CHAPTER - 7

RESONANCE

Lecture - 01

SERIES RLC RESONANCE CIRCUIT :

RESONANCE :

→ Resonance describes the circuit condition in which (i) Inductive & capacitive features neutralize with each other.

(ii) Powerfactor of circuit will be unity.

(iii) Impedance of circuit will become resistive in nature.

(iv) Input voltage and input current will be in phase.

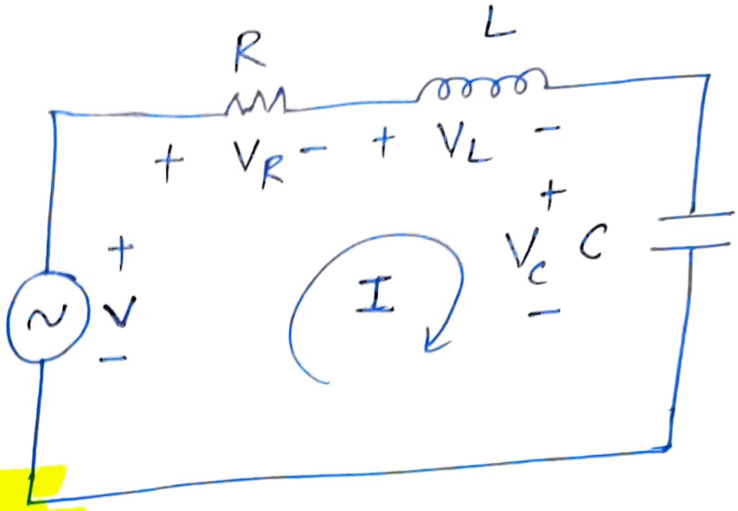
→ Resonance describes the energy transfer between the capacitor and inductor at a constant freq rate.

→ The freq rate at which energy transfer will happen between capacitor & inductor is called as "resonance frequency" (OR) "frequency of oscillation".

1. Series RLC Network

$V = V_R + j(V_L - V_C) \rightarrow ①$

$Z = R + j(X_L - X_C) \rightarrow ②$



Condition of Resonance:

$Img[V] = 0$ (OR) $Img(Z) = 0$

$V_L - V_C = 0$

$V_L = V_C$

$I X_L = I X_C$

$\boxed{X_L = X_C}$.

