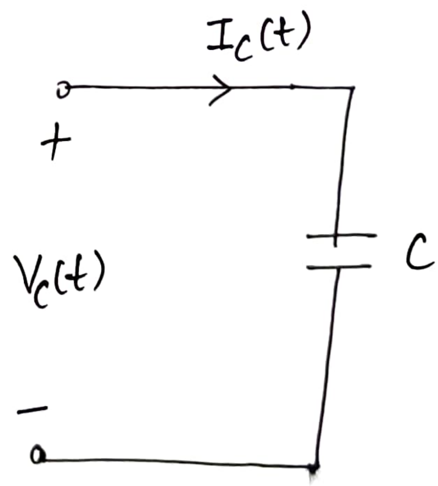
$$\boxed{V_L(s) = LS\, I_L(s)}$$

Transform Impedance $= \boxed{\dfrac{V_L(s)}{I_L(s)} = LS = Z(s)}$

NOTE:

If there is no information about the initial current of the inductor then by default we will consider the unenergised inductor.

Capacitor:



$$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} I_C(t)\,dt$$

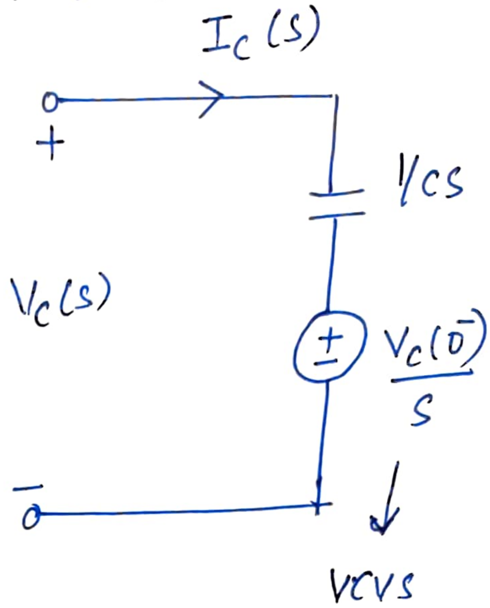
$$V_C(t) = \frac{1}{C}\int_{-\infty}^{0} I_C(t)\,dt + \frac{1}{C}\int_{0}^{t} I_C(t)\,dt$$

$$\boxed{V_C(t) = V_C(0^-) + \frac{1}{C}\int_{0}^{t} I_C(t)\,dt}$$

Applying laplace transform on both sides.

$$\boxed{V_c(s) = \frac{V_c(0^-)}{s} + \frac{I_c(s)}{Cs}}$$

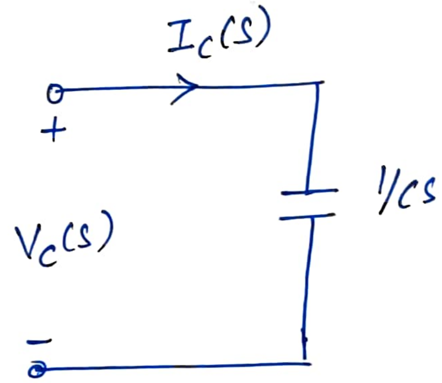
represents.



Consider unenergized capacitor.

here $V_c(0^-) = 0$ volt

$E_c(0^-) = 0$ J



i.e, $$\boxed{V_c(s) = \frac{I_c(s)}{Cs}}$$

i.e, $$\boxed{\frac{V_c(s)}{I_c(s)} = \frac{1}{Cs} = Z(s)}$$ = Transform impedance.

NOTE:

If there is no information of initial voltage of capacitor by default we will consider uncharged capacitor. (or) unenergized capacitor.

