INTEGRAL & DIFFERENTIAL CALCULUS :

Lecture : 1 : BASICS OF INTEGRAL CALCULUS :

Integral Calculus :

a) Definite Integrals :

$$I = \int_a^b f(x)\,dx = \left[g(x)\right]_a^b = \left[g(b) - g(a)\right].$$

Q). find $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

soln. $\displaystyle\int_{-\infty}^{\infty} \left(\frac{1}{1+x^2}\right) dx = \left[\tan^{-1}(x)\right]_{-\infty}^{\infty} = \left[\tan(+\infty) - \tan(-\infty)\right]$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi.$$

$$\boxed{\therefore \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \pi}$$

Properties Of Definite Integral :

1. $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

2. $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(y)\,dy$

3. $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

(if $a < c < b$)

4. $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

** 5. $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ **

* 6. $$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\displaystyle\int_0^a f(x)\,dx & ; \text{ if } f(x) \text{ is even} \\ 0 & ; \text{ if } f(x) \text{ is odd.} \end{cases}$$

Even functn :

$$\boxed{f(x) = f(-x)}$$

Eg: $\cos x$

Odd function :

$$\boxed{f(-x) = -f(x)}$$

Eg: $\sin x$.

7. $$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & ; \text{ if } f(2a-x) = f(x). \\ 0 & ; \text{ if } f(2a-x) = -f(x). \end{cases}$$

WORKBOOK PROBLEMS:

Pg 16 OBJECTIVE TYPE QUESTIONS:

Q1) If $f(x)$ is an even function and 'a' is a +ve real number, then $\int_{-a}^{a} f(x)\,dx$ equals

A) 0 B) a C) 2a D) $2\int_0^a f(x)\,dx$.

Soln: Clearly from properties if $f(x)$ is even functn then

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx.$$

Ans: (D)

Q2) What is the value of the definite integral,

$$\int_0^a \left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\right) dx \; ?$$

A) 0 B) $\frac{a}{2}$ C) a D) 2a

Soln: We have $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

$$I = \int_0^a \left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\right) dx \longrightarrow (1)$$

$$I = \int_0^a \left(\frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}\right) dx \longrightarrow (2)$$

(1) + (2)

$$2I = \int_0^a (1)\,dx$$

$$2I = a$$

$$I = \frac{a}{2}$$

$$\boxed{\therefore \int_0^a \left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\right) dx = \frac{a}{2}}$$

Ans: (B)

Q3) Given $i = \sqrt{-1}$, what will be the evaluation of the definite integral $\int_0^{\pi/2} \left(\frac{\cos x + i\sin x}{\cos x - i\sin x}\right) dx$

A) 0 B) 2 C) $-i$ D) i

soln:

$$I = \int_0^{\pi/2} \left(\frac{\cos x + i\sin x}{\cos x - i\sin x}\right) dx$$

$$e^{ix} = \cos x + i\sin x \quad ; \quad e^{-ix} = \cos x - i\sin x$$

$$I = \int_0^{\pi/2} \left(\frac{e^{ix}}{e^{-ix}}\right) dx$$

$$I = \int_0^{\pi/2} \left(e^{2ix}\right) dx = \left[\frac{e^{2ix}}{2i}\right]_0^{\pi/2}$$

$$I = \frac{1}{2i}\left[e^{i\pi} - e^{0}\right] = \frac{1}{2i}[-1-1] = \frac{-2}{2i}$$

$$I = \frac{-1}{i} = \frac{-1 \times i}{i \times i} = \frac{-i}{i^2} = \frac{-i}{-1} = i$$

$$\boxed{\therefore\ I = \int_0^{\pi/2} \left(\frac{\cos x + i\sin x}{\cos x - i\sin x}\right) dx = i}$$

Ans: (D)

Practice Questions:

Pg: 18 :

Q2) The value of the integral $\int_0^2 \left[\frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}\right] dx$

A) 3 B) 0 C) -1 D) -2

soln. We have $\int_0^a f(x) = \int_0^a f(a-x)\, dx$

$$I = \int_0^2 \left[\frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}\right] dx \longrightarrow ①$$

$$I = \int_0^2 \left(\frac{(2-x-1)^2 (\sin(2-x-1))}{[(2-x-1)^2 + \cos(2-x-1)]}\right) dx.$$

$$I = \int_0^2 \left(\frac{(1-x)^2 \sin(1-x)}{(1-x)^2 + \cos(1-x)}\right) dx$$

$$I = -\int_0^2 \left(\frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}\right) dx \longrightarrow ②$$