$X_L - X_C = 0$

$\boxed{X_L = X_C}$

For the case of resonance $\Rightarrow \boxed{Z = R}$

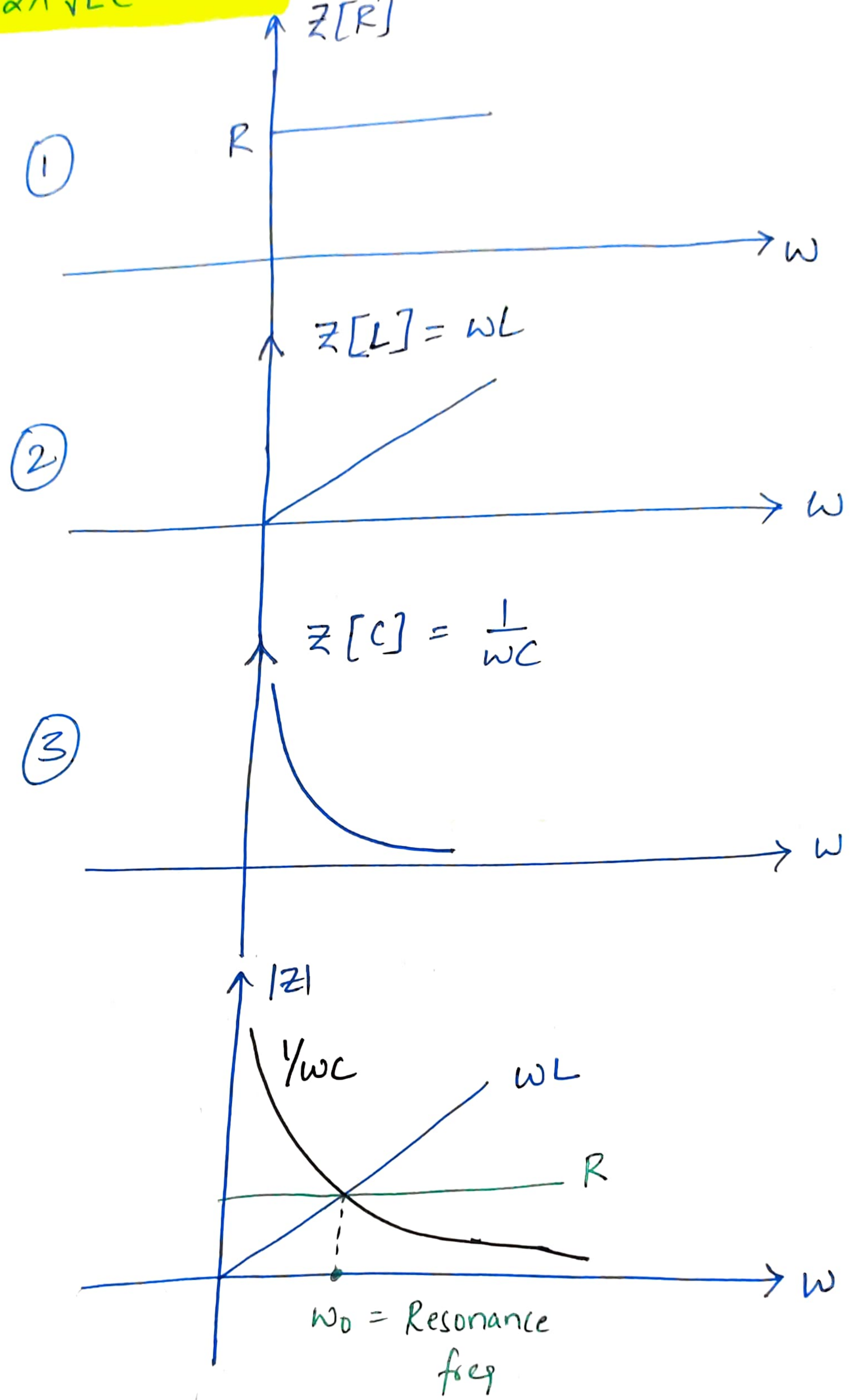
Now, We have

$X_L = X_C$

$\omega_0 L = \frac{1}{\omega_0 C}$

$\omega_0^2 = \frac{1}{LC} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \ Hz.$$



If we combine 1, 2 & 3. The graph obtained is drawn as follows :