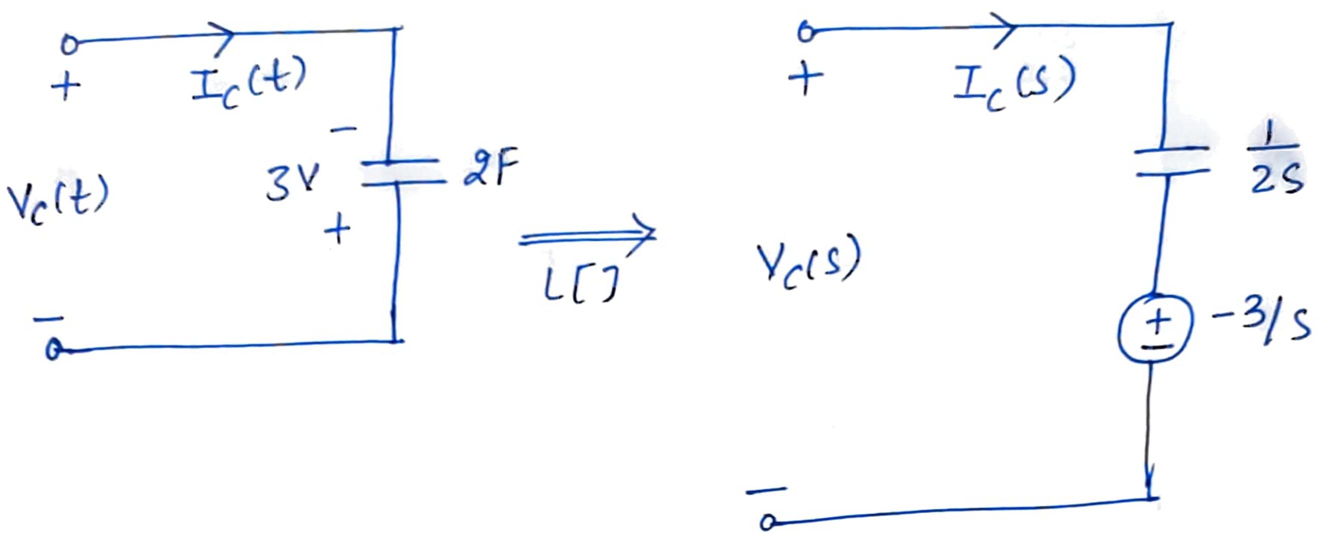
Transform Impedance of Inductor $Z[L] = Ls$

Transform Impedance of Capacitor $Z[C] = \frac{1}{Cs}$

VVIMP

Time Domain

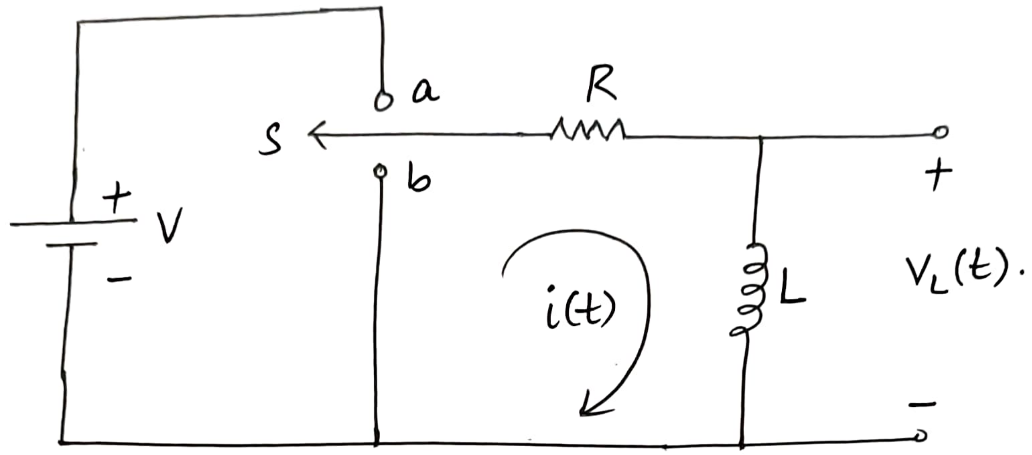
Laplace Transform Domain



Lecture - 5

Questions Based On RL - Network(1):

Q1) Consider the network shown. At $t=0$ the switch is moving from position 'a' to 'b'



Find (i) $i(t)$; $t \geq 0$

(ii) $V_L(t)$; $t \geq 0$.