① + ②

$$2I = 0$$

$$\boxed{\therefore\ I = \int_0^2 \left(\frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}\right) dx = 0}$$

Ans: (B)

Q4) Consider the following definite integral :

$$I = \int_0^1 \left(\frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}} \right) dx$$. The value of I is

A) $\frac{\pi^3}{24}$ B) $\frac{\pi^3}{12}$ C) $\frac{\pi^3}{48}$ D) $\frac{\pi^3}{64}$

Soln.

$$I = \int_0^1 \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}} dx$$

let $t = \sin^{-1} x$

$dt = \frac{1}{\sqrt{1-x^2}} dx$

$t = 0 \;;\; x = 0$

$t = \frac{\pi}{2} \;;\; x = 1$

$$I = \int_0^{\pi/2} t^2 dt = \left(\frac{t^3}{3} \right)_0^{\pi/2} = \frac{1}{3} \left[\frac{\pi}{2} \right]^3$$

$$I = \frac{1}{3} \times \frac{\pi^3}{8} = \frac{\pi^3}{24}$$

$$\boxed{\therefore \quad I = \int_0^1 \left(\frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}} \right) dx = \frac{\pi^3}{24}}$$

Ans: (A)

Q5) $\displaystyle\int_0^{\pi/4} \left(\frac{1-\tan x}{1+\tan x}\right) dx$ evaluates to

A) 0 B) 1 C) ln 2 D) $\frac{1}{2}\ln 2$.

Soln.

$$I = \int_0^{\pi/4} \left[\frac{1-\tan x}{1+\tan x}\right] dx. \longrightarrow (1)$$

We have $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

$$I = \int_0^{\pi/4} \left[\frac{1-\tan\left(\frac{\pi}{4}-x\right)}{1+\tan\left(\frac{\pi}{4}-x\right)}\right] dx$$

We have $\tan\left(\frac{\pi}{4}-x\right) = \dfrac{1-\tan x}{1+\tan x}$.

$$I = \int_0^{\pi/4} \left[\frac{1-\left(\frac{1-\tan x}{1+\tan x}\right)}{1+\left(\frac{1-\tan x}{1+\tan x}\right)}\right] dx$$

$$I = \int_0^{\pi/4} \left[\frac{(1+\tan x)-(1-\tan x)}{(1+\tan x)+(1-\tan x)}\right] dx$$

$$I = \int_0^{\pi/4} \left(\frac{2\tan x}{2}\right) dx = \int_0^{\pi/4} \tan x \, dx.$$

$$I = \left(\ln|\sec x|\right)_0^{\pi/4} = \ln|\sec \pi/4| - \ln|\sec 0|$$

$$I = \ln\sqrt{2} - \ln 1 = \ln\sqrt{2} - 0 = \ln\sqrt{2}$$

$$I = \ln\sqrt{2} = \frac{1}{2}\ln 2$$

$$\boxed{\therefore\ I = \int_0^{\pi/4} \left(\frac{1-\tan x}{1+\tan x}\right) dx = \frac{1}{2}\ln 2}$$

Ans: (D)

SPECIAL FUNCTIONS FOR SINGLE INTEGRAL
(GAMMA & BETA FUNCTIONS)

1. GAMMA FUNCTION ($\Gamma(n)$) :

$$\boxed{\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx \; ; \text{ for } (n > 0) \text{ \& } n = \text{integers}}$$

STANDARD RESULTS :

1. $\Gamma(1) = 1$

** *2. * $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

3. $\Gamma(n+1) = n\Gamma n$; This is valid only when "n" is a fraction

4. $\Gamma(n+1) = n!$; This is valid only when 'n' is an integer.