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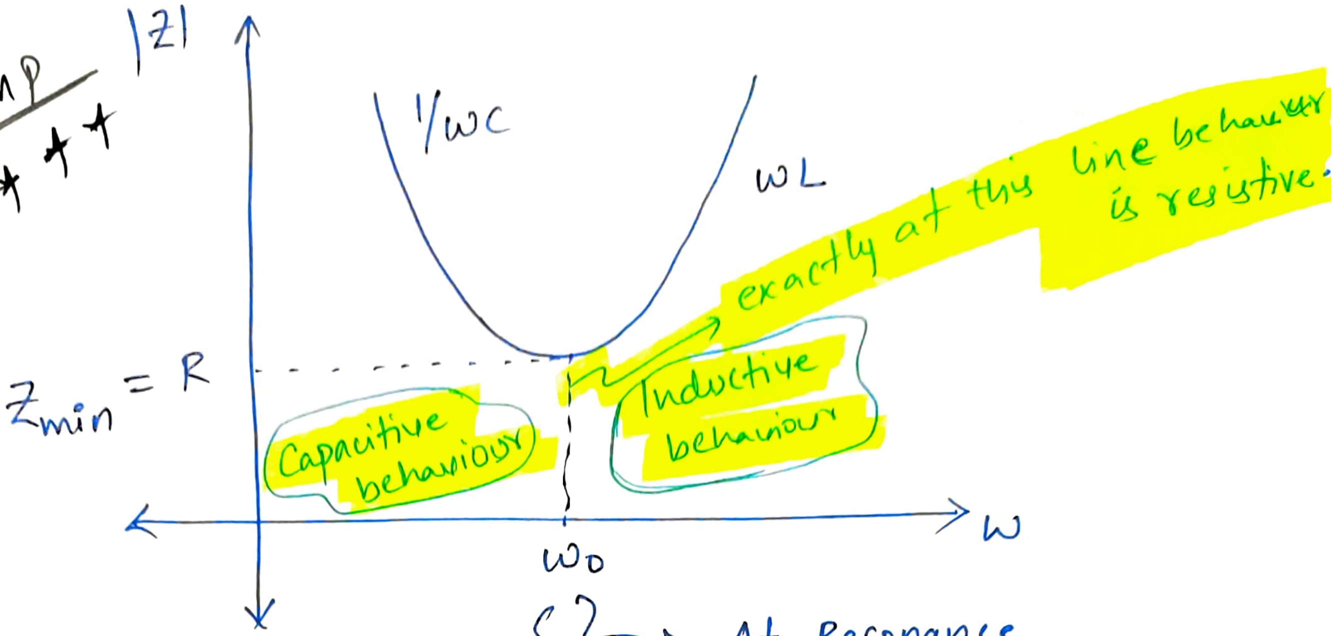

$\to$ At Resonance

Here the behaviour is resistive in nature.

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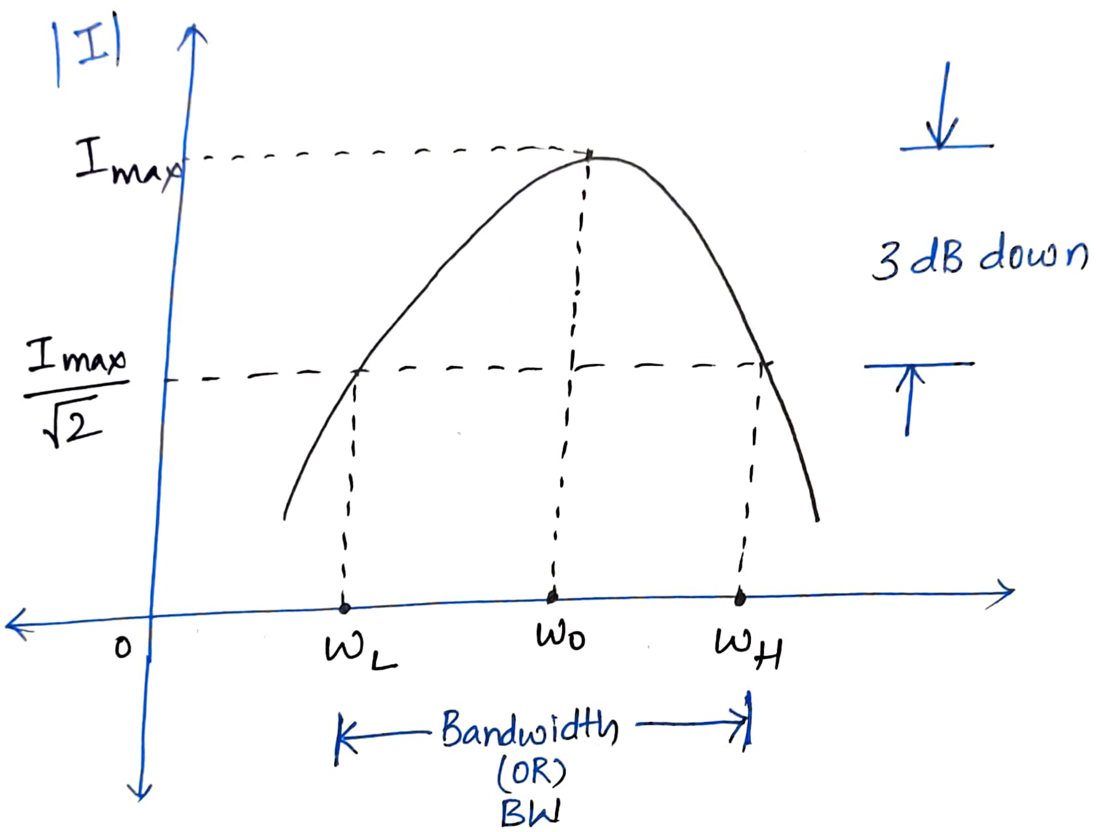
ω	R/L/C	Power Factor
$\omega < \omega_0$	Capacitive (C)	Leading
$\omega > \omega_0$	Inductive (L)	Lagging
$\omega = \omega_0$	Resistive (R)	Unity Power factor (UPF)