Soln) Method - I : (Time Domain Analysis)

Step-1: At $t=0^- / t<0 / -\infty<t<0 /$ S.S

switch will be in position "a"

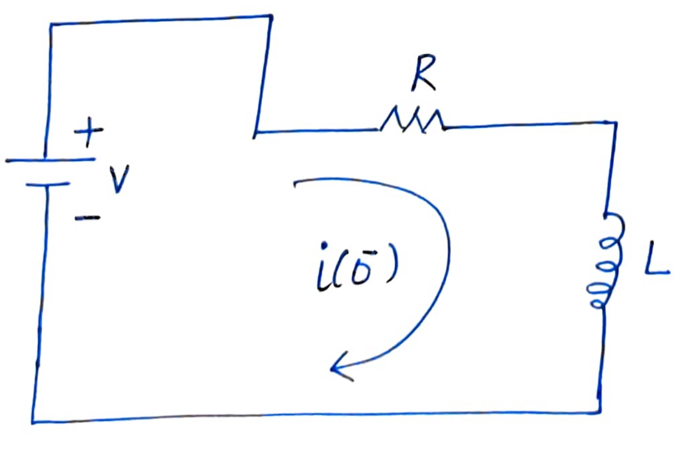
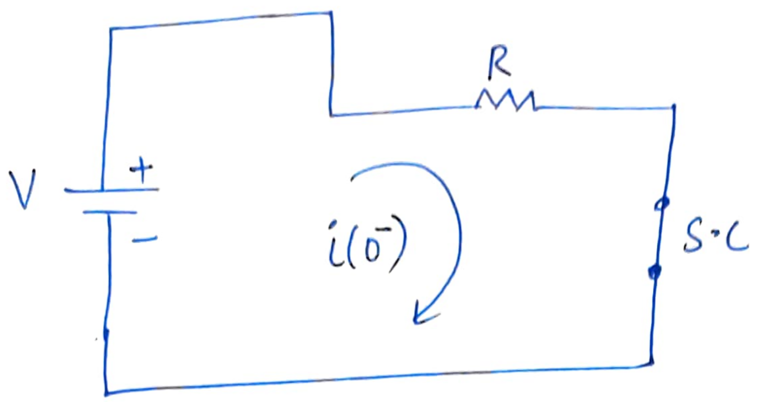


fig at "$t=0^-$"

At steady state (SS) inductor will be short circuit



By KVL

$$V - i(0^-)R = 0$$

$$i(0^-) = \frac{V}{R} \text{ volt}$$

Initial current

$$\boxed{i(0^-) = I_L(0^-) = \frac{V}{R} \text{ volt}}$$

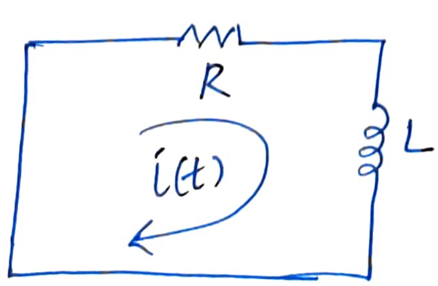
Initial Energy

$$E_L(0^-) = \frac{1}{2} L\, i^2(0^-) = \frac{1}{2} L \times \frac{V^2}{R^2}.$$

$$E_L(0^-) = \frac{LV^2}{2R^2} \text{ Joules.}$$

$$\boxed{V_L(0^-) = 0 \text{ volt} \quad \text{i.e. S.C, It is fixed}}$$

<u>Step-2:</u> At $t = 0$, the switch is moving from a to b. Which means switch will at '<u>b</u>'.



→ Source free RL N/w

It is discharging n/w

The conclusions from above n/w

(i) Source free RL n/w

(ii) Discharging n/w

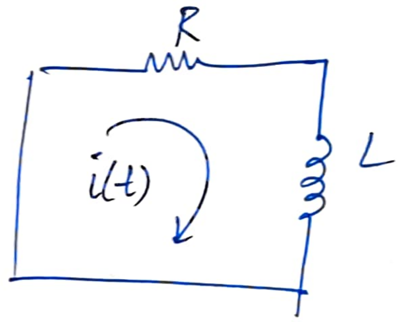
(iii) $i(t) \downarrow$ (decreasing) $\quad i(\infty) = \text{min}/0$

(iv) $E_L(t) \downarrow$ (decreasing) $\quad E_L(\infty) = \text{min}/0.$

(v) $V_L(t) = L \frac{d}{dt} i(t) = \text{Negative.}$

(vi) $V_L(\infty) = L \frac{d}{dt}(\text{min/const}) = 0 \text{ volt} / \text{S.C.}$

