

27/06/19

## Chapter - 3 : NETWORK THEOREMS :

### Lecture 1 : INTRODUCTION OF THEVENIN'S THEOREM :

Purpose of learning Network Theorems :

To simplify the network parameters very easily & with less number of steps.

3.1

#### Thevenin's Theorem :

1. This theorem is valid for both independent and dependent sources  $\text{m/w}'s$ .
2. Equivalent of thevenins  $\text{m/w}$  is same as practical voltage equivalent  $\text{m/w}$ . The practical voltage equivalent  $\text{m/w}$  is given as

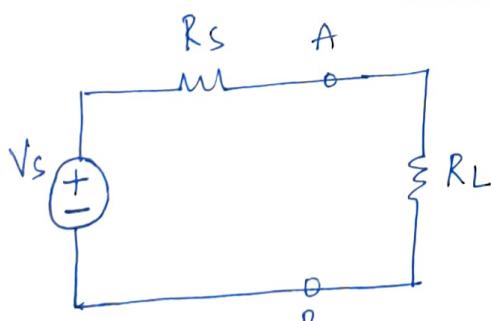
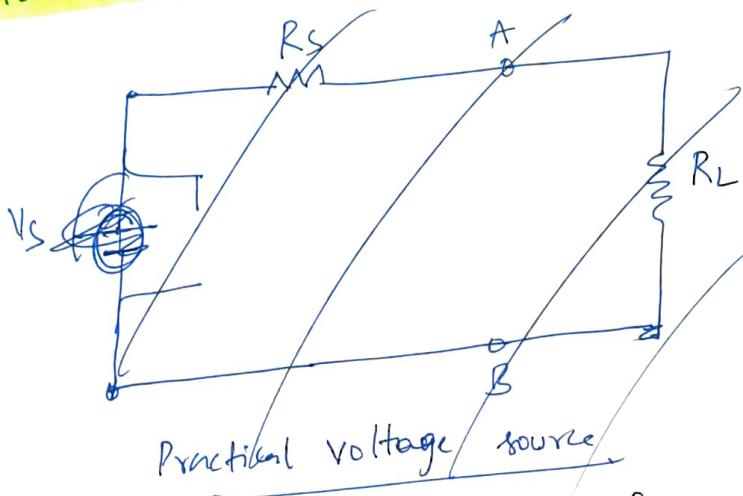
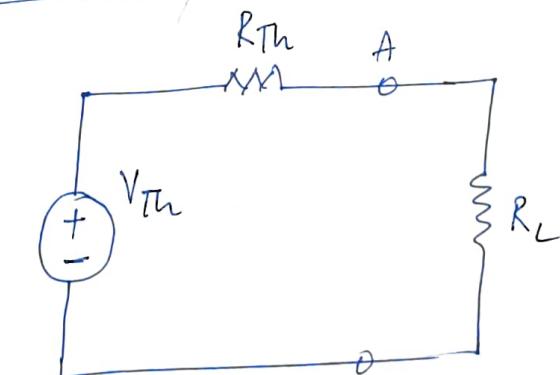


fig: Practical Voltage Source equivalent



Equivalent ckt of Thevenin

3. Thevenin voltage is referred as open ckt voltage across the load terminal, i.e., current across the load terminal is '0' (zero).

i.e.  $V_{DC} = V_{Th}$

$$V_{Th} = V_{oc} = \text{ideal voltmeter reading (Vm)}$$

4. Calculation of  $R_{Th}$  depends on the behaviour of the network.

4(a) When nw contains only independent sources then

$$R_{Th} = \text{Req across the load terminal with all independent sources value is zero}$$

Independent sources value is zero

i.e.

Voltage source = 0



Short ckt



Current source = 0



Open ckt

$R_{Th} = \text{Req when voltage source is short circuited (or) current source is open ckt}$

4b) When n/w contains both dependent and independent sources

$$R_{Th} = \frac{V_{dc}}{I_{dc}}$$

when independent sources value is zero, which means voltage source is S.C & current source is O.C

where  $V_{dc}$  = dc voltage across the load terminal.

$I_{dc}$  = dc current which flowing from  $V_{dc}$ .

4c) When n/w contains only dependent source:

$$R_{Th} = \frac{V_{dc}}{I_{dc}}$$

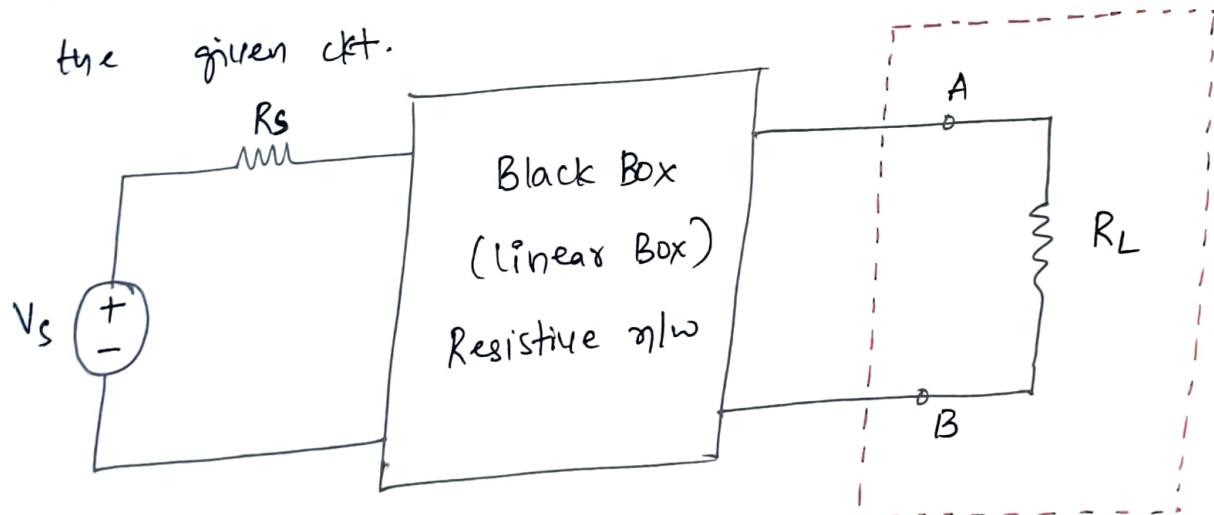
→ special case of n/w

$$V_{Th} = 0' \text{ volt}$$

$V_{Th}$  = (function of independent source)

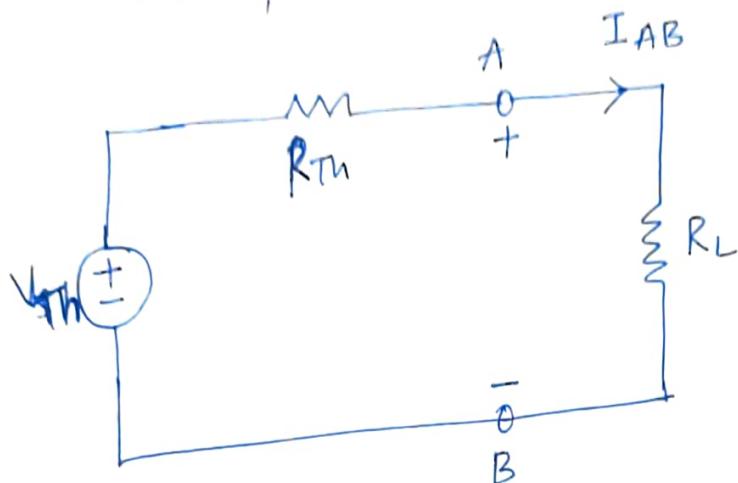
↓  
no conditions.

Example: find the thevenin eq of the dotted part of the given ckt.



Soln: Let us consider the dotted part of the given n/w.