$$I = \frac{V}{Z} = \frac{V\angle 0^\circ}{R + j(\omega L - 1/\omega C)}$$

$$|I| = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



ω_L : lower 3 dB freq ; BW = Bandwidth.

ω_H : upper (or) higher 3 dB freq. ;

At $\omega = \omega_0 \Rightarrow$ (i) $Z_{min} = R$

(ii) $I_{max} = \frac{V_{min}}{R}$.

Series RLC resonance ckt is also called as acceptor ckt because it accepts maximum current at resonance

Bandwidth $= \omega_H - \omega_L$

Now $x \xrightarrow{dB} 20 \log_{10} x$

$\frac{1}{\sqrt{2}} \xrightarrow{dB} 20 \log \frac{1}{\sqrt{2}} = -3.010\, dB$

If the I_{max} decreases to $\frac{I_{max}}{\sqrt{2}}$ (or) $0.707\, I_{max}$ then there will be 3dB down (ie, −3dB) from the max current I_{max} (in dB).

Q) Here the ckt is in series, but how the current changes in series?

Ans: The current changes w.r.t angular frequency "w". If 'w' changes the impedance (Z) changes. If impedance(Z) changes then the current changes. So, this is the answer.

Calculation of 3dB frequency :

3-dB frequency :

The frequency at which current response will be 70.7% or $\frac{1}{\sqrt{2}}$ times of its max current value is called as 3dB frequency

$$|I| = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At $I = I_{max}$; $P_0 = I_{max}^2 R$

At $I = \frac{I_{max}}{\sqrt{2}}$; $P_0' = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R = \frac{I_{max}^2 R}{2}$

At $I = 0.707\, I_{max}$; $P_0' = \frac{P_0}{2}$

→ If the max current I_{max} changes from I_{max} to $\frac{I_{max}}{\sqrt{2}}$ then the maximum power changes from P_0 to $\frac{P_0}{2}$ (i.e it will be halved).

→ That is why we call the

ω_L : lower half power frequency

ω_H : higher half power frequency

$$|I| = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{I_{max}}{\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega_{3\text{-}dB} L - \frac{1}{\omega_{3\text{-}dB} C}\right)^2}}$$