By KVL



$$R \times i(t) + L \frac{d}{dt} i(t) = 0$$

$$\frac{d}{dt} i(t) + \frac{R}{L} i(t) = 0$$

$$i(t) = K e^{-R/L\, t}$$

$$i(0) = K e^{-0} = K$$

$$i(t) = i(0) e^{-Rt/L}$$

$$\boxed{\therefore \; i(t) = \frac{V}{R} e^{-Rt/L}}$$

$i(0^-) = \frac{V}{R}$; $\boxed{\cancel{i(0^-) = i(0) = i(0^+)}}$

$$\boxed{i(0^-) = i(0) = i(0^+) = \frac{V}{R}}$$ → Vimp eqn.

$$E_L(t) = \frac{1}{2} L\, i^2(t)$$

$$E_L(t) = \frac{1}{2} L \times \left[\frac{V}{R} e^{-Rt/L}\right]^2$$

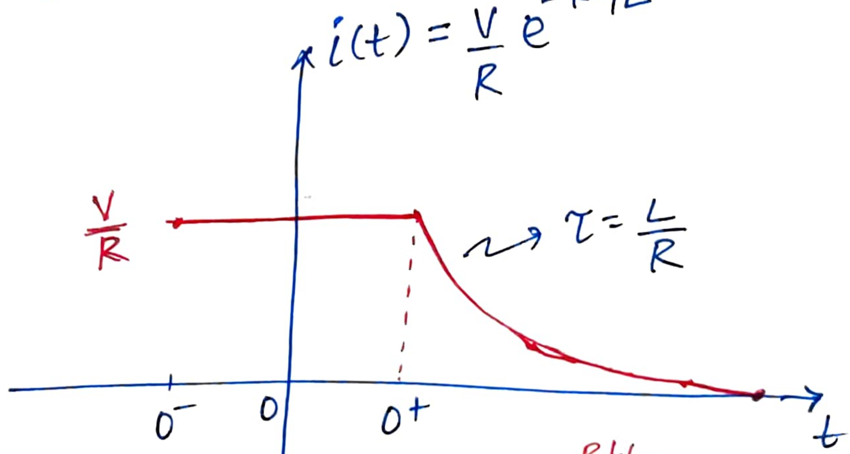
$$\boxed{E_L(t) = \frac{LV^2}{2R^2} e^{-2Rt/L}}$$

$$V_L(t) = L \frac{d}{dt} i(t) = L \frac{d}{dt}\left[\frac{V}{R} e^{-Rt/L}\right]$$

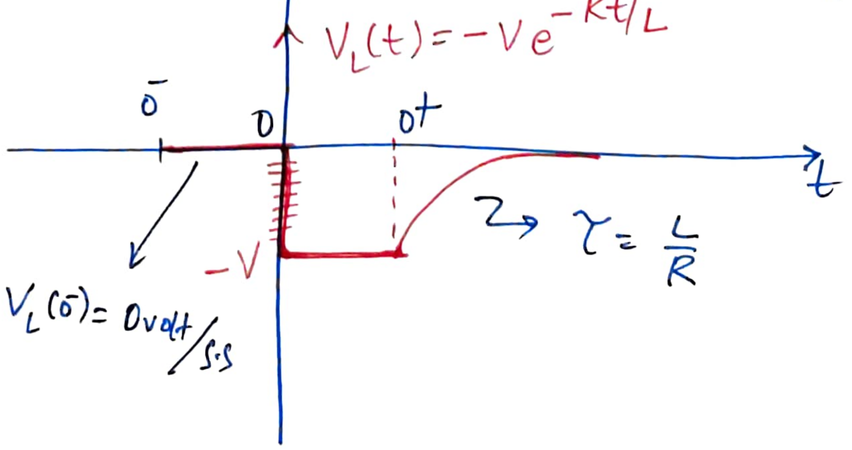
$$= \frac{LV}{R} \times \left(-\frac{R}{L}\right) \times e^{-Rt/L}$$