5. Gamma Trigonometric form :

$$\boxed{\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}}$$

Example:

find a) $\Gamma 7$ b) $\Gamma \frac{7}{2}$

a) $\Gamma 7$

Soln. We have $\Gamma(n+1) = n!$ (if n is integer)

here $n = 6$

$\Rightarrow \boxed{\Gamma 7 = 6! = 720}$

b) $\Gamma \frac{7}{2}$

Soln We have $\Gamma(n+1) = n\Gamma n$ (if n is fraction)

$n = \frac{5}{2}$ $\Gamma\left(\frac{5}{2}+1\right) = \frac{5}{2}\Gamma\frac{5}{2}$

$\Gamma\frac{7}{2} = \frac{5}{2}\left[\Gamma\left(\frac{3}{2}+1\right)\right] = \frac{5}{2}\times\frac{3}{2}\Gamma\frac{3}{2}$

$= \frac{5}{2}\times\frac{3}{2}\cdot\Gamma\left(\frac{1}{2}+1\right) = \frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\Gamma\frac{1}{2}$

$\Gamma\frac{7}{2} = \frac{5\times 3}{2\times 2\times 2}\times\sqrt{\pi} = \frac{15}{8}\sqrt{\pi}$

$$\boxed{\therefore \Gamma\left(\frac{7}{2}\right) = \frac{15\sqrt{\pi}}{8}}$$

2. BETA FUNCTION :

$$\boxed{\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} \, dx}$$

STANDARD RESULTS :

1. $\beta(m,n) = \beta(n,m)$

2. Trigonometric form. of Beta functn.

$$\boxed{\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta}$$

Relation b/w $\beta(m,n)$ Beta function & $\Gamma(n)$ Gamma Functn:

3. $$\boxed{\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}}$$ *** VVIMP.

WORKBOOK PROBLEMS :

OBJECTIVE TYPE QUESTIONS :

Q5) The value of the integral $I = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(\frac{-x^2}{8}\right) dx$ is ____

A) 1 B) π C) 2 D) 2π

Soln: We have

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx.$$

$$I = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(\frac{-x^2}{8}\right) dx = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-x^2/8} dx.$$

Let $t = \frac{x^2}{8} \Rightarrow x^2 = 8t$ & $x = 2\sqrt{2t}$.

$dt = \frac{x}{4} dx. \Rightarrow dt = \frac{\sqrt{2t}}{2} dx = \sqrt{\frac{t}{2}} dx.$

$x = 0$; $t = 0$

$x = \infty$; $t = \infty$

$$I = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t} \times \sqrt{\frac{2}{t}} dt = \frac{\sqrt{2}}{\sqrt{2}\sqrt{\pi}} \int_0^\infty e^{-t} t^{-1/2} dt$$

$$I = \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-t} t^{\left(\frac{1}{2}\right)-1} dt = \frac{1}{\sqrt{\pi}} \times \Gamma\left(\frac{1}{2}\right)$$

$$I = \frac{1}{\sqrt{\pi}} \times \sqrt{\pi} = 1$$

$$\boxed{\therefore \quad I = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(\frac{-x^2}{8}\right) dx = 1}$$

Ans: (A)

Q6) The integral $\int_0^{\pi} \sin^3\theta \, d\theta$ is given by

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{4}{3}$ D) $\frac{8}{3}$.

Sol: $I = \int_0^{\pi} \sin^3\theta \, d\theta = \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta$

$$I = \int_0^{\pi} (1-\cos^2\theta) \sin\theta \, d\theta$$

Let $t = \cos\theta$ | $\theta = 0$; $t = 1$

$dt = -\sin\theta \, d\theta$ | $\theta = \pi$; $t = -1$

$$I = \int_1^{-1} (1-t^2)(-dt) = \int_{-1}^{1} (1-t^2)\, dt$$

$$I = \left(t - \frac{t^3}{3}\right)_{-1}^{1} = \left[\left(1-\frac{1}{3}\right) - \left(\frac{-1}{1} + \frac{1}{3}\right)\right]$$

$$I = \left[1 - \frac{1}{3} + 1 - \frac{1}{3}\right] = 2 - \frac{2}{3} = \frac{4}{3}$$

$$\boxed{\therefore \; I = \int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}}$$

Ans: (C)

Q7) Assuming $i=\sqrt{-1}$ and t is a real number,

$\int_0^{\pi/3} e^{it}\,dt$ is ______

A) $\frac{\sqrt{3}}{2}+\frac{i}{2}$ B) $\frac{\sqrt{3}}{2}-\frac{i}{2}$ C) $\frac{1}{2}+\frac{i\sqrt{3}}{2}$ D) $\frac{1}{2}+i\left(1-\frac{\sqrt{3}}{2}\right)$

Soln.

$$I=\int_0^{\pi/3} e^{it}\,dt=\frac{1}{i}\left[e^{it}\right]_0^{\pi/3}$$

$$I=\frac{1}{i}\left[\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)-(1)\right]$$

$$I=\frac{1}{i}\left[\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}-1\right)\right]=\frac{1}{i}\left[-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right]$$

$$I=-i\left[\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right]=\frac{i}{2}+\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}+\frac{i}{2}$$

$$\boxed{\therefore\ I=\int_0^{\pi/3} e^{it}\,dt=\frac{\sqrt{3}}{2}+\frac{i}{2}}$$

Ans: (A)

Q8) The following definite integral evaluates to

$\int_{-\infty}^{0} e^{-(x^2/20)}\,dx$

A) $\frac{1}{2}$ B) $\sqrt{5\pi}$ C) $\sqrt{10}$ D) π

Soln: Given $I=\int_{-\infty}^{0} e^{-(x^2/20)}\,dx.$

We have $\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx.$

$$I = \int_{-\infty}^{0} e^{-(x^2/20)} dx$$

Let $t = \frac{x^2}{20} \Rightarrow x^2 = 20t \Rightarrow x = 2\sqrt{5}\sqrt{t}$

$dt = \frac{x}{10} dx \Rightarrow dx = \frac{10}{x} dt = \frac{10 \, dt}{2\sqrt{5}\sqrt{t}} = \sqrt{\frac{5}{t}} dt$

$\Rightarrow dx = \sqrt{\frac{5}{t}} dt$

$x = -\infty \; ; \; t = \infty$

$x = 0 \; ; \; t = 0$

$$I = \int_{\infty}^{0} e^{-t} \sqrt{\frac{5}{t}} dt = -\sqrt{5} \int_0^{\infty} e^{-t} t^{-1/2} dt$$

$$I = -\sqrt{5} \int_0^{\infty} e^{-t} t^{(\frac{1}{2})-1} dt = -\sqrt{5}\,\Gamma\tfrac{1}{2}$$

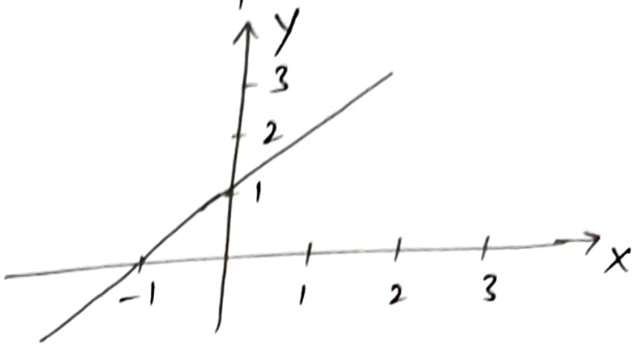
$$I = -\sqrt{5} \times \sqrt{\pi} = -\sqrt{5\pi} \; ; \; I = |-\sqrt{5\pi}|$$

$\therefore$ I is always $\boxed{\therefore I = \sqrt{5\pi}}$

Ans: (B)

Q9) The following plot shows a function y which varies linearly with x. The value of the integral

$I = \int_1^2 y\,dx$ is ______



A) 1 B) 2.5 C) 4 D) 5

Soln.
$(0,1)$ & $(-1,0)$

$(y-1) = \left(\frac{0-1}{-1-0}\right)(x-0)$

$y-1 = x$

$\boxed{y = x+1}$

$I = \int_1^2 y\,dx = \int_1^2 (x+1)\,dx = \left(\frac{x^2}{2}+x\right)_1^2$

$I = \left[(2+2)-\left(\frac{1}{2}+1\right)\right] = \left[4-\frac{3}{2}\right] = \frac{5}{2} = 2.5$

$$\boxed{\therefore\ I = \int_1^2 y\,dx = 2.5}$$

Ans: (B)

Q10) The value of the integral $\int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2}\,dx\,dy$ is ______

A) $\sqrt{\frac{\pi}{2}}$ B) $\sqrt{\pi}$ C) π D) $\frac{\pi}{4}$.

soln:

$$I = \int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2} \, dx \, dy$$

$$I = \left(\int_0^\infty e^{-x^2} dx \right) \left(\int_0^\infty e^{-y^2} dy \right)$$

$$I = I_1 \, I_2$$

$$I_1 = \int_0^\infty e^{-x^2} \, dx$$

Let $t = x^2 \Rightarrow x = \sqrt{t}$

$dt = 2x \, dx \Rightarrow dx = \frac{dt}{2x} = \frac{dt}{2\sqrt{t}}$

$\Rightarrow dx = \frac{dt}{2\sqrt{t}}$

$x = 0 \;;\; t = 0$

$x = \infty \;;\; t = \infty$

$$I_1 = \int_0^\infty e^{-t} \frac{dt}{2\sqrt{t}} = \frac{1}{2} \int_0^\infty e^{-t} t^{-1/2} \, dt$$

$$I_1 = \frac{1}{2} \int_0^\infty e^{-t} t^{\left(\frac{1}{2}\right)-1} \, dt = \frac{1}{2} \Gamma \frac{1}{2} = \frac{1}{2} \times \sqrt{\pi}$$

$$I_1 = \frac{\sqrt{\pi}}{2}$$

|||ly $\quad I_2 = \frac{\sqrt{\pi}}{2}$.

$$I = I_1 I_2$$

$$I = \frac{\sqrt{\pi}}{2} \times \frac{\sqrt{\pi}}{2} = \frac{\pi}{4}.$$

$$\boxed{\therefore \; I = \int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2}\, dx\, dy = \frac{\pi}{4}}$$

Ans: (D)

Practice Question: Pg-18 :

Q1) If 'a' is a constant, then the value of the integral $a^2 \int_0^\infty x e^{-ax} dx$ is ____

A) $\frac{1}{a}$ B) a C) 1 D) 0

Soln: We have $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$.

$$I = a^2 \int_0^\infty x e^{-ax} dx$$

Let $t = ax \Rightarrow x = t/a$

$dt = a\,dx \Rightarrow dx = dt/a$

$x = 0$ then $t = 0$; $x = \infty$ then $t = \infty$

$$I = a^2 \int_0^\infty \left(\frac{t}{a}\right) e^{-t} \left(\frac{dt}{a}\right)$$

$$I = a^2 \times \frac{1}{a^2} \int_0^\infty t\, e^{-t}\, dt$$

$$I = \int_0^\infty e^{-t} t^1\, dt = \int_0^\infty e^{-t}\, t^{2-1}\, dt$$

$$I = \Gamma(2) = \Gamma(1+1) = 1! = 1$$

$$\boxed{\therefore\ I = a^2 \int_0^\infty x\, e^{-ax}\, dx = 1}$$

Ans: (C)

INTEGRAL & DIFFERENTIAL CALCULUS :

Lecture - 3 :

CHANGE OF ORDER (DOUBLE INTEGRAL)

MULTIPLE INTEGRAL : Generally we have two types of Integral. They are :

a) Double Integral (or) Surface Integral

b) Triple Integral (or) Volume Integral.

Examples of Double Integral (or) Surface Integral :

1. $I = \int_a^b \int_c^d f(x,y)\, dx\, dy.$

2. $I = \int_0^a \int_{x^2}^{\sqrt{x}} f(x,y)\, dx\, dy$

Examples of Triple Integral (or) Volume Integral :

1. $I = \int_0^a \int_0^b \int_0^c f(x,y,z)\, dx\, dy\, dz$

2. $I = \int_0^1 \int_x^{x^2} \int_{x-y}^{x+y} f(x,y,z)\, dx\, dy\, dz$

Q14. $I = \int_0^1 \int_{3y}^{3} e^{x^2}\,dx\,dy$ is

A) $\frac{1}{6}(e^9 - 1)$ B) $\frac{1}{6}(e^6 + 1)$ C) $\frac{1}{6}(e^9 + 1)$ D) $\frac{1}{9}(e^6 - 1)$

Soln:

$$I = \int_0^1 \int_{3y}^{3} e^{x^2}\,dx\,dy.$$

$$I = \int_{y=0}^{y=1} \left(\int_{x=3y}^{x=3} e^{x^2}\,dx \right) dy.$$

Let $x^2 = t \Rightarrow x = \sqrt{t}$

$2x\,dx = dt$

$dx = \frac{1}{2x}\,dt = \frac{1}{2\sqrt{t}}\,dt$

If $x = 3y$; $t = 9y^2$

If $x = 3$; $t = 9$.