$$I_{max} = \frac{V}{R}$$

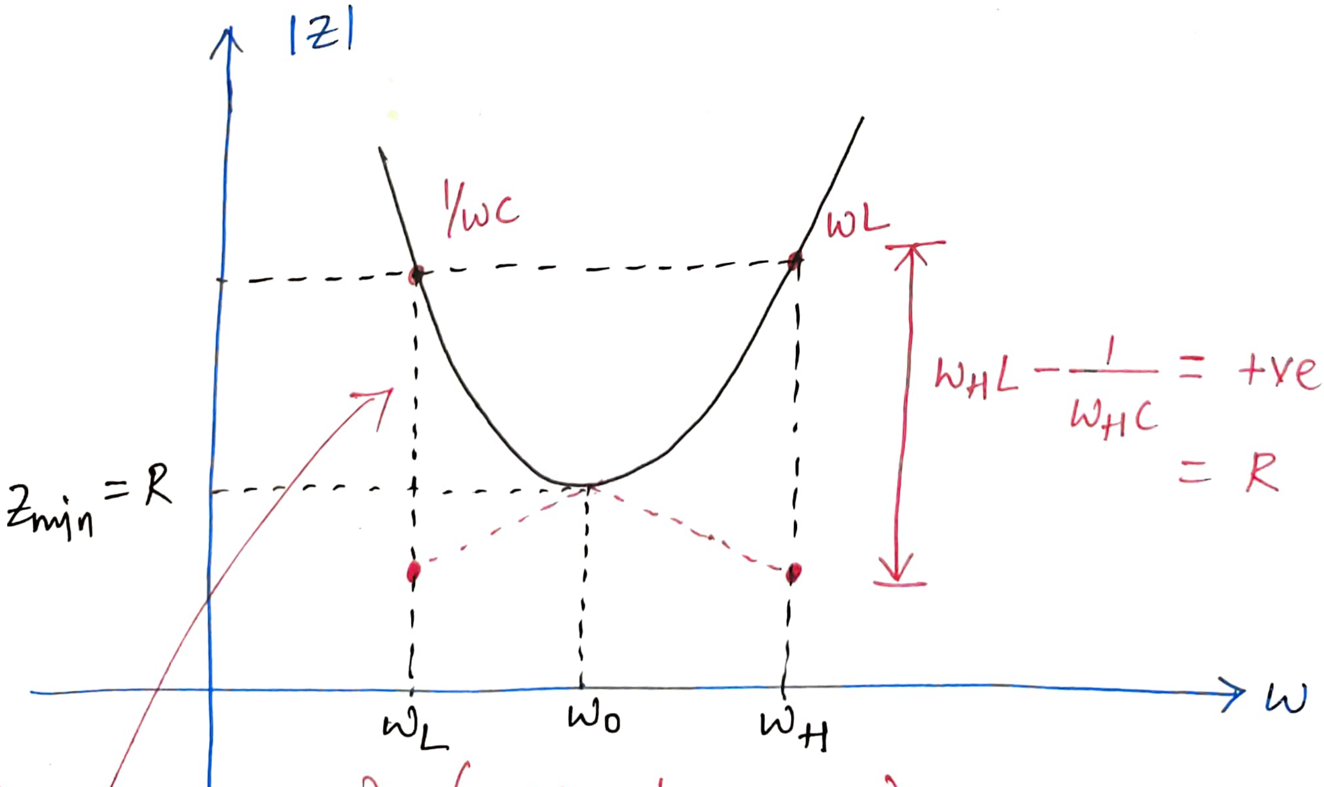
$$\frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega_{3\text{-dB}}L - \frac{1}{\omega_{3\text{-dB}}C}\right)^2}}$$

$$R^2 + \left(\omega_{3\text{-dB}}L - \frac{1}{\omega_{3\text{-dB}}C}\right)^2 = 2R^2$$

$$\boxed{\left(\omega_{3\text{-dB}}L - \frac{1}{\omega_{3\text{dB}}C}\right) = \pm R} \rightarrow \text{I}$$

$$\omega_{3\text{-dB}}L - \frac{1}{\omega_{3\text{dB}}C} = R \quad \text{(OR)} \quad \omega_{3\text{-dB}}L - \frac{1}{\omega_{3\text{-dB}}C} = -R$$

$$\underbrace{\left(\omega_H L - \frac{1}{\omega_H C}\right) = R}_{\text{why ?}} \rightarrow 1 \qquad \underbrace{\left(\omega_L L - \frac{1}{\omega_L C}\right) = -R}_{\text{why ?}} \rightarrow 2$$



$$\left(\frac{1}{\omega_L C} - \omega_L L = R\right) \Rightarrow \left(\omega_L L - \frac{1}{\omega_L C} = -R\right)$$