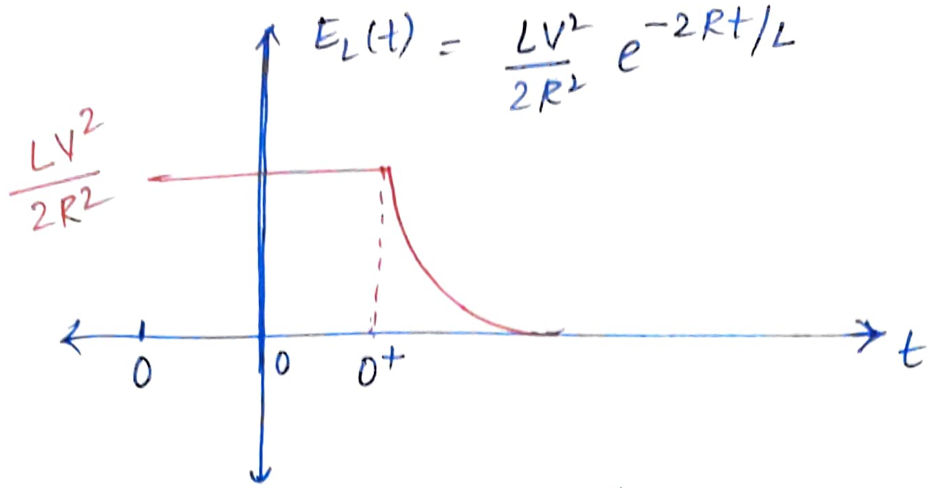
$$\boxed{V_L(t) = -V e^{-Rt/L}}$$ = negative.

