CHAPTER : NUMERICAL METHODS :

Lecture 01 : METHODS TO SOLVE NON LINEAR ALGEBRAIC EQNS :

Solution Of Non Linear Equations : (EC + EE)

1. Bisection Method
2. Regular Falsi / False position Method.
3. Secant Method.
4. ** Newton Raphson Method. *** VVIMP for GATE EC

Solution of Differential Eqns (Single OR Multiple Steps) : (EC+EE)

1. Euler's Method (RK order 1)
2. Runge Kutta method (order 2, 3, 4)

Numerical Integrals : (For Mech, Civil)

1. Trapezoidal Method
2. Simpsons Methods.

   (2a) $\frac{1}{3}$ rd Method (Default)

   (2b) $\frac{3}{8}$ th Method

Errors :

Tables, Basic.

Solution of Non-Linear Algebraic Equations :

Method - 1 : BISECTION METHOD

THEORGY :

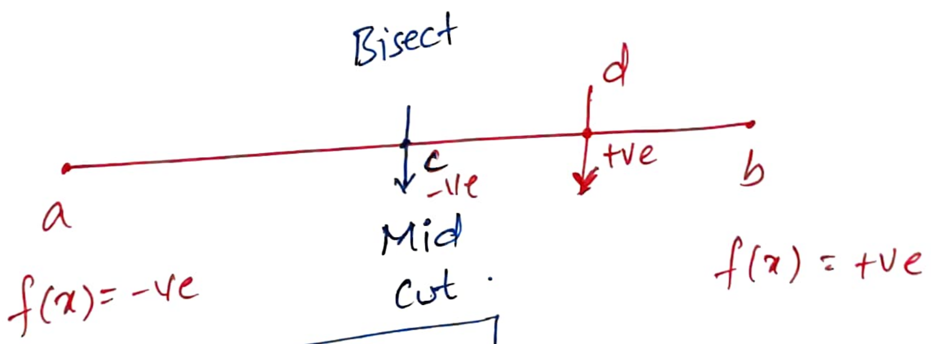
1. This method is guarenteed to converge.
2. The order of convergence is 1. (linear order)

WORKING RULE :

$$f(x) = 0 \longrightarrow (1) \qquad x \in [a, b]$$

$f(a) = -ve$; $f(b) = +ve$.

x lies b/w 'a' and 'b'.



Iterative Formula : $\boxed{c = \frac{a+b}{2}}$

$f(c) = -ve$; $x \in (c, b)$ the root lies b/w c & b.

NOTE:

For Bisection Method. of n. iterations & 'ϵ' as accuracy. Accuracy is given as

$$\boxed{\text{Accuracy} (\epsilon) = \frac{b-a}{2^n} \leq \epsilon_p}$$

NOTE:

The convergence rate of bisection method is low/slow.

1. BISECTION METHOD:

$$\boxed{x = \frac{a+b}{2} \quad ; \quad x \in (a,b)}$$

* Minimum number of iterations required to achieve accuracy (ϵ_p) in $[a, b]$ using bisection method is

$\log_2 \left[\frac{b-a}{\epsilon_p}\right]$.

$$\boxed{\frac{b-a}{2^n} \leq \epsilon_p}$$

2. REGULAR - FALSI METHOD:

→ Guaranteed convergent

→ Order is linear i.e, 1.

→ Faster than Bisection method.

For eq.n $f(x) = 0$; if $x \in (x_0, x_1)$

where $f(x_0) = -ve$, $f(x_1) = +ve$

$$\boxed{x_2 = x_0 - \left[\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right] f(x_0)}$$

$$\boxed{x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}}$$

3. SECANT METHOD :

→ This method need not to satify Intermediate Theorem.

→ This method doesn't guarantee convergence.

→ This method is not reliable.

$$\boxed{x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}}$$

4. VVIMP NEWTON RAPHSON METHOD : **

VVIMP

$$f(x) = 0 \longrightarrow (1)$$

Let x_0 be the initial approximation which is very close to the exact root.

Iterative Formula :

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Initially $n = 0$

first approximation : $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

→ It is also called as "TANGENT method" or "GRADIENT Method" (or) "SLOPE method" graphically.

→ This method is faster than bisection & regular falsi method.

→ In the previous two method we cannot locate complex roots.

→ But ~~by~~ Newton-Raphson's method we can locate complex roots.