Now from ① & ②

ω_H from ①

$$\omega_H^2 LC - 1 = \omega_H RC$$

$$\omega_H^2 LC - \omega_H RC - 1 = 0$$

$$\omega_H^2 - \omega_H \frac{R}{L} - \frac{1}{LC} = 0$$

$$\omega_H = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{\cancel{4}1}{LC}}}{2}$$

$$\omega_H = \frac{\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 + \frac{\cancel{4}1}{LC}}}{2}$$

$$\boxed{\omega_H = \left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right) \text{ rad/sec}}$$

Similarly

$$\boxed{\omega_L = \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right) \text{ rad/sec}}$$

$$\text{Bandwidth} = BW = \omega_H - \omega_L$$

$$\therefore BW = \frac{R}{2L} - \left(-\frac{R}{2L}\right) = \frac{R}{L}$$

$\therefore$ ~~Bandwidth~~ $= \cancel{\frac{R}{L}}$

$$\boxed{\text{Bandwidth} = \frac{R}{L}\ \text{rad/sec} = \frac{R}{2\pi L}\ \text{Hz}}$$

$$\boxed{\begin{array}{l}\text{Bandwidth} = \text{function}(R, L) \\ \text{Bandwidth} \neq \text{function}(C)\end{array}}$$

VVIMP

Now $$\omega_H \omega_L = \left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)\left(\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)$$

$$\omega_H \omega_L = -\left(\frac{R}{2L}\right)^2 + \left[\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}\right]$$

$$\boxed{\omega_H \omega_L = \frac{1}{LC}} \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}\ \text{rad/sec}}$$