$$\boxed{i(t) = \frac{V}{R} e^{-Rt/L}}$$



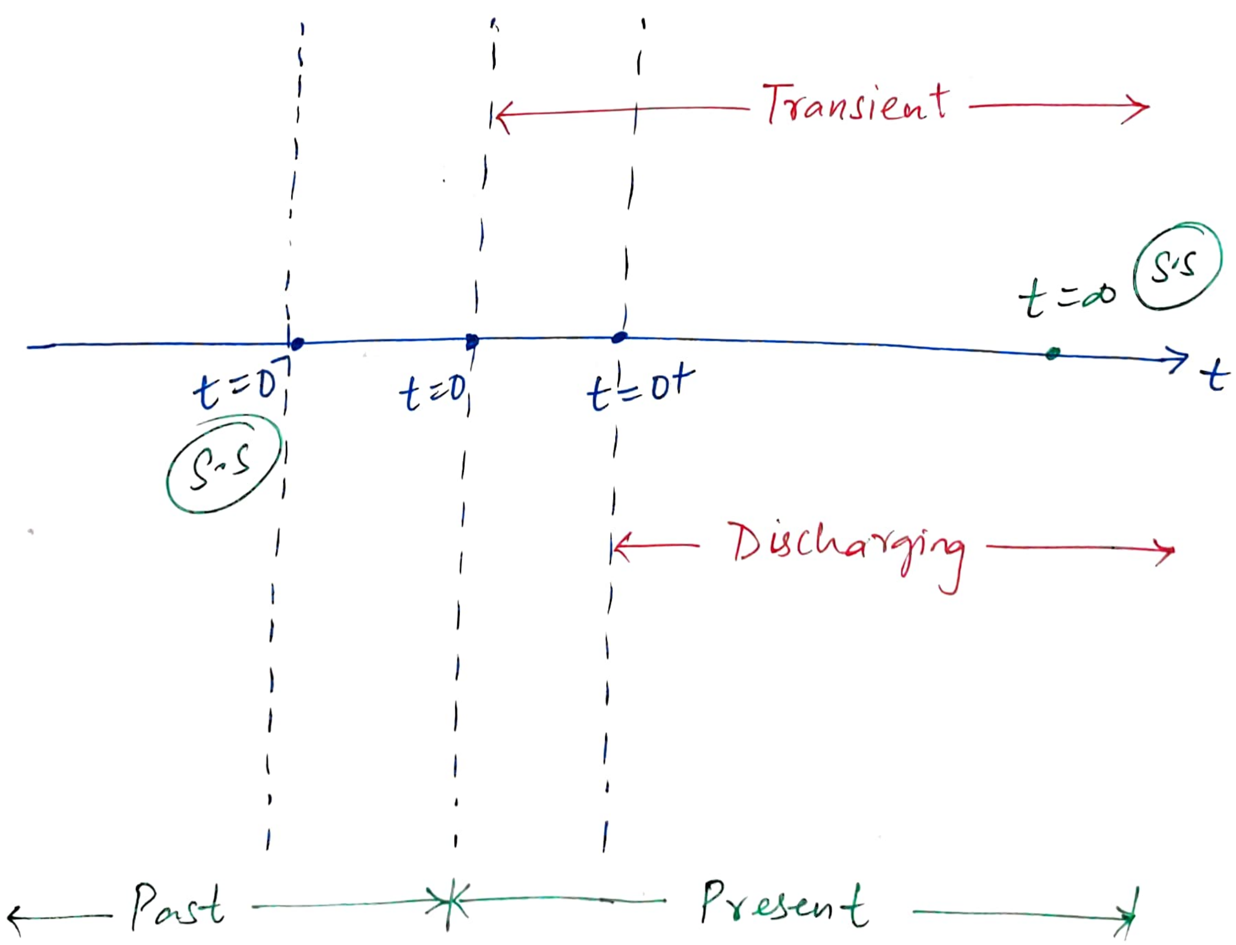
$$\boxed{V_L(t) = -V e^{-Rt/L}}$$





$$\boxed{\therefore\ E_L(t) = \frac{LV^2}{2R^2} e^{-2Rt/L}}$$

$$\boxed{\text{Time const of energy} = \frac{L}{2R}} \quad ; \quad \boxed{\text{Time const of energy} = \frac{1}{2}\left(\text{Time const of voltage}\right)}$$



Important Conclusions :

$$E_L(0) = E_L(0^+)$$

$$i(0) = i(0^+)$$

$$V_L(0) = V_L(0^+)$$

$$I_R(0) = I_R(0^+)$$

$$V_R(0) = V_R(0^+)$$

Table - 1

W.K.T

$$i(0^-) = \frac{V}{R}$$

$$E_L(0^-) = \frac{LV^2}{2R^2}.$$

$$V_L(0^-) = 0 \text{ volt}.$$

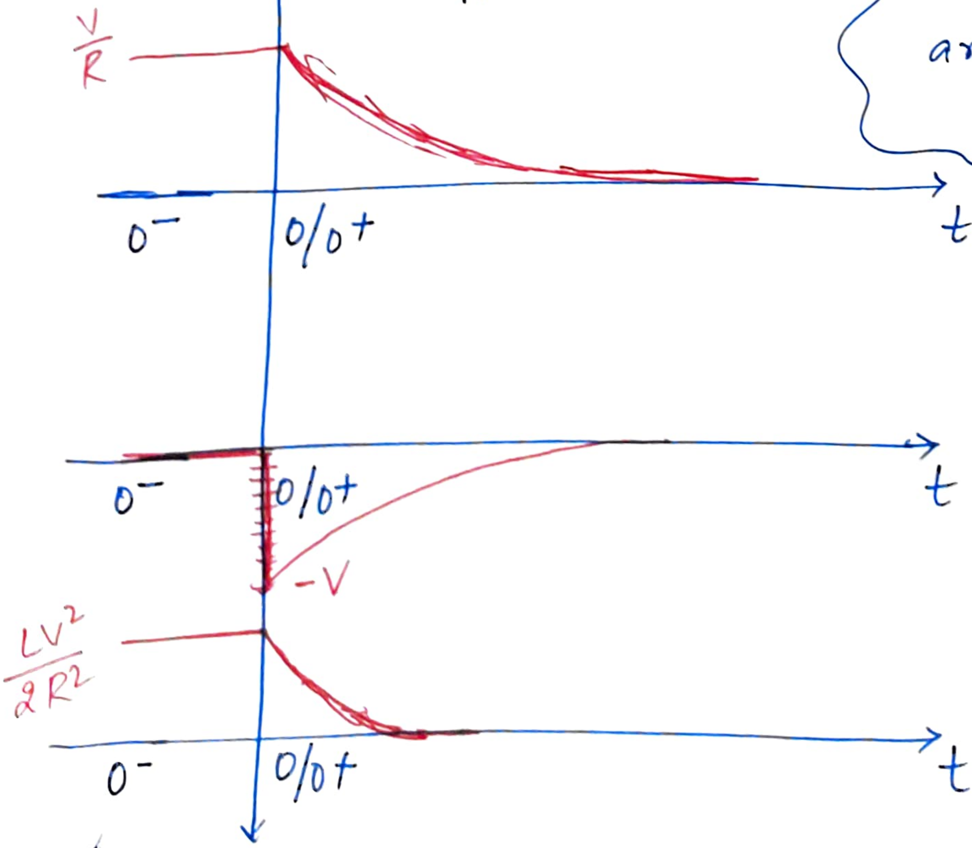
$$i(0^+) = \frac{V}{R} \text{ A}$$

$$E_L(0^+) = \frac{LV^2}{2R^2}$$

$$V_L(0^+) = -V.$$

Clearly. we can say that there will be no difference between 0 & 0^+.