$$I = \int_{y=0}^{y=1} \left(\frac{1}{2} \int_{t=9y^2}^{9} e^{t}\, t^{-1/2}\,dt \right) dy.$$

$$I = \int_{y=0}^{1} \left(\frac{1}{2} \times \Gamma\left(\tfrac{1}{2}\right) \right) dy = \left(\int_{y=0}^{1} \right.$$

This problem cannot be solved by using general methods. To solve these types of problems we need to apply change of order concept.

Steps to working rule of Change of Order :

Step-1: Identification of limits.

Step-2: Trace the limits for the given boundary.

Step-3: Identification of the strip.

strip may be

a) Vertical strip (If the strip is along Y-axis)

(or)

b) Horizontal strip (If the strip is along X-axis)

Step-4: Change the strip i.e, change the strip to vertical strip if it is horizontal strip & if it is horizontal strip then change it into vertical strip. Here we are changing the order.

Now let us solve the Q.14) problem using the change of order concept and applying the above working rules step by step.

SATYA PREPARATION IS VERY GOOD & AWESOME

ALL THE VERY BEST

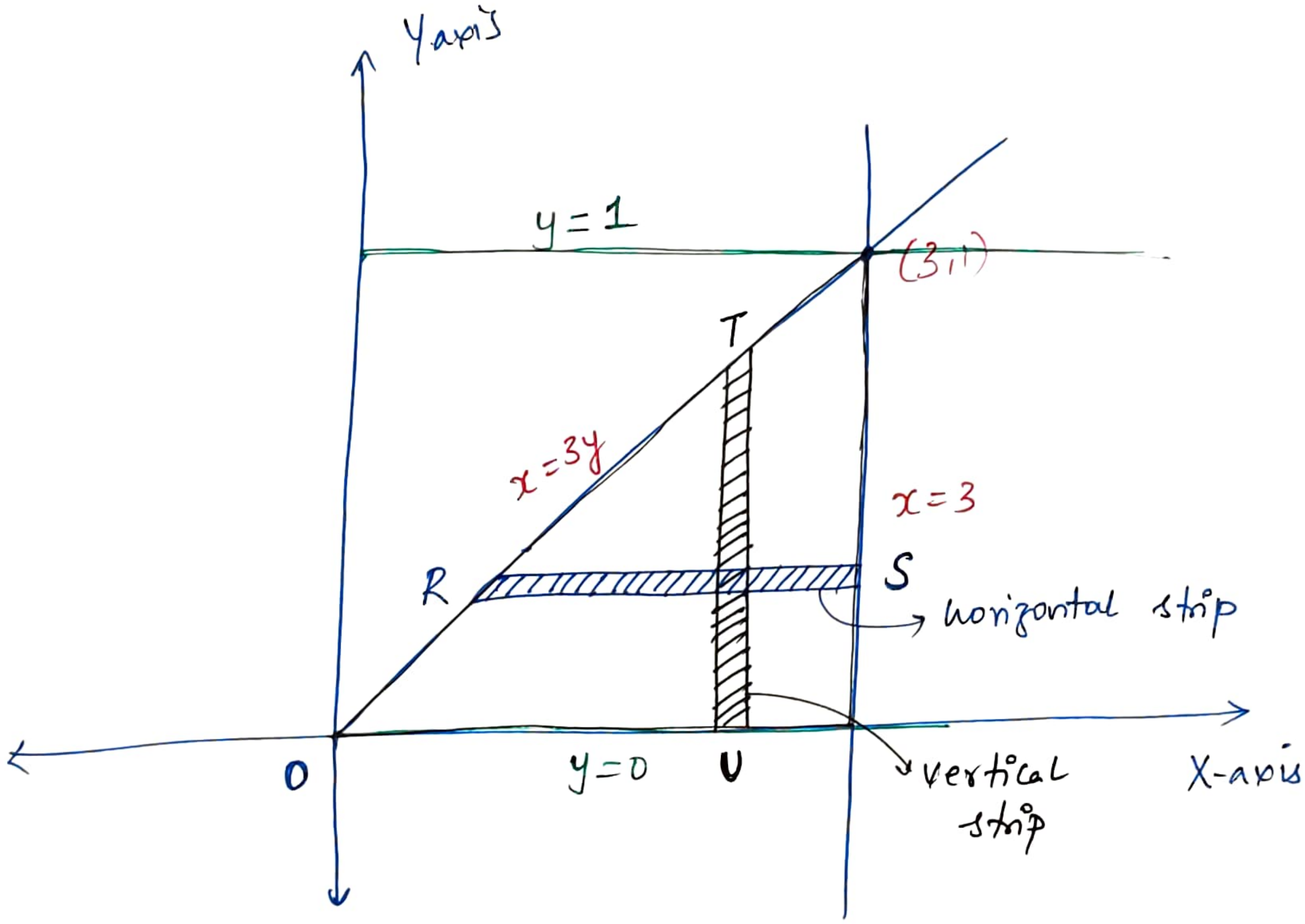
Now $I = \int_{0}^{1} \int_{3y}^{3} e^{x^2} \, dx \, dy$.

$y \uparrow \quad \uparrow x$

soln: Tracing the limits:

① $x = 3y$ (line), $x = 3$ (line ||le to y axis)

② $y = 0$ (x-axis), $y = 1$ (line ||le to x-axis)



here from the question we have to take the horizontal strip.

Always the horizontal strip ~~moves~~ limits ~~on~~ are $x = k_1$ & $x = k_2$

here the left hand limit of horizontal strip is R'
and the right hand limit of strip is S.

here $R : x = 3y$

$S : x = 3.$

Now the horizontal strip must be moved from $y = 0$ to $y = 1$ in the vertical direction.

Now change of order of the integration must be applied.

Change of order of integration implies to change the horizontal strip to the vertical strip.

Now the vertical strip is denoted with the black ink as shown in fig. In vertical strip the lower & upper limits are U & T respectively

$U : y = 0$

$T : y = \frac{x}{3} \quad (\because x = 3y)$

Now the vertical strip must be moved from the $x = 0$ (Left hand limit) to $x = 3$ (right hand limit)

$x = 0$
to
$x = 3$

By applying the change of order of integration.

we get

$$I = \int_{x=0}^{x=3} \int_{y=0}^{y=x/3} e^{x^2} \, dy \, dx$$ change of order.

$$I = \int_{x=0}^{x=3} e^{x^2} \left(\int_{y=0}^{y=x/3} dy \right) dx.$$

$$I = \int_{x=0}^{3} e^{x^2} \times \frac{x}{3} \, dx.$$

$$I = \frac{1}{3} \int_{x=0}^{3} e^{x^2} x \, dx$$

$$I = \frac{1}{6} \int_{x=0}^{3} e^{x^2} (2x \, dx)$$

$t = x^2$; $x = 0$; $t = 0$

$dt = 2x \, dx$; $x = 3$; $t = 9$

$$I = \frac{1}{6} \int_{t=0}^{9} e^{t}\, dt$$

$$I = \frac{1}{6} \left(e^{t}\right)_0^9 = \frac{1}{6}\left[e^9 - e^0\right] = \frac{1}{6}\left[e^9 - 1\right].$$

$$\boxed{\therefore\ I = \int_0^1 \int_{3y}^{3} e^{x^2}\, dx\, dy = \frac{1}{6}\left[e^9 - 1\right]}$$

$\boxed{\text{Ans: (A)}}$

WORKBOOK QUESTIONS: (OBJECTIVE TYPE QUESTIONS):

Q12) The double integral $\int_0^a \int_0^y f(x,y)\, dx\, dy$ is equivalent to

A) $\int_0^x \int_0^y f(x,y)\, dx\, dy$

B) $\int_0^a \int_x^y f(x,y)\, dx\, dy$

c) $\int_0^a \int_x^a f(x,y)\, dx\, dy$

D) $\int_0^a \int_0^a f(x,y)\, dx\, dy$

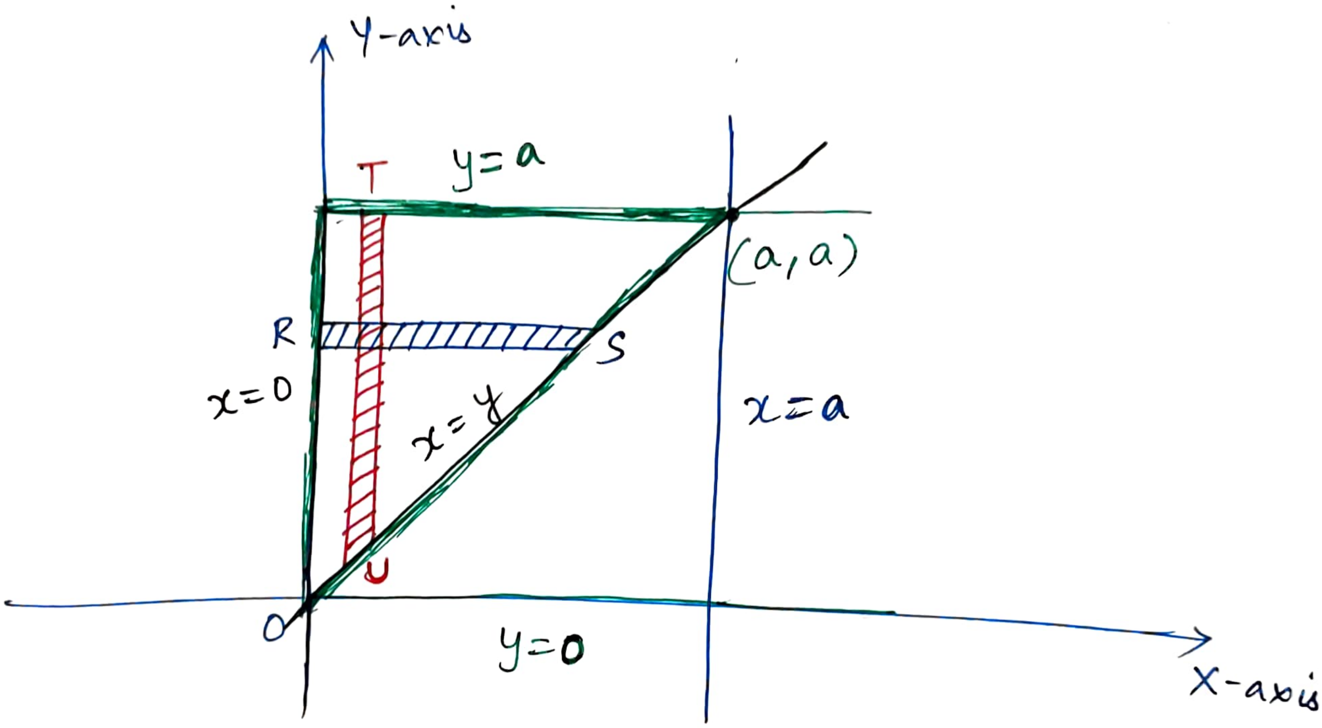
Soln:

$$I = \int_{y=0}^{a} \int_{x=0}^{y} f(x,y)\, dx\, dy.$$

Tracing the limits:

$x = 0$ to $x = y$

$y = 0$ to $y = a$



By changing the order of integration. Which implies changing horizontal strip to the vertical strip here in this problem.

for horizontal strip:

R : $x = 0$ to $x = y$: S (In horizontal direction)

$y = 0$ to $y = a$ (In vertical direction) for vertical strip:

In horizontal direction: $x = 0$ to $x = a$

In vertical direction: $y = x$ to $y = a$.

Now by applying change of order of Integration

$$\boxed{I = \int_{x=0}^{x=a} \int_{y=x}^{y=a} f(x,y)\, dy\, dx.}$$

Ans: (C)

Q13) Change the order of integral

V.GOOD Problem (Refer)

$$I = \int_0^1 \int_{x^2}^{2-x} f(x,y)\, dy\, dx$$

AWESOME PROBLEM ☺

A) $\int_0^2 \int_0^{\sqrt{y}} f(x,y)\, dx\, dy$

B) $\int_0^1 \int_0^{\sqrt{y}} f(x,y)\, dx\, dy + \int_1^2 \int_0^{2-y} f(x,y)\, dx\, dy$

C) $\int_0^1 \int_0^{2-y} f(x,y)\, dx\, dy$

D) $\int_1^2 \int_0^{2-y} f(x,y)\, dx\, dy.$