$$\boxed{\omega_0^2 = \omega_H \omega_L = \frac{1}{LC}}$$

$$\omega_H = \frac{BW}{2} + \sqrt{\left(\frac{BW}{2}\right)^2 + \omega_0^2}$$

$$\omega_L = -\frac{BW}{2} + \sqrt{\left(\frac{BW}{2}\right)^2 + \omega_0^2}$$

If $\omega_0 >>> BW$

$\rightarrow \omega_H \approx \frac{BW}{2} + \omega_0$

$\rightarrow \omega_L \approx -\frac{BW}{2} + \omega_0$

SATYA

PREPARATION

IS IN THE FINAL

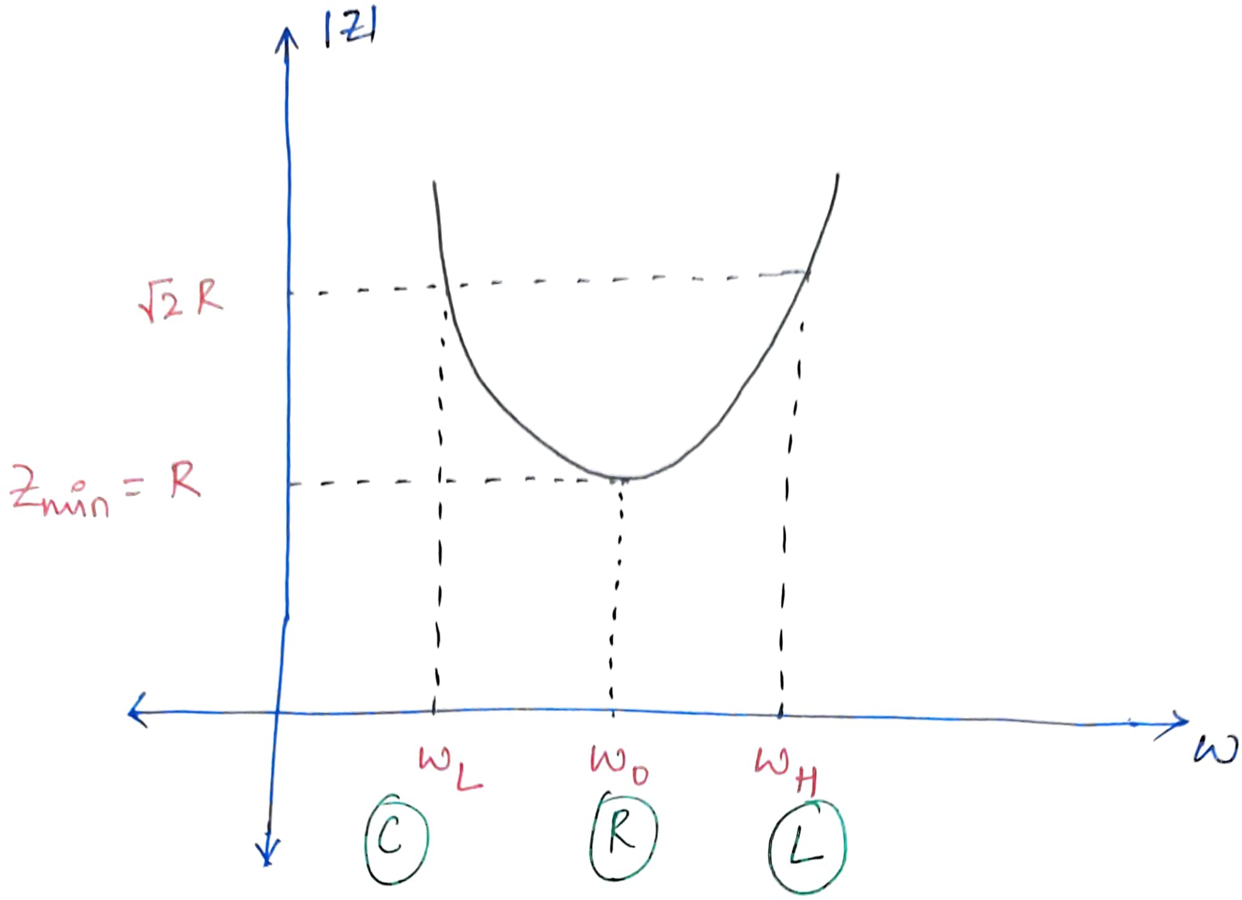
PHASE ... ☺ ☺ ☺

GOOD N AWESOME

23/07/20

ω	$\omega L - \frac{1}{\omega C}$	$\lvert Z\rvert = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$	$\angle Z = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$	$I = \frac{V}{Z}$	Power factor
$\omega = \omega_H$ $(\omega > \omega_0)$	$\omega_H L - \frac{1}{\omega_H C}$	$\lvert Z\rvert = \sqrt{R^2+R^2} = \sqrt{2}R$	$\angle Z_H = \tan^{-1}(1) = 45^\circ$	$I_H = \frac{V}{\sqrt{2}R}\angle -45^\circ$	$\cos 45^\circ = 0.707$ logging
$\omega = \omega_L$ $(\omega < \omega_0)$	$\omega_L L - \frac{1}{\omega_L C}$	$\lvert Z_L\rvert = \sqrt{2}R$	$\angle Z_L = -45^\circ$	$I_L = \frac{V}{\sqrt{2}R}\angle 45^\circ$	$\cos 45^\circ = 0.707$ leading.
$\omega = \omega_0$	$\omega_0 L - \frac{1}{\omega_0 C} = 0$	$Z_{min} = R$	$\angle Z_0 = 0^\circ$	$I_0 = \frac{V}{R}\angle 0^\circ$	Unity power factor i.e. PF = 1.

Don't Remember this table. Its VIMP.
Create It



3-dB Impedance Phasor Diagram :