$$i(t) = \frac{V}{R} e^{-Rt/L}$$



0 & 0^+ states results are equal always

$$\boxed{t \geq 0 \ \& \ t \geq 0^+ \ \text{are same.}}$$

from table - 1:

$$E_L(0^-) = E_L(0^+)$$

$$i(0^-) = i(0^+)$$

$$V_L(0^-) \neq V_L(0^+)$$

$$I_R(0^-) \neq I_R(0^+)$$

$$V_R(0^-) = V_R(0^+)$$

In special cases $I_R(0^-) = I_R(0^+)$

Method - II : (Transform Domain Analysis)

Step-1: $t<0$ / s.s / $-\infty < t < 0$ / $t = 0^-$

$$i(0^-) = \frac{V}{R} \text{ Amp}$$

$$E_L(0^-) = \frac{LV^2}{2R^2} \text{ J}$$

step-2:

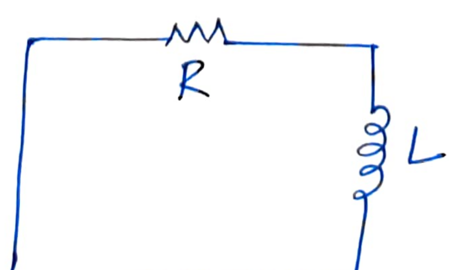
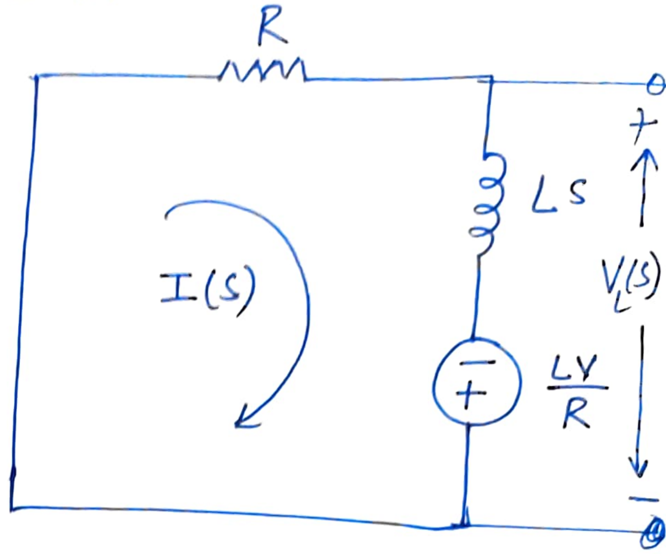


fig: $t \geq 0$

Here the inductor has initial current i.e, the inductor is energized inductor.

$$-I(s)[R+Ls] + \frac{LV}{R} = 0$$



$$I(s) = \left[\frac{LV/R}{R+Ls}\right]$$

$$I(s) = \frac{LV/R}{L(s+R/L)} = \left[\frac{V/R}{s+R/L}\right]$$

$$I(s) = \left[\frac{V/R}{s+R/L}\right]$$

$$\Downarrow L^{-1}[\,]$$

$$\boxed{i(t) = \frac{V}{R} e^{-Rt/L}} \quad \left(\tau = \frac{L}{R} \text{ sec}\right)$$

$$\boxed{E_L(t) = \frac{1}{2} L i^2(t) = \frac{LV^2}{2R^2} e^{-2Rt/L}}$$

$$\boxed{V_L(t) = L\frac{d}{dt} i(t) = -Ve^{-Rt/L}}$$

Method-III: (Concept of Transient Eq.n):

Step-1:

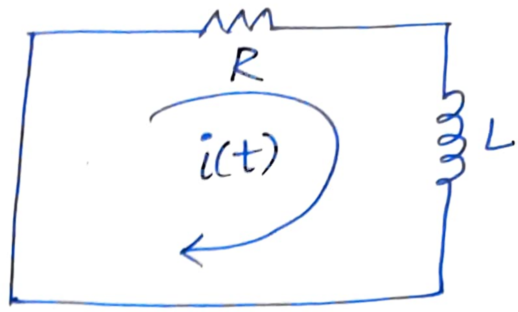
$$i(0^-) = \frac{V}{R}$$

$$i(0^-) = i(0^+) = \frac{V}{R}$$

Step-2: Now the switch is moved from a to b.