Thevenin's equivalent of the dotted part of n/w is as follows



Applying KVL for loop ABA

$$V_{Th} - I_{AB} R_{Th} - I_{AB} R_L = 0$$

$$V_{Th} = I_{AB} (R_{Th} + R_L).$$

$$\boxed{I_{AB} = \frac{V_{Th}}{R_{Th} + R_L}} \rightarrow \textcircled{1}$$

We know by Ohm's law

$$V_{AB} = I_{AB} R_L = \frac{V_{Th} R_L}{R_{Th} + R_L}$$

$$\boxed{V_{AB} = \frac{V_{Th} R_L}{R_{Th} + R_L}} \rightarrow \textcircled{2}$$

### Conclusion:

1. If we know thevenin voltage ( $V_{Th}$ ) & thevenin resistance ( $R_{Th}$ ) of the given n/w, we can find the load current & load voltage of that particular

n/w. This is the main intention or essence of the thevenins theorem.

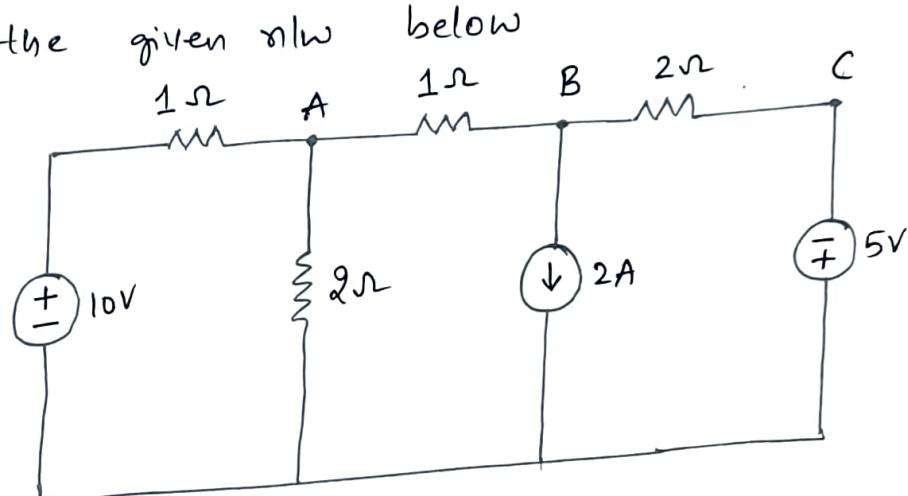
2. We can find voltage & current of any branch of n/w if we know thevenins voltage & thevenins resistance of the n/w.

### Lecture-2

#### Example Based On Thevenins Theorem (1) :

Example: Consider the given n/w below

(i) find the thevenin equivalent across 'AB'



(ii) find the thevenin equivalent across 'BC'

Soln: (i) Thevenin Equivalent Across AB :

Now (II) To find  $V_{Th}$ :

According to the property -

→ Remove the branch resistance across AB. (Open ckt)

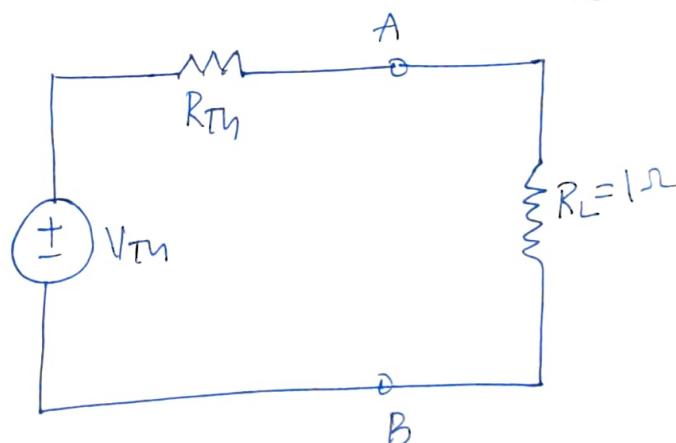
→ Find the ~~for~~ voltage

across AB (i.e  $V_{AB}$ ) . It

is also called as ~~V<sub>AB</sub>~~

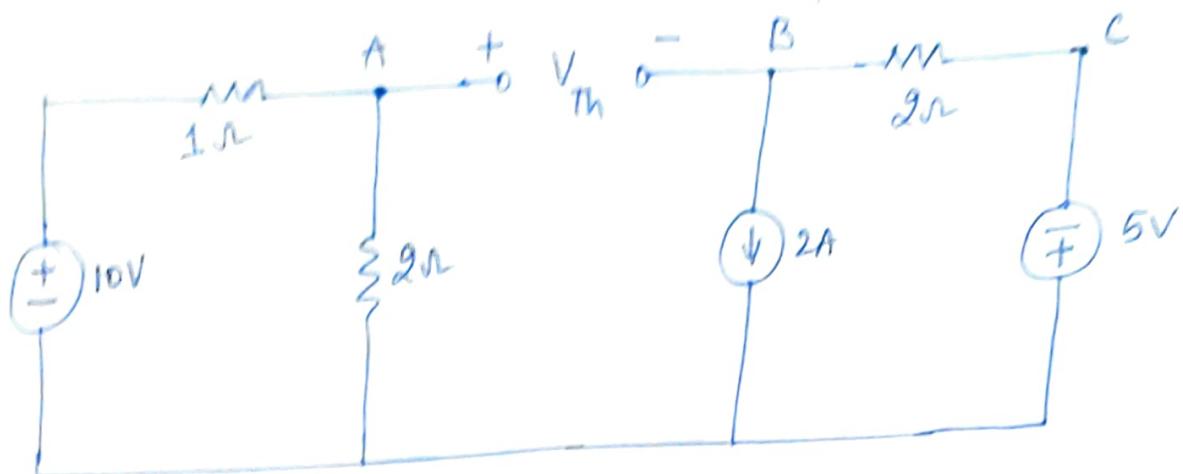
open ckt voltage

Consider thevenin eq~~s~~ across AB .



$$V_{AB} = V_{OC} = V_{Th}$$

$V_{AB}$  is known as Thevenin voltage of ~~across~~ AB.



$$V_{AB} = V_{Th} = V_A - V_B$$

(a)  $V_A$ :

By VDR  $V_A = \frac{2}{3} \times 10 = \frac{20}{3}$  Volts;  $\boxed{V_A = \frac{20}{3} \text{ Volts}}$

(b)  $V_B$ :

By KCL  $\rightarrow 5 + (V_B \times 2)$

By KCL at "B"

$$\frac{V_B + 5}{2} + 2 = 0$$

$$\frac{V_B + 5 + 4}{2} = 0$$

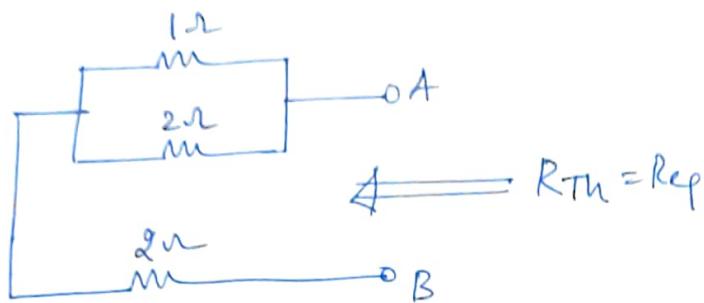
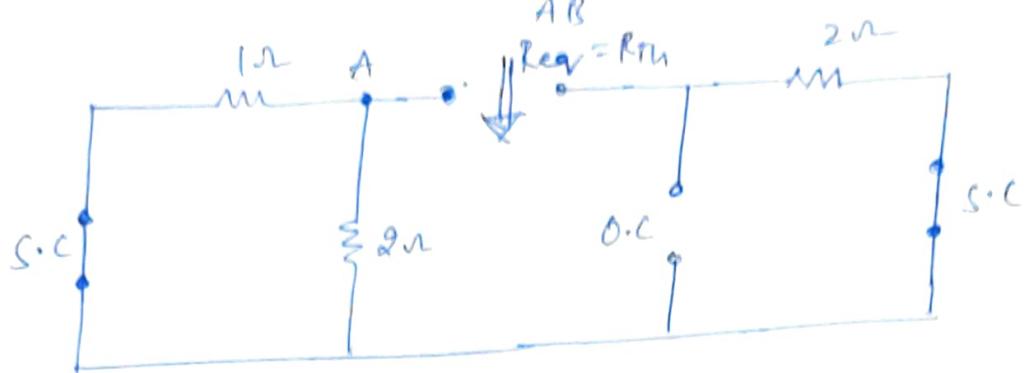
$$\boxed{V_B = -9 \text{ Volts}}$$

$$V_{Th} = V_{AB} = \frac{20}{3} + 9 = \frac{47}{3} \text{ Volts}$$

To find  $R_{Th}$ :