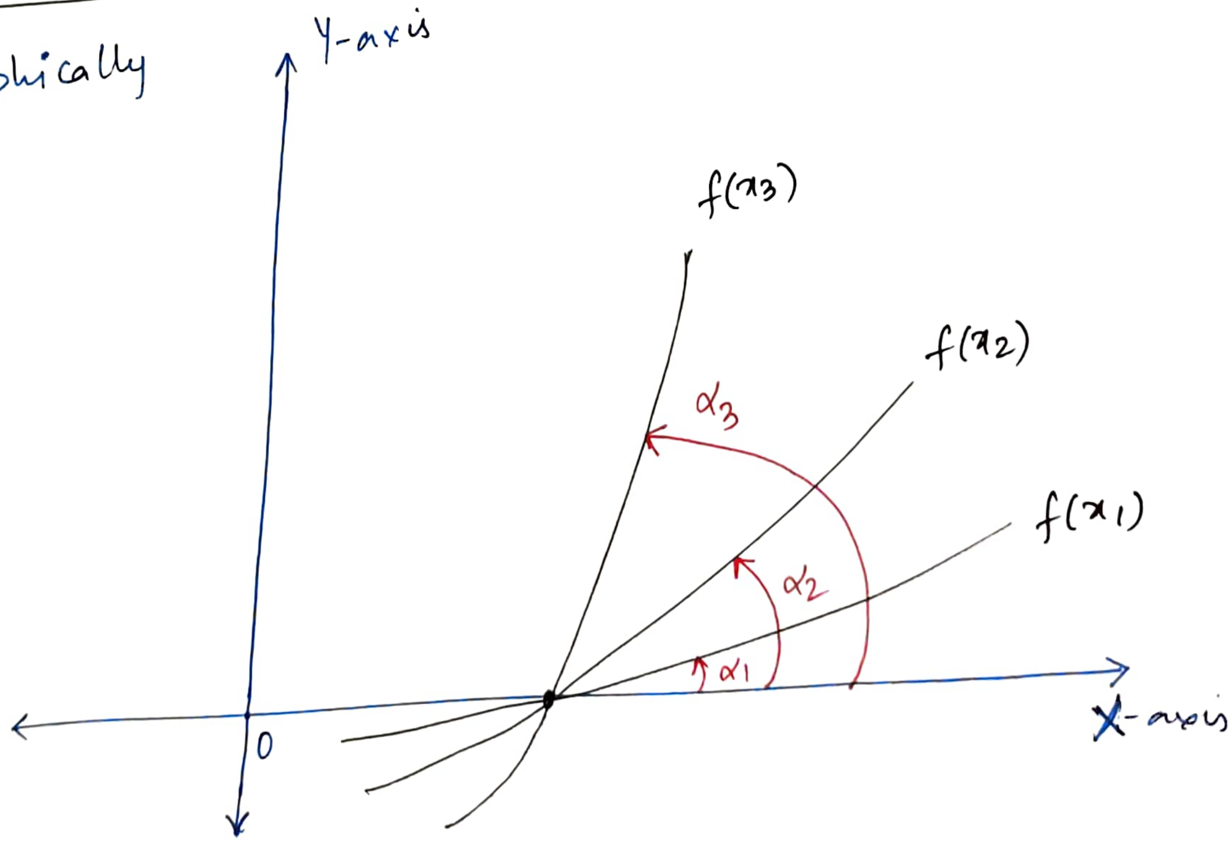
→ * This method is very sensitive about the <u>initial guess</u>.

→ By this method we can calculate root value.

→ ~~If~~ Newton Raphson method fails when $f'(x_0) = 0$.

<u>Geometrical Interpretation of Newton Raphson's Method:</u>

Graphically



<u>NOTE</u>:

~~If stop~~

If slope increases, iterative steps decreases & vice versa.

→ If highest slope = 90° ⇒ Exact ans ⇒ SUCCESS.

→ If slope decreases → Toward failure.

→ If slope = 0° ⇒ tangent is $\parallel^{le}$ X-axis ⇒ Test fails.

(*) Find the next iteration of $x_n = \sqrt{N}$, i.e, find the value of x_n using Newton Raphson's method?

soln:

$$x_n = \sqrt{N}$$

$$f(x_n) = x_n^2 - N = 0 \longrightarrow (1)$$

$$f(x_n) = x_n^2 - N.$$

We have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

$$f'(x_n) = 2x_n$$

$$x_{n+1} = x_n - \left(\frac{x_n^2 - N}{2x_n}\right) = x_n - \frac{x_n}{2} + \frac{N}{2x_n}.$$

$$x_{n+1} = \frac{x_n}{2} + \frac{N}{2x_n}.$$

$$\boxed{x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]}$$

General Results of Newton Raphson's Method:

1. $x_n = \sqrt[p]{N} \Rightarrow x_{n+1} = \frac{1}{p}\left[(p-1)x_n + \frac{N}{x_n^{p-1}}\right]$

2. $x_n = \frac{1}{\sqrt[p]{N}} \Rightarrow x_{n+1} = \frac{x_n}{p}\left[(p+1) - N x_n^p\right]$