$\therefore$ K.V.K.T

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right] e^{-t/\tau}; \quad t \geq 0$$



Here our job is to find

a) $i(\infty)$ at steady state (Final Value)

b) $i(0^+)$ (Initial Value).

c) τ = time const.

time constant can be in the below four forms.

(i) $\tau = \frac{L}{R}$ sec

(ii) $\tau = \frac{L}{R_{Th}}$ sec

(iii) $\tau = \frac{L_{eq}}{R}$ sec

(iv) $\tau = \frac{L_{eq}}{R_{Th}}$ sec.

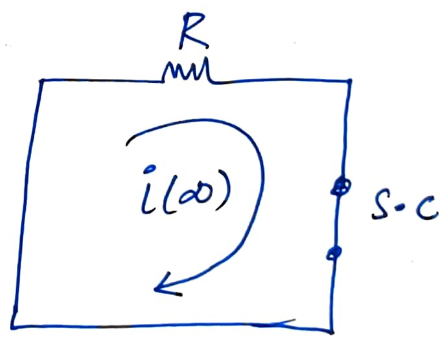


fig $t=\infty$

Now we have

$$i(0^-) = i(0^+) = \frac{V}{R} \text{ Amp} \longrightarrow (1)$$

$$i(\infty) = 0 \longrightarrow (2)$$

Because at $t=\infty$ i.e. S.S the inductor becomes S.C. In this n/w at $t=\infty$ there is no source. So, clearly current $i(\infty) = 0$.

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]\, e^{-t/\tau} \qquad \tau = \frac{L}{R}$$

$$i(t) = 0 + \left[\frac{V}{R} - 0\right] e^{-t/\tau}$$

$$\boxed{i(t) = \frac{V}{R} e^{-t/\tau}}$$

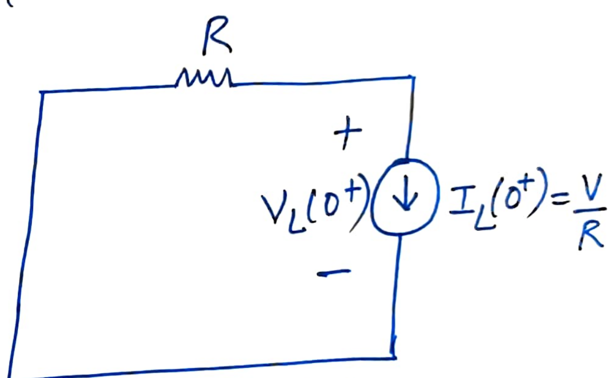
***** To find $V_L(t)$ using the transient ~~eqns~~ VVIMP!:**

$$V_L(t) = V_L(\infty) + [V_L(0^+) - V_L(\infty)]\, e^{-t/\tau} \quad ; \; t \geq 0 \rightarrow \text{I}$$

(i) $\tau = \frac{L}{R}$ sec $\rightarrow$ ①

(ii) $V_L(\infty) =$ '0' volt → ② (fixed) ($\because$ at S.S inductor is S.C).

(iii) $V_L(0^+) = \,?$



"fig $t = 0^+$"

By KVL

$$-R \times \frac{V}{R} - V_L(0^+) = 0$$

$$V + V_L(0^+) = 0$$

$$V_L(0^+) = -V \text{ volt} \rightarrow ③$$

put ①, ② + ③ in I

$$V_L(t) = 0 + [-V - 0]\, e^{-t/\tau}$$

$$\boxed{V_L(t) = -V e^{-t/\tau}}$$

NOTE:

$I_L(t)$	$V_L(t)$	$I_R(t)$	$V_R(t)$
$I_L(0^+) = I_L(0^-)$ fig is not required	$V_L(0^+) = ?$ fig is required.	$I_R(0^+) = ?$ fig is required.	$V_R(0^+) = ?$ fig is required.
$I_L(\infty) = ?$ fig is required.	This is always fixed $V_L(\infty) = 0$ fig is not required	$I_R(\infty) = ?$ fig is required.	$V_R(\infty) = ?$ fig is required.

Always $V_L(0^-) = 0V$ & $V_L(\infty) = 0V$ as it is steady state