Here we observe only independent sources. So we have to use first case

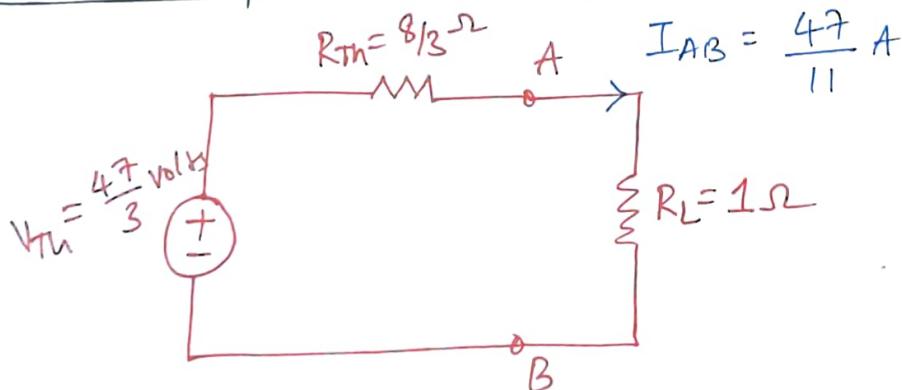
i.e.  $R_{Th} = Req$  / when voltage sources are S.C  
of current sources are O.C



$$R_{Th} = (1 \parallel 2) + (2) = \left(\frac{1 \times 2}{1+2}\right) + (2) = \frac{2}{3} + 2 = \frac{8}{3} \Omega$$

$$\therefore R_{Th} = \frac{8}{3} \Omega$$

Thevenin eq across AB:



IAB = ?

$$\frac{47}{3} - I_{AB} \times \frac{8}{3} - I_{AB} \times 1 = 0$$

$$I_{AB} \left[ 1 + \frac{8}{3} \right] = \frac{47}{3} \Rightarrow I_{AB} \left[ \frac{11}{3} \right] = \left[ \frac{47}{3} \right]$$

$$I_{AB} = \frac{47}{11} A$$

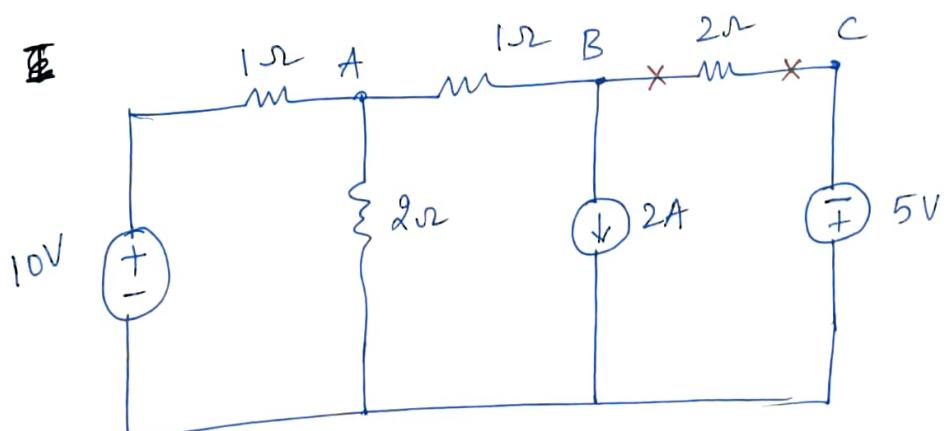
$$V_{AB} = I_{AB} \times R_L = \frac{47}{11} \times 1 = \frac{47}{11} \text{ Volts}$$

$$\therefore V_{AB} = \frac{47}{11} \text{ Volts}$$

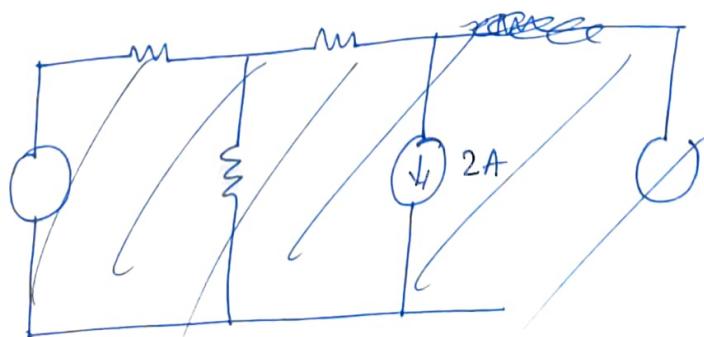
NOTE:

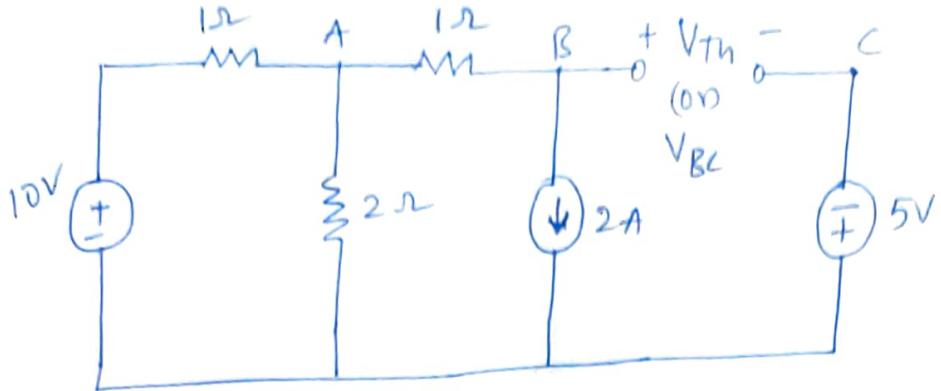
- We can find  $I_{AB}$  using nodal analysis also.
- But here we've used Thévenin's theorem to find  $I_{AB}$ .
- Both methods can be used according to the convenience of student. ~~exp~~ &

(ii) Thevenin Equivalent across BC:



(I) To find  $V_{Th}$ :





By applying nodal analysis at 'A' #.

$$\left(\frac{V_A - 10}{1}\right) + \left(\frac{V_A}{2}\right) + \left(\frac{V_A - V_B}{1}\right) = 0$$

$$2.5V_A - V_B = 10 \longrightarrow ①$$

# By applying nodal analysis at 'B'

$$\left(\frac{V_B - V_A}{1}\right) + 2 = 0$$

$$V_A - V_B = 2 \longrightarrow ②$$

$$\text{Solving } ① \text{ & } ② \quad 1.5V_A = 8 \Rightarrow V_A = \frac{8 \times 10}{15} = \frac{8 \times 2}{3} = \frac{16}{3}$$

$$\therefore V_A = \frac{16}{3} \text{ Volts}$$

$$— ③$$

$$V_B = V_A - 2 = \frac{16}{3} - 2 = \frac{16-6}{3} = \frac{10}{3} \text{ Volts}$$

$$\therefore V_B = \frac{10}{3} \text{ Volts}$$

$$— ④$$

We have from ckt

$$\therefore V_C = -5 \text{ Volts}$$

$$\rightarrow ⑤$$

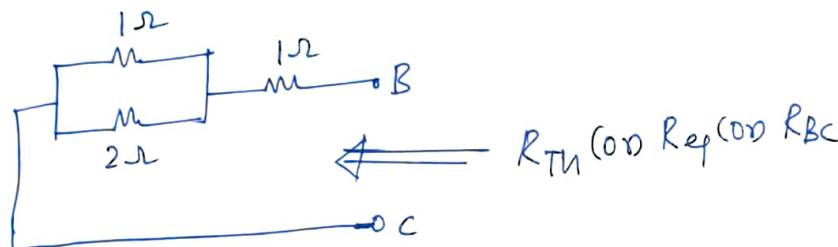
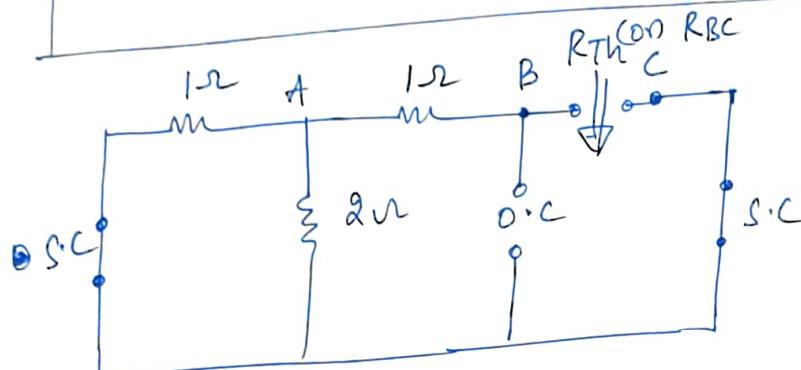
$$V_{Th} = V_{BC} = V_B - V_C = \frac{10}{3} + 5 = \frac{25}{3} \text{ Volts}$$

$$\rightarrow ⑥$$

To find  $R_{Th}$ :

The given ckt consists only independent sources.  
We have to use 1st case, i.e.

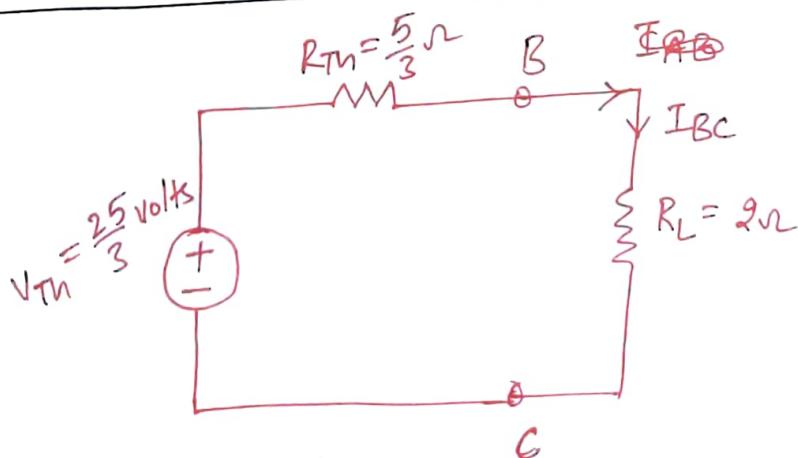
$$\boxed{\text{# } R_{Th} = \text{Req} \Big|_{\text{at BC}} \quad \begin{array}{l} \text{when} \\ \text{Voltage source is S.C} \\ \text{current source is O.C} \end{array}}$$



$$R_{Th} = (1||2) + 1 = \frac{2}{3} + 1 = \frac{5}{3} \Omega$$

$$\therefore R_{Th} = \frac{5}{3} \Omega$$

Thevenins eq. across BC:



Applying KVL in above loop

$$\frac{25}{3} - \left( I_{BC} \times \frac{5}{3} \right) - \left( I_{BC} \times 2 \right) = 0$$

$$I_{BC} \left( 2 + \frac{5}{3} \right) = \frac{25}{3}$$

$$I_{BC} \left[ \frac{11}{3} \right] = \frac{25}{3}$$

$$I_{BC} = \frac{25}{11} \text{ A}$$

from ohms law

$$V_{BC} = I_{BC} \times R_L$$

$$V_{BC} = \frac{25}{11} \times 2 = \frac{50}{11} \text{ Volts}$$

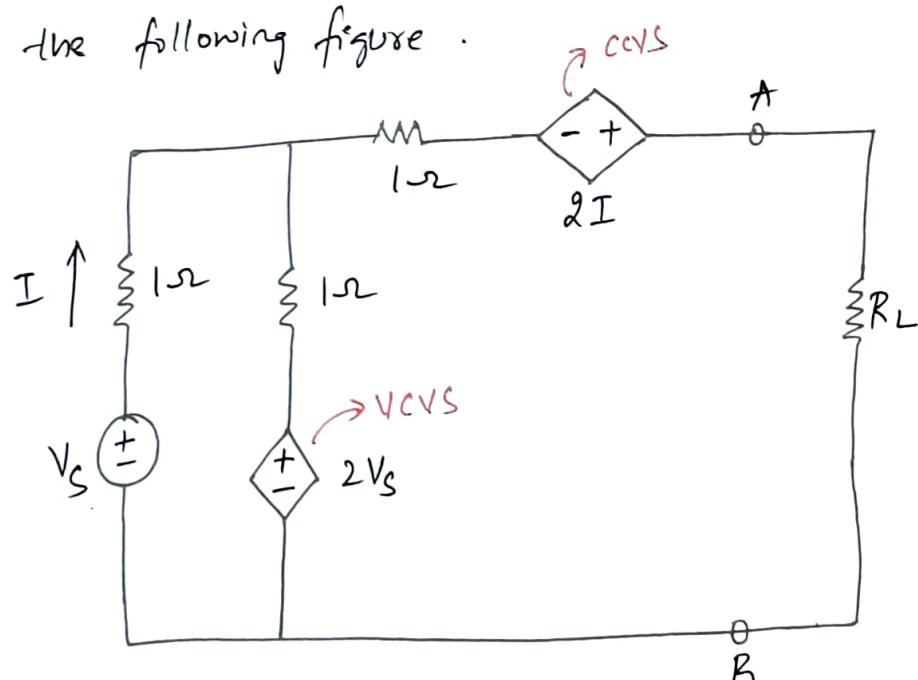
$$\therefore V_{BC} = \frac{50}{11} \text{ Volts}$$

### Lecture-3 :

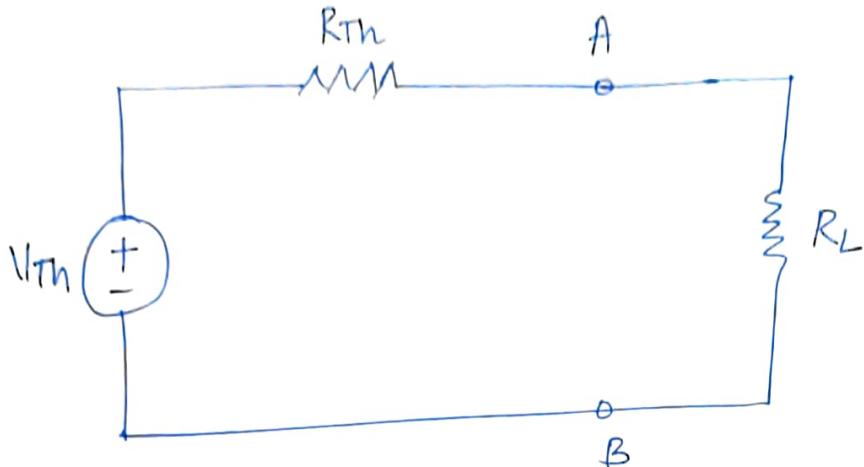
#### Example Based On Thevenins Theorem (2)

Example: Consider the following figure.

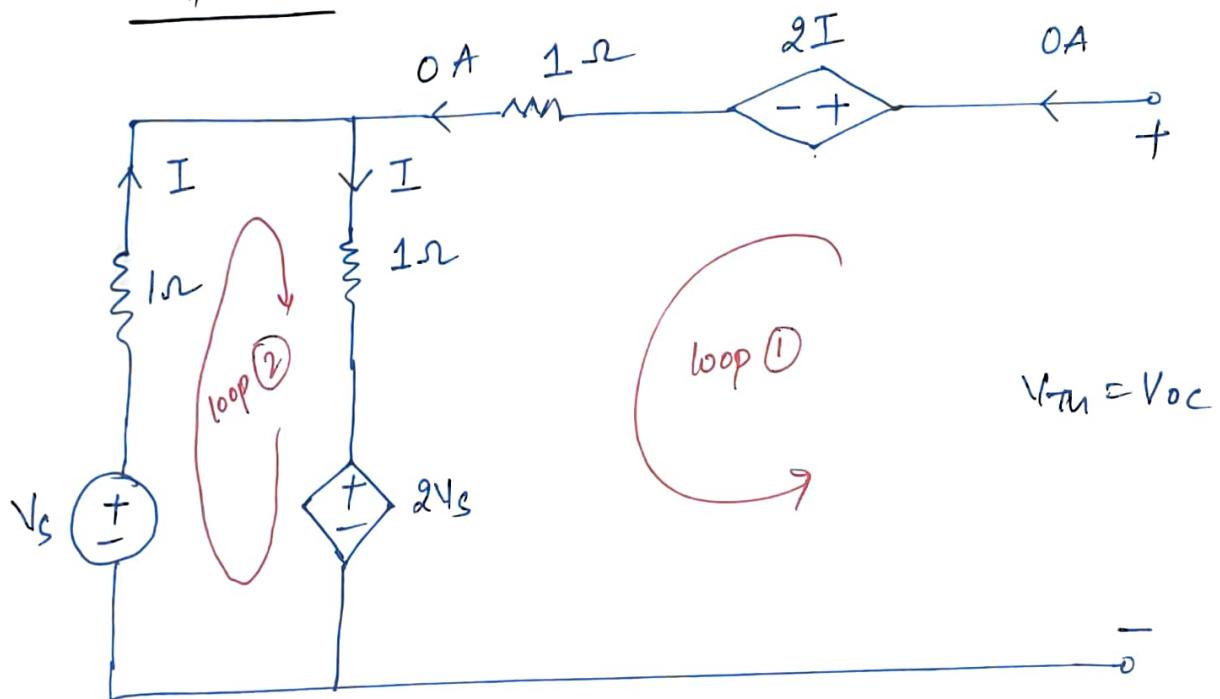
find the thevenins  
eq. i.e. thevenin  
voltage ( $V_{TH}$ ) &  
thevenin resistance  
( $R_{TH}$ ) across the  
terminal AB ?



Sol. The Thevenin eq. of given ckt across AB is



Now To find  $V_{Th}$ :



Applying KVL in the loop ①..

$$V_{Th} - 2I - I - 2V_s = 0$$

$$V_{Th} = 2V_s + 3I \quad \rightarrow \textcircled{1}$$

Apply KVL in loop ②

$$V_s - I - I - 2V_s = 0$$

$$-V_s - 2I = 0$$

$$V_s = -2I \quad \cancel{\text{or}} \quad \textcircled{2}$$

$$V_S = -2I$$

$$I = \frac{-V_S}{2} \rightarrow (2)$$

put (2) in (1)

$$V_{Th} = 2V_S + 3\left[\frac{-V_S}{2}\right]$$

$$V_{Th} = 2V_S - \frac{3V_S}{2} = \frac{V_S}{2}$$

$$\boxed{\therefore V_{Th} = \frac{V_S}{2} \text{ volts}}$$

NOTE:

finding the  $V_{Th}$  is in same style for both  
(only independent sources) & (combination of independent  
4 dependent sources).

Steps to find  $V_{Th}$ :

1. Open the branch which is our area of interest & find the voltage across that particular branch.
2. i.e. open circuit voltage is to be evaluated for the branch which is our area of interest it is equal to  $V_{Th}$  (Therminin voltage).

$$\boxed{V_{Th} = V_{oc}}$$

To find  $R_{Th}$ :

Steps to calculate  $R_{Th}$  in a n/w with both independent & dependent sources:

Step-1: Remove the branch b/w the terminals where  $R_{Th}$  is to be evaluated.

Step-2: Place the dc voltage ' $V_{DC}$ ' across the terminals where  $R_{Th}$  is to be evaluated.

Step-3: Let "  $I_{DC}$ " current be flown ~~from~~ the dc voltage ' $V_{DC}$ ' .

Step-4: Make the independent sources value as '0'  
i.e. ~~for a voltage an independent source~~  
voltage source apply (S.C) on it.  
i.e. for an independent current source apply  
(O.C) on it. (or remove)

Step-4: Do not disturb any dependent source.  
Note that any dependent sources must not be removed.

Step-5: For in some special cases, like if  
the variable in dependent sources depends  
on the variable of independent sources;

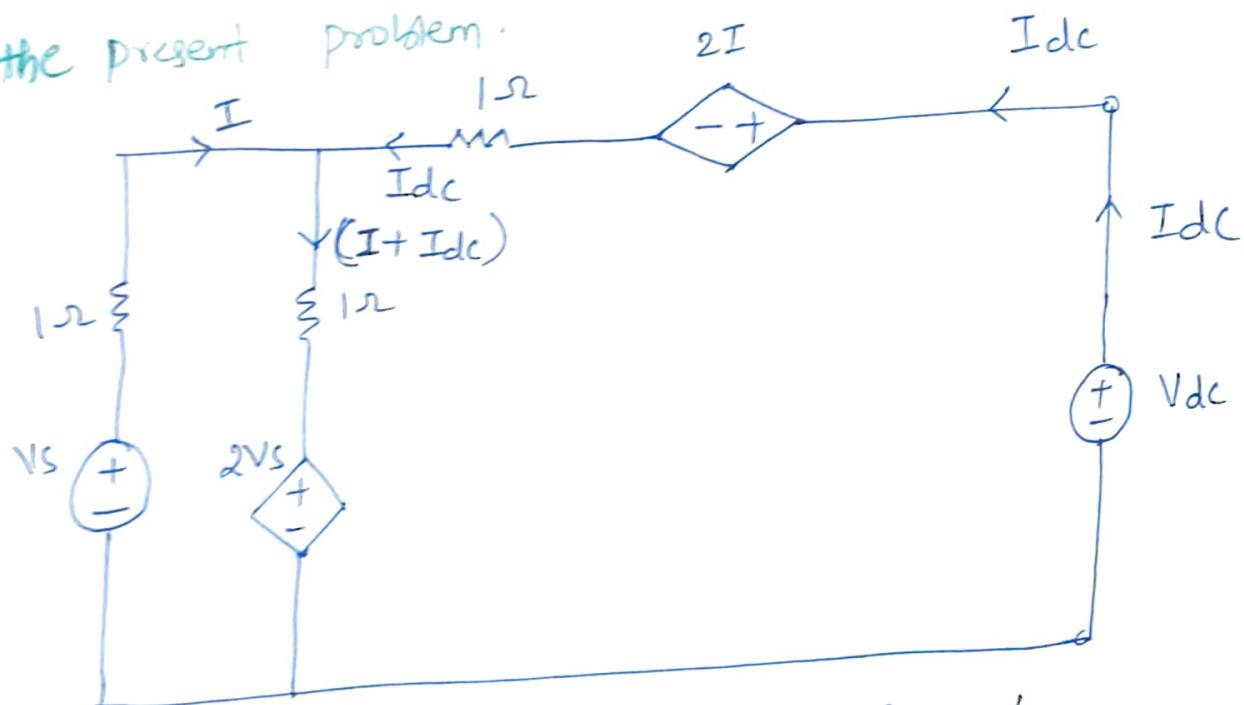
In that case that particular dependent source value should be made '0'.

i.e. the voltage of dependent  $\text{P}$  should be made as  $(S)$  & the current of dependent should be made  $(0 \cdot I)$ .

Step 6: The ratio of  $V_{dc}$  &  $I_{dc}$  is called as  $R_{Th}$  (Thevenin resistance)

$$R_{Th} = \frac{V_{dc}}{I_{dc}}$$

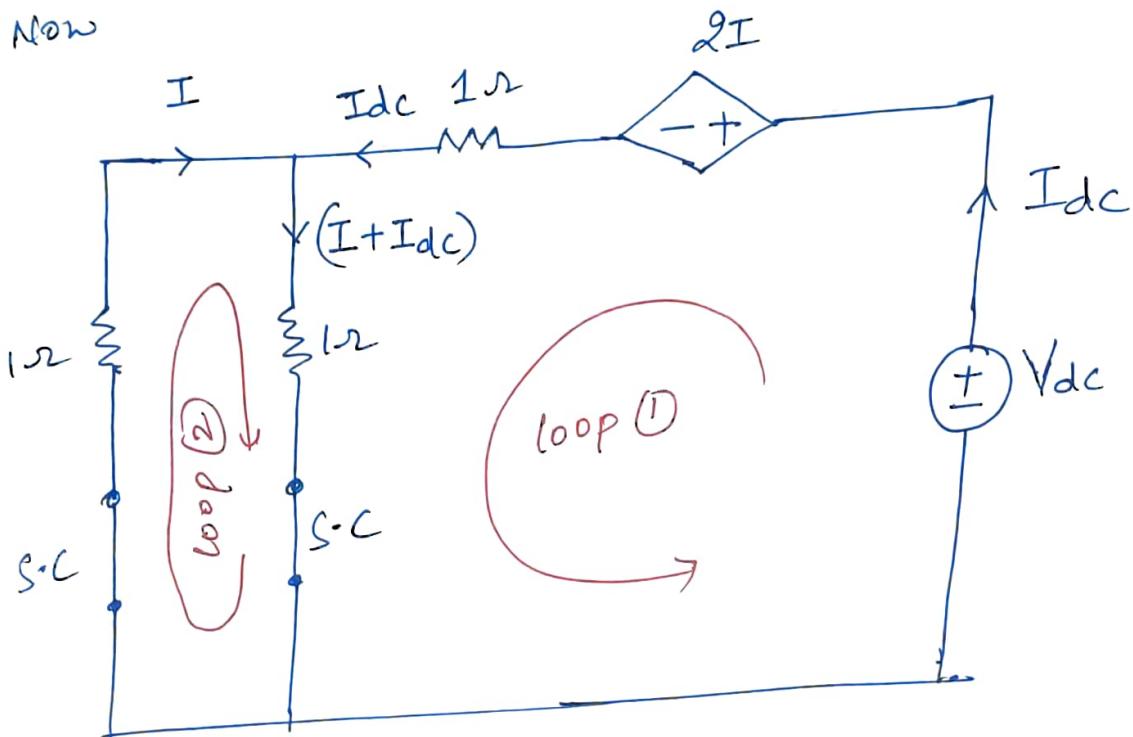
Now let us apply the above working rule for the present problem.



Now make ' $V_s$ ' voltage (independent) '0' i.e. s.c.  
" $2V_s$ " ~~independ~~ dependent voltage source must  
~~not~~ be made s.c because its magnitude depends  
on the independent voltage source variable  $V_s$ .

So, ' $2V_s$ ' dependent voltage source is made S.C  
 $\rightarrow$  ' $2I$ ' dependent voltage source must not be made  
 S.C because ' $I$ ' variable does not depend  
 on the independent voltage variable  $V_s$ .  
 $\rightarrow$  ' $2I$ ' dependent voltage source is not made S.C

Now



Apply KVL in loop ①

$$V_{dc} - 2I - I_{dc} - (I + I_{dc}) = 0$$

$$V_{dc} - 2I - I_{dc} - I - I_{dc} = 0$$

$$V_{dc} = 3I + 2I_{dc} \quad \rightarrow \quad ①$$

Apply KVL in loop ②

$$-I - (I + I_{dc}) = 0$$

$$-2I - I_{dc} = 0$$

$$I_{dc} = -2I$$

$$I = -\frac{I_{dc}}{2} \rightarrow ②$$

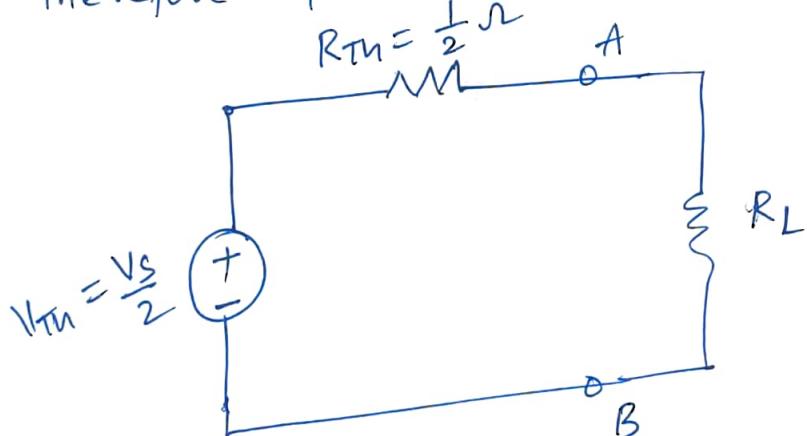
put ② in ①

$$V_{dc} = 3\left[-\frac{I_{dc}}{2}\right] + 2I_{dc} = \frac{-3I_{dc} + 4I_{dc}}{2}$$

$$V_{dc} = \frac{I_{dc}}{2}$$

$$\boxed{\frac{V_{dc}}{I_{dc}} = R_{Th} = \frac{1}{2} \Omega}$$

Therefore final thevenin equivalent is as follows

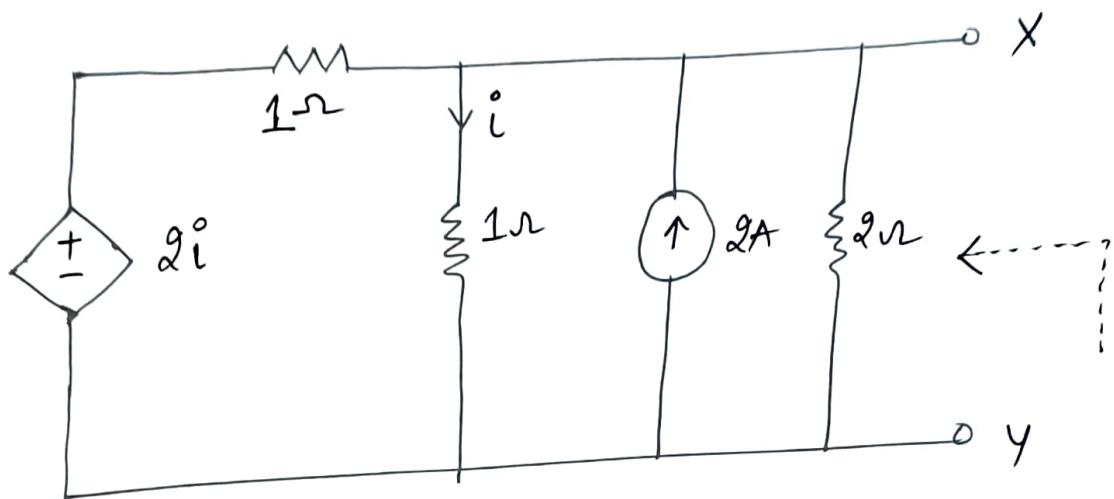


## Lecture - 4 :

### Questions Based On Thevenin's Theorem (1)

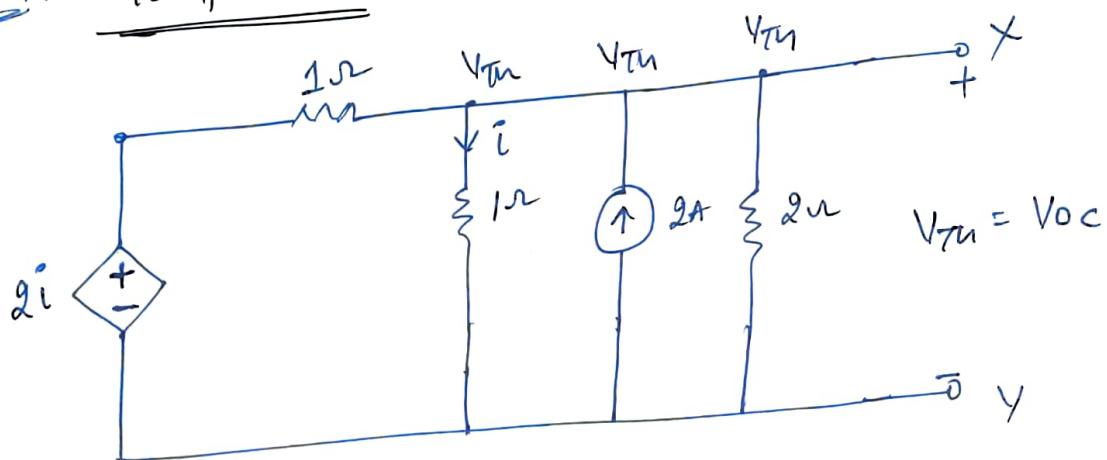
GATE  
2007

Q) For the ckt shown in figure the thevenin voltage and resistance looking into X-Y are



- A)  $\frac{4}{3}V, 2\Omega$     B)  $4V, \frac{2}{3}\Omega$     C)  $\frac{4}{3}V, \frac{2}{3}\Omega$     D)  $4V, 2\Omega$

Soln: To find  $V_{Th}$ :



Applying KCL Nodal analysis at  $V_{Th}$ .

$$\frac{V_{Th} - 2i}{1} + \frac{V_{Th}}{1} - 2 + \frac{V_{Th}}{2} = 0$$

$$\frac{5}{2}V_{Th} = 2i + 2$$

$$5V_{Th} = 4i + 4 \quad \text{--- (1)}$$

$$V_{Th} = i \times 1 = i \quad \text{--- } (2)$$

put (2) in (1).

$$5V_{Th} = 4V_{Th} + 4$$

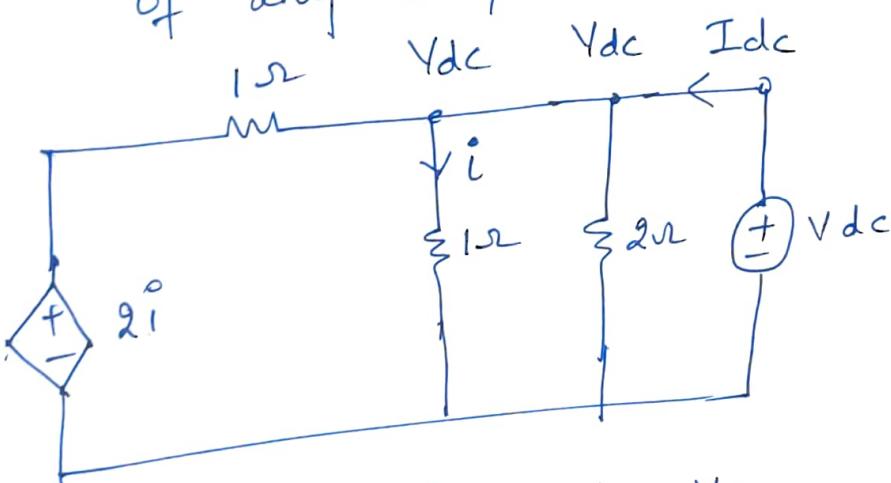
$V_{Th} = 4 \text{ volts}$

To find  $R_{Th}$ :

↑ 2A independent current source should be 0°C

↓  $2i$  should be dependent voltage source should not be removed because ' $i$ ' is independent

of any independent source.



Apply Nodal analysis at  $V_{dc}$ .

$$\left( \frac{V_{dc} - 2i}{1} \right) + \frac{V_{dc}}{1} + \frac{V_{dc}}{2} - Idc = 0$$

$$\frac{5}{2}V_{dc} - 2i = Idc$$

$$5V_{dc} = 2Idc + 4i \quad \text{--- } (1)$$

$$V_{dc} = 4i \quad \text{--- } (2)$$

put ② in ①

$$5V_{dc} = 2I_{dc} + 4V_{dc}$$

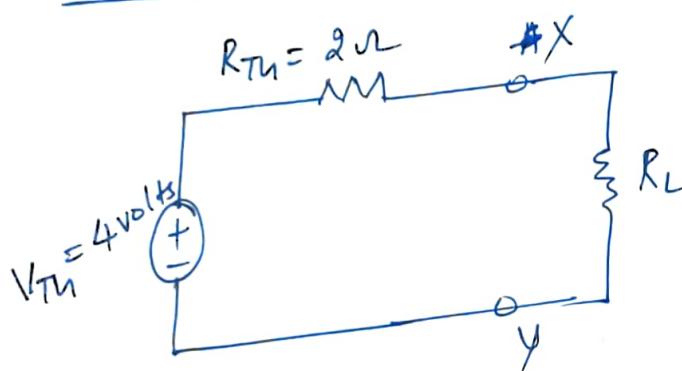
$$V_{dc} = 2I_{dc}$$

$$R_{Th} = \frac{V_{dc}}{I_{dc}} = 2\Omega$$

option (D)

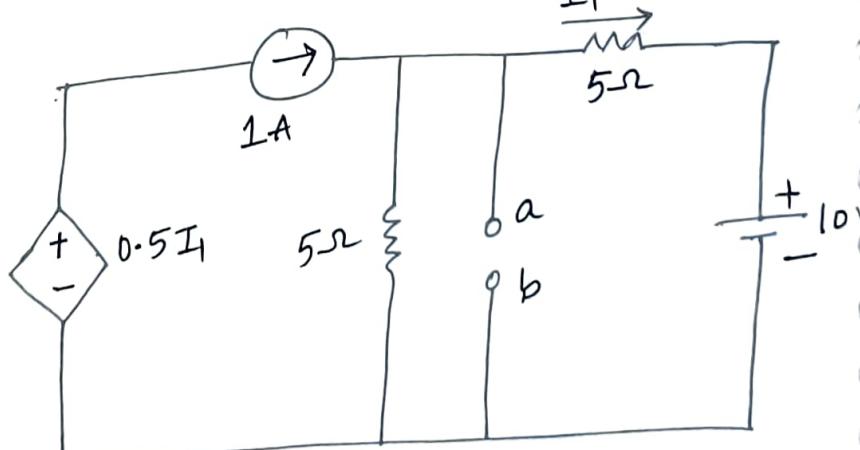
is correct.

Thevenin equivalent:



- GATE Q) For the ckt shown in fig , Thevenin's voltage & Thevenin's equivalent resistance at terminal a-b is

- A) 5V & 2Ω
- B) 7.5V & 2.5Ω
- C) 4V & 2Ω
- D) 3V & 2.5Ω



Soln: To find  $V_{Th}$ :

Consider the fig given below &

Apply KCL at  $V_{Th}$

$$\frac{V_{Th}}{5} - 1 + \frac{V_{Th} - 10}{5} = 0$$

$$2V_{Th} - 10 = 5$$

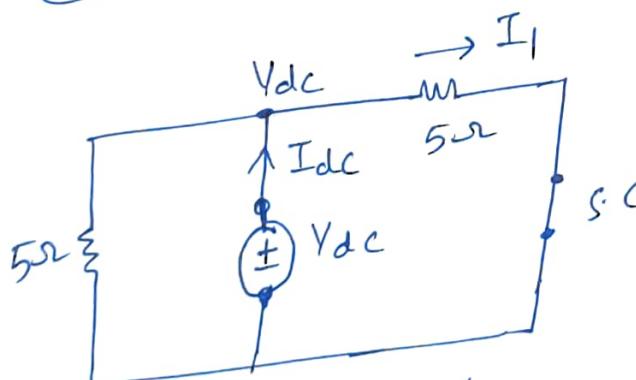
$$2V_{Th} = 15$$

$$\boxed{V_{Th} = 7.5 \text{ volts}}$$

To find  $R_{Th}$ :

~~Open~~ Here we have both dependent & independent source. So, we have to consider  $V_{dc}$  &  $I_{dc}$  at a & b terminal & let  $I_{dc}$  current flow from  $V_{dc}$ .

(i) Open  $\rightarrow$  & (ii) ~~open~~ S.C. the  $\pm$  source



Apply nodal at  $V_{dc}$

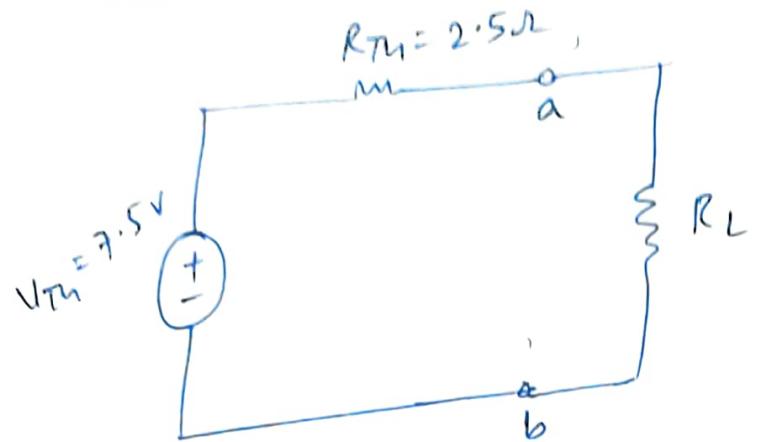
$$\frac{V_{dc}}{5} - I_{dc} + \frac{V_{dc}}{5} = 0$$

$$2V_{dc} = 5I_{dc}$$

$$\frac{V_{dc}}{I_{dc}} = R_{Th} = 2.5 \Omega$$

$$\therefore R_{Th} = 2.5 \Omega$$

## Equivalent Thevenin :



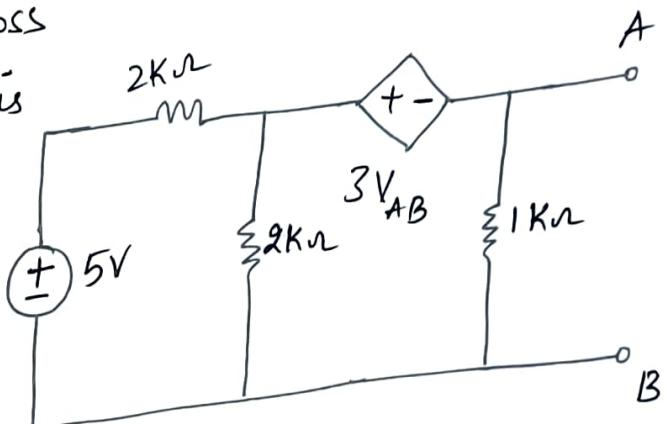
$\therefore$  Option (B) is correct.

## Lecture - 5:

### Questions Based on Thevenin Theorem (2)

Q1) Thevenin's resistance across the terminals A and B is

- A)  $0.5\text{ k}\Omega$
- B)  $0.2\text{ k}\Omega$
- C)  $1\text{ k}\Omega$
- D)  $0.11\text{ k}\Omega$



Q2) The thevenin voltage across the terminals A & B is

- A)  $1.25\text{ V}$
- B)  $0.25\text{ V}$
- C)  $1\text{ V}$
- D)  $0.5\text{ V}$

Soln To find  $V_{Th}$ :

Consider the fig as follows

$$\frac{V_a - 5}{2K} + \frac{V_a}{2K} + \frac{V_{Th}}{1K} = 0$$

$$V_a - 5 + V_a + 2V_{Th} = 0$$

$$2V_a + 2V_{Th} = 5$$

$$V_a + V_{Th} = 2.5 \rightarrow ①$$

Now KVL at Supernode

$$V_a - 3V_{Th} - V_{Th} = 0$$

$$V_a = 4V_{Th} \rightarrow ②$$

put ② in ①

$$5V_{Th} = 2.5$$

$$V_{Th} = \frac{2.5}{50} = \frac{1}{2} = 0.5 \text{ Volts}$$

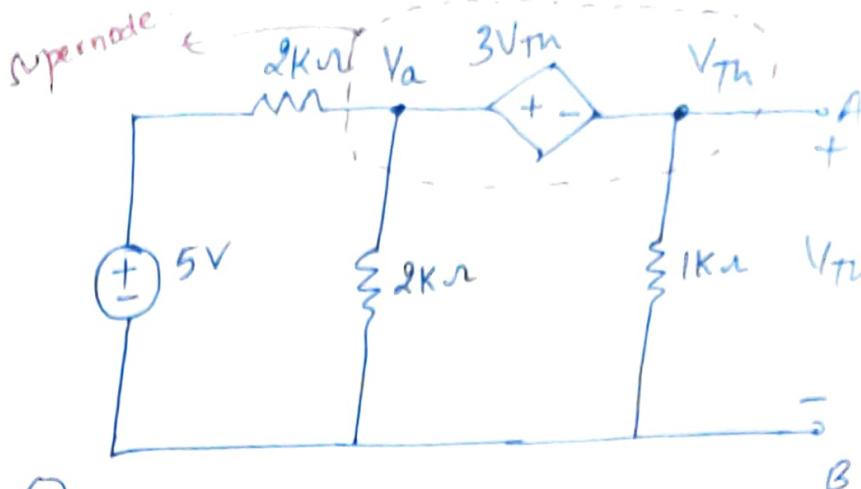
$$\boxed{\therefore V_{Th} = 0.5 \text{ Volts}}$$

To find  $R_{Th}$ :

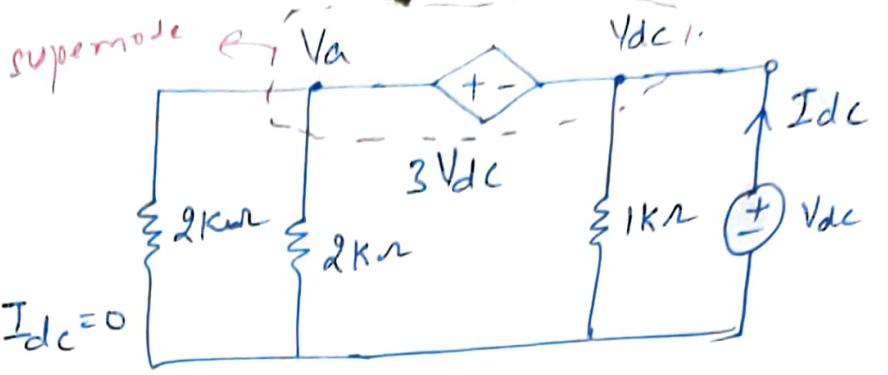
→ Clearly  $\oplus 5V$  becomes  $5.0C$

→ Apply  $\oplus V_{dc}$  b/w 'A & B' and allow  $I_{dc}$  current to flow from  $V_{dc}$ .

→ The q.m/w will be modified as follows



Applying supermode analysis



$$\frac{V_a}{2k} + \frac{V_a}{2k} + \frac{V_{dc}}{1k} - I_{dc} = 0$$

$$2V_a + 2V_{dc} = (2k) \times I_{dc}$$

$$V_a + V_{dc} = (1k \times I_{dc}) \rightarrow \textcircled{1}$$

$$\text{Now } V_a - 3V_{dc} - V_{dc} = 0$$

$$V_a = 4V_{dc} \rightarrow \textcircled{2}$$

put \textcircled{2} in \textcircled{1}

$$4V_{dc} + V_{dc} = (1k \times I_{dc})$$

$$5V_{dc} = 1k \times I_{dc}$$

$$\frac{V_{dc}}{I_{dc}} = \frac{1}{5} k\Omega = R_{Th} = 0.2 k\Omega$$

$$\therefore R_{Th} = 0.2 k\Omega$$

Thevenin Equivalent :

