CHAPTER : LIMITS & SERIES EXPANSION

Lecture - 1.

LIMITS

STANDARD RESULTS OF LIMITS :

1. $\lim\limits_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1$ (θ is in radians)

2. $\lim\limits_{\theta \to 0} \cos\theta = 1$ (θ is in radians)

3. $\lim\limits_{\theta \to 0} \dfrac{\tan\theta}{\theta} = 1$ (θ is in radians)

4. $\lim\limits_{x \to a} \left(\dfrac{x^n - a^n}{x - a}\right) = n a^{n-1}$.

5. $\lim\limits_{x \to \infty} \left(\dfrac{1}{x}\right) = 0$

6. $\lim\limits_{x \to 0} (1+x)^{1/x} = \lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^{x} = e$

7. $\lim\limits_{x \to 0} \dfrac{\log(1+x)}{x} = 1$

8. $\lim\limits_{x \to 0} \left(\dfrac{a^x - 1}{x}\right) = \ln(a)$

9. $\lim_{x \to \infty} \frac{\sin x}{x} = \lim_{x \to \infty} \frac{\cos x}{x} = \frac{(\text{any number b/w } -1 \text{ \& } 1)}{\infty} = 0$

10. If $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$

then $\lim_{x \to a} \{f(x)\}^{g(x)} = \lim_{x \to a} e^{g(x)\{f(x)-1\}}$

11. If $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$, then

$$\lim_{x \to a} \{1 + f(x)\}^{\frac{1}{g(x)}} = e^{\lim_{x \to a} \frac{f(x)}{g(x)}}.$$

12. If $\lim_{x \to a} f(x) = A > 0$ and $\lim_{x \to a} g(x) = B$,

then $\lim_{x \to a} \{f(x)\}^{g(x)} = A^{B}$.

L-Hospitals Rule :

L-hospitals rule is applicable only for $\frac{0}{0}$ or $\frac{\infty}{\infty}$ indeterminate form. "If $f(a) = g(a) = 0$ or $f(a) = g(a) = \infty$ then.

$$\boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \lim_{x \to a} \frac{f''(x)}{g''(x)} \cdots \text{ so on.